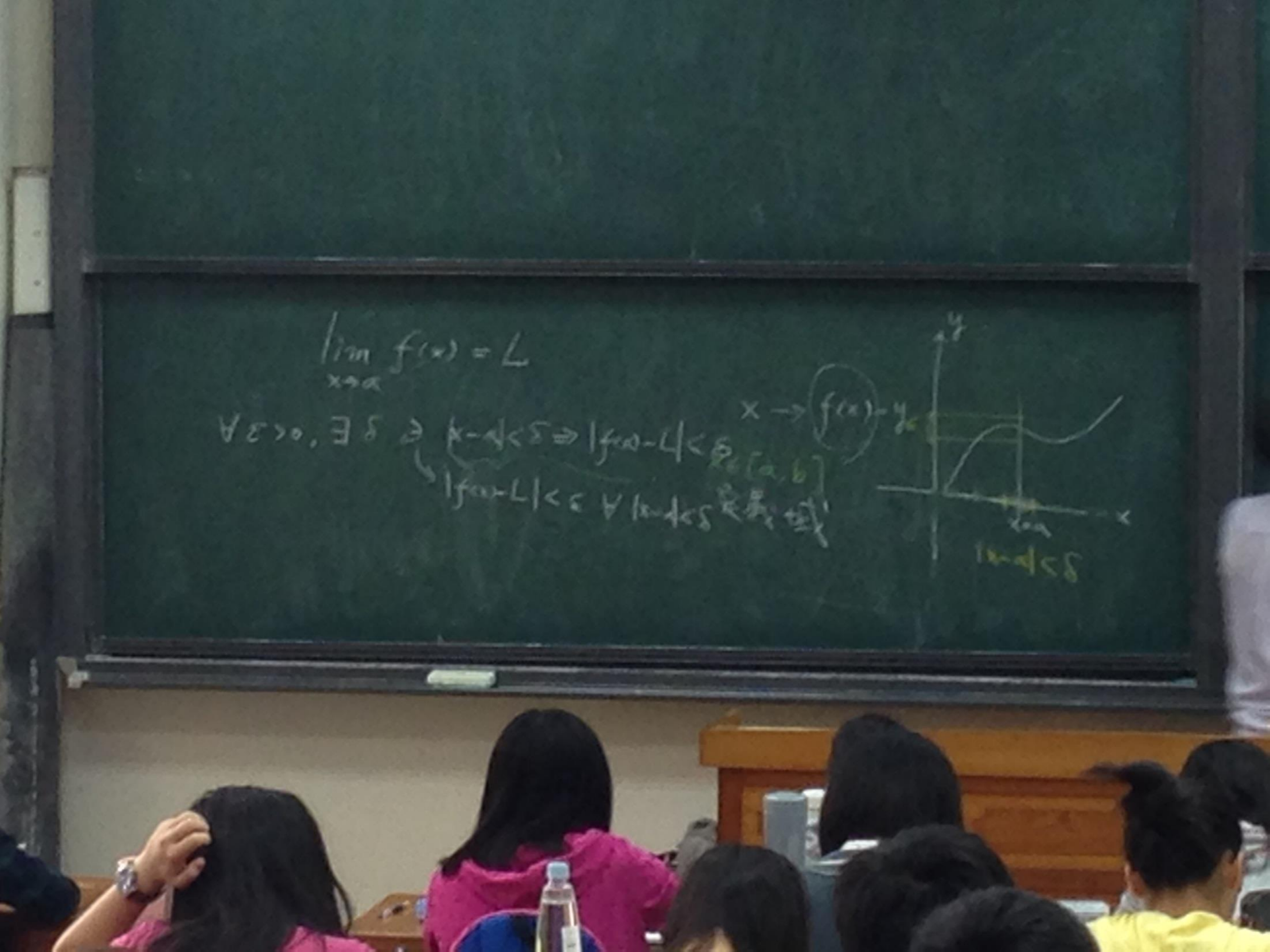
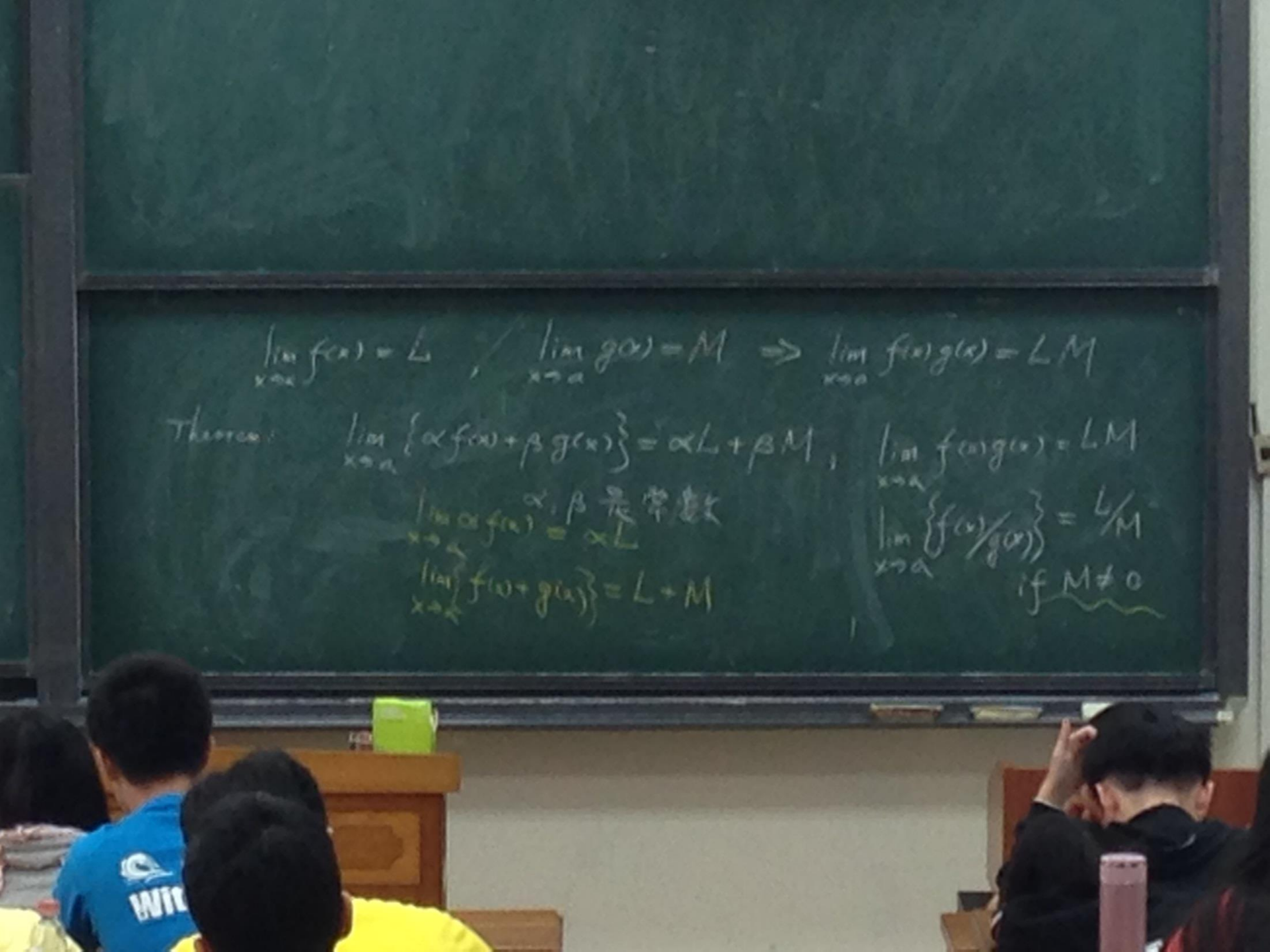
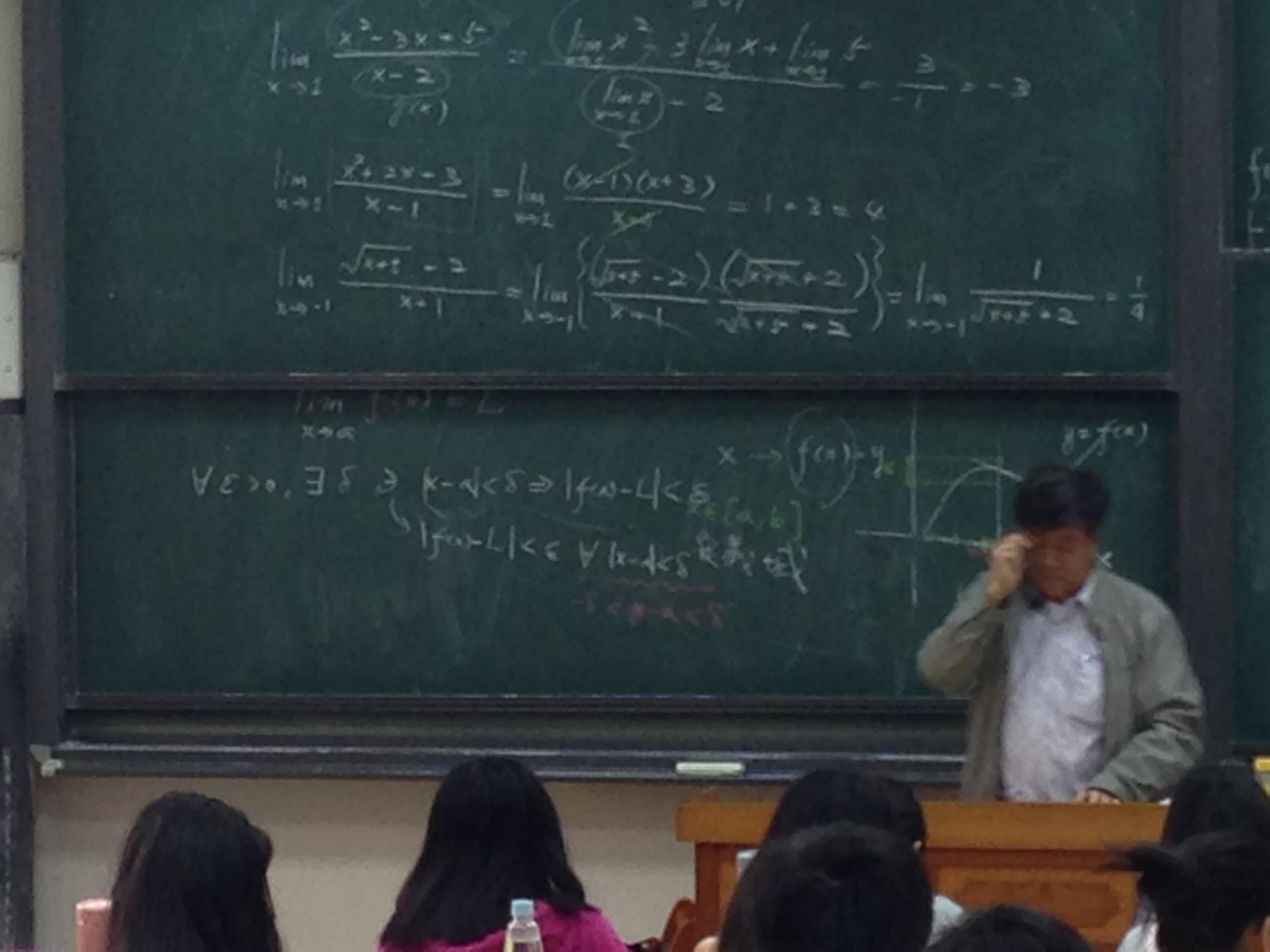


$\lim_{x \to a} f(x) = L$
$\forall \varepsilon > 0, \exists \delta \ni |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$
$|f(x) - L| < \varepsilon \ \forall |x-a| < \delta$
$x \to f(x) \to y$
$x \in [a, b]$
y
x
$|x-a| < \delta$


lim x→a f(x) = L , lim x→a g(x) = M ⇒ lim x→a f(x)g(x) = LM
Theorem: lim x→a {αf(x) + βg(x)} = αL + βM , lim x→a f(x)g(x) = LM
α, β 是常數
lim x→a αf(x) = αL
lim x→a {f(x) + g(x)} = L + M
lim x→a {f(x)/g(x)} = L/M
if M ≠ 0


lim x→1 (x² - 3x + 5)/(x - 2) = (lim x² - 3 lim x + lim 5)/(lim x - 2) = 3/-1 = -3
lim x→1 (x² + 2x - 3)/(x - 1) = lim x→1 (x-1)(x+3)/(x-1) = 1 + 3 = 4
lim x→-1 (√(x+5) - 2)/(x + 1) = lim x→-1 1/(√(x+5) + 2) = 1/4
∀ε>0, ∃δ ∋ |x-a|<δ ⇒ |f(x)-L|<ε
|f(x)-L|<ε ∀ |x-a|<δ
x → f(x) → y
y = f(x)

$\lim_{x\to a} f(x) = L$, $\lim_{x\to a} g(x) = M \Rightarrow \lim_{x\to a} f(x)g(x) = LM$

Theorem: $\lim_{x\to a}\{\alpha f(x) + \beta g(x)\} = \alpha L + \beta M$; $\lim_{x\to a} f(x)g(x) = LM$

α, β 是常數

$\lim_{x\to a} \alpha f(x) = \alpha L$

$\lim \{f(x) + g(x)\} = L + M$

$\lim_{x\to a}\{f(x)/g(x)\} = L/M$ if $M \neq 0$

$f(x) = \sqrt{4 - x^2}$

$-2 \le x \le 2$

$\lim_{x\to 0} \frac{x}{|x|}$

$$\lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x) = \frac{df(x)}{dx}$$

One-sided limit

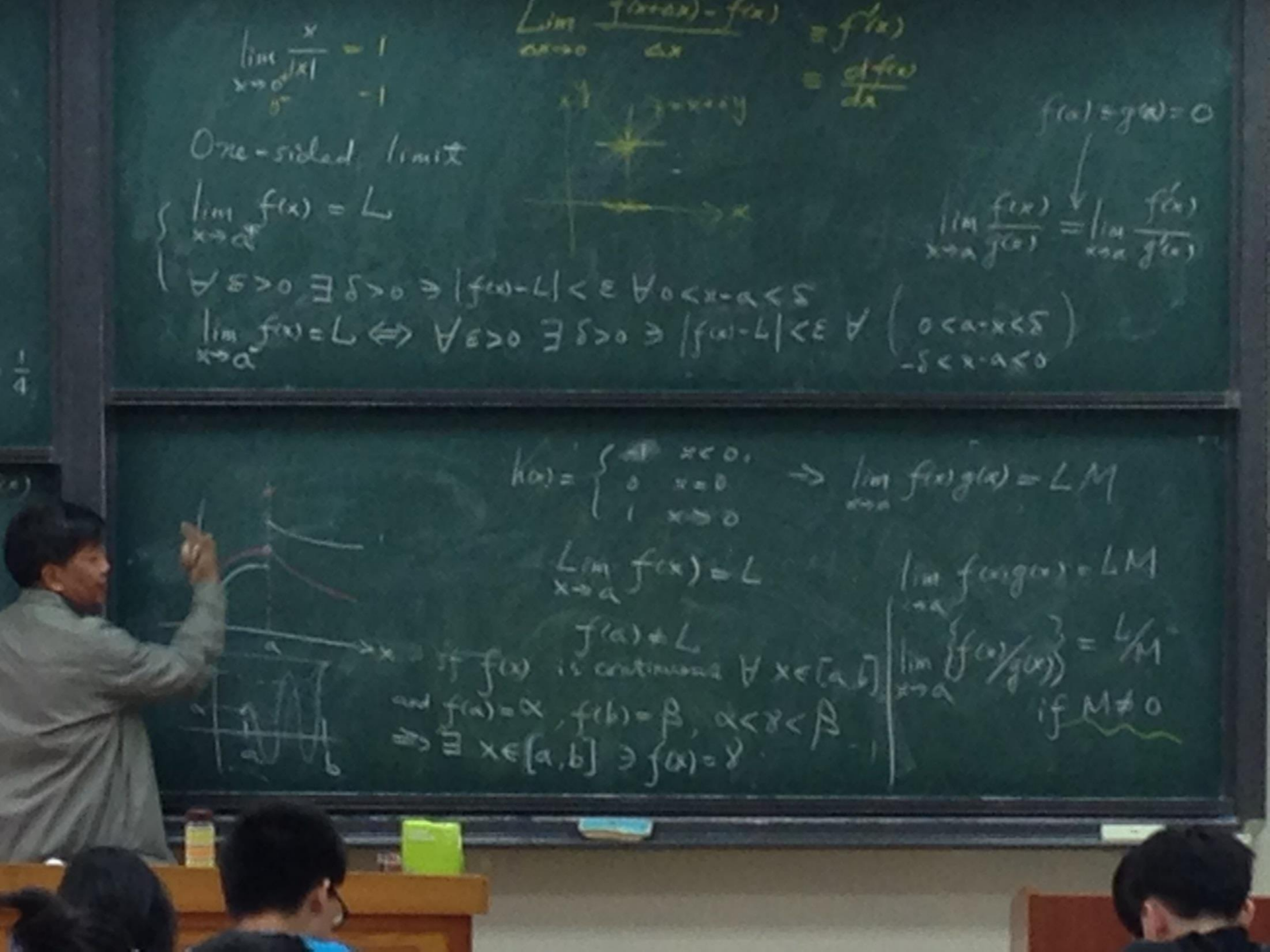
$$\begin{cases} \lim_{x\to a^+} f(x) = L \\ \forall \varepsilon > 0, \exists \delta > 0 \ni |f(x) - L| < \varepsilon \ \forall\, 0 < x - a < \delta \end{cases}$$

a^+

$$\lim_{x\to a^-} f(x) = L \Leftrightarrow \forall \varepsilon > 0 \ \exists \delta > 0 \ni |f(x) - L| < \varepsilon \ \forall \begin{pmatrix} 0 < a - x < \delta \\ -\delta < x - a < 0 \end{pmatrix}$$

$f(a) = g(a) = 0$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$


lim x→0+ x/|x| = 1
0− −1
Lim Δx→0 (f(x+Δx) − f(x))/Δx ≡ f′(x)
≡ df(x)/dx
f(a) = g(a) = 0
One-sided limit
lim x→a+ f(x) = L
∀ ε>0 ∃ δ>0 ∋ |f(x)−L| < ε ∀ 0<x−a<δ
lim x→a− f(x) = L ⟺ ∀ ε>0 ∃ δ>0 ∋ |f(x)−L| < ε ∀ (0<a−x<δ, −δ<x−a<0)
lim x→a f(x)/g(x) = lim x→a f′(x)/g′(x)
h(x) = { x<0, 0 x=0, 1 x>0
→ lim f(x)g(x) = LM
Lim x→a f(x) = L
f(a) ≠ L
if f(x) is continuous ∀ x∈[a,b]
and f(a)=α, f(b)=β, α<γ<β
⇒ ∃ x∈[a,b] ∋ f(x)=γ
lim x→a f(x)g(x) = LM
lim x→a {f(x)/g(x)} = L/M
if M≠0

$\lim_{x\to 1} \frac{\cdots}{(x-2)g(x)} = \cdots = \frac{\cdots}{\lim_{x\to 1} x - 2} = \frac{3}{-1} = -3$

$$\lim_{x\to 1}\frac{x^2+2x-3}{x-1} = \lim_{x\to 1}\frac{(x-1)(x+3)}{x-1} = 1+3 = 4$$

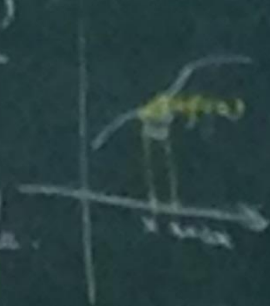
$$\lim_{x\to -1}\frac{\sqrt{x+5}-2}{x+1} = \lim_{x\to -1}\left\{\frac{(\sqrt{x+5}-2)}{x+1}\cdot\frac{(\sqrt{x+5}+2)}{\sqrt{x+5}+2}\right\} = \lim_{x\to -1}\frac{1}{\sqrt{x+5}+2} = \frac{1}{4}$$

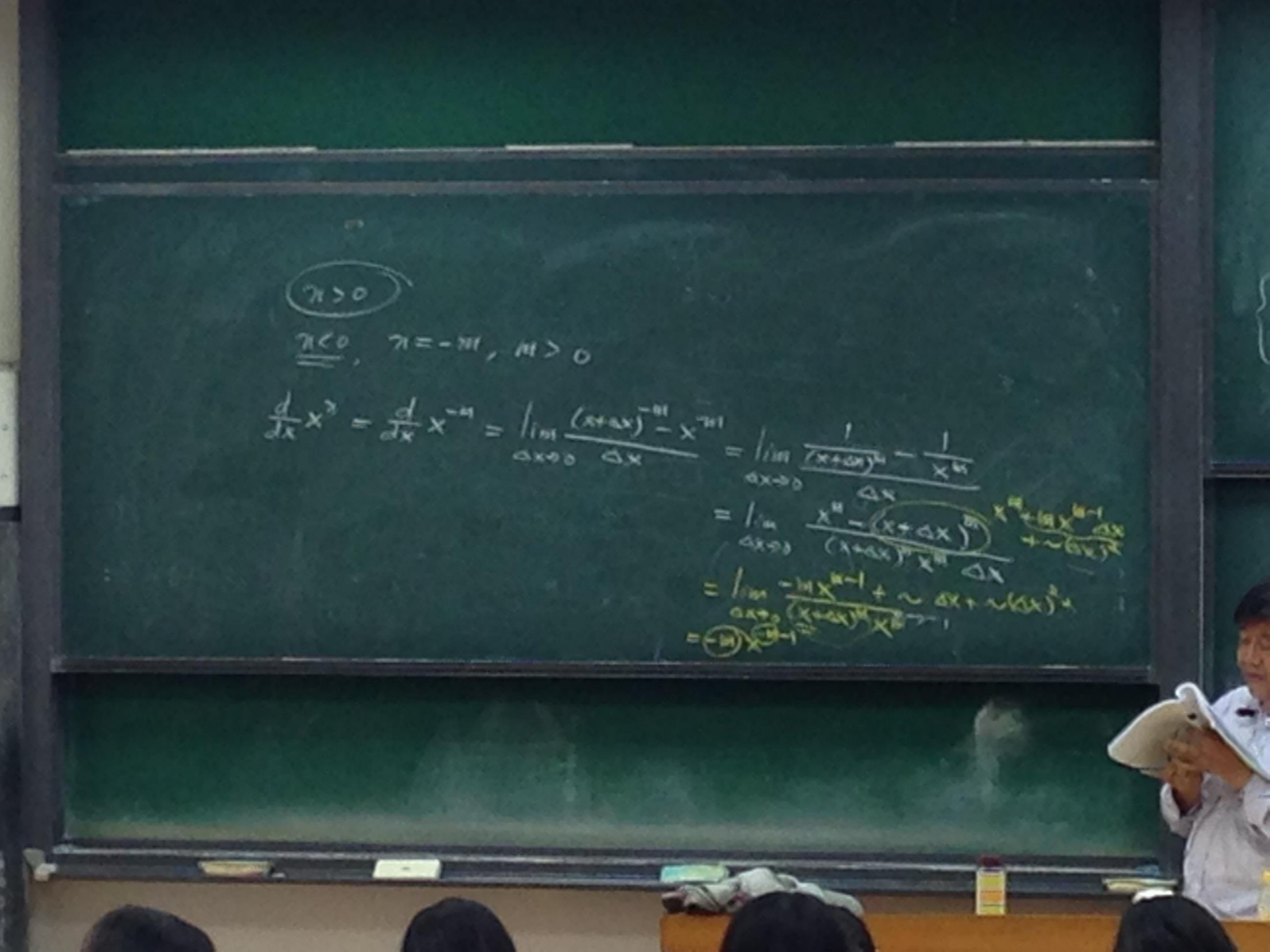
$$\begin{cases} g(x) = x^2+1 \\ f(x) = \sqrt{x} \end{cases}$$

$$f(g(x)) = \sqrt{x^2+1}$$

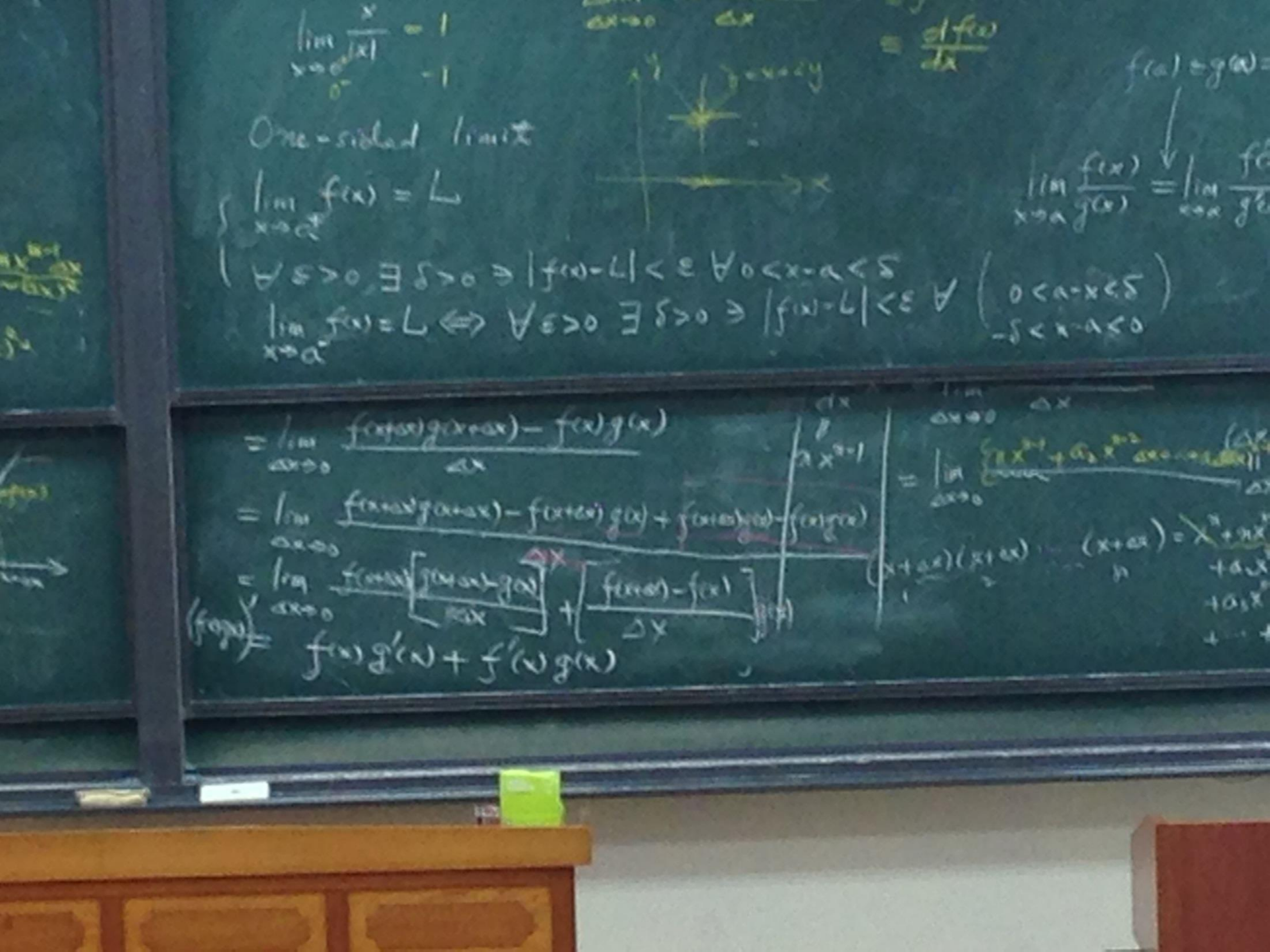
$$\frac{df(x)}{dx} = \lim_{\Delta x\to 0}\frac{\Delta f(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$\frac{df(g(x))}{dx} = \frac{df(g(x))}{dg(x)}\,\frac{dg(x)}{dx}$$ chain rule.





$n > 0$
$n < 0$, $n = -m$, $m > 0$
$\frac{d}{dx} x^n = \frac{d}{dx} x^{-m} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^{-m} - x^{-m}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)^m} - \frac{1}{x^m}}{\Delta x}$
$= \lim_{\Delta x \to 0} \frac{\frac{x^m - (x+\Delta x)^m}{(x+\Delta x)^m x^m}}{\Delta x}$
$x^m + m x^{m-1} \Delta x + \sim (\Delta x)^2$
$= \lim_{\Delta x \to 0} \frac{-m x^{m-1} + \sim \Delta x + \sim (\Delta x)^2 +}{(x+\Delta x)^m x^m}$
$= -m x^{-m-1}$



One-sided limit
lim f(x) = L
∀ε>0 ∃δ>0 ∋ |f(x)-L| < ε ∀ 0<x-a<δ
lim f(x) = L ⟺ ∀ε>0 ∃δ>0 ∋ |f(x)-L| < ε ∀ (0<a-x<δ, -δ<x-a<0)
= lim f(x+Δx)g(x+Δx) - f(x)g(x) / Δx
= lim f(x+Δx)g(x+Δx) - f(x+Δx)g(x) + f(x+Δx)g(x) - f(x)g(x) / Δx
= lim f(x+Δx)[g(x+Δx)-g(x) / Δx] + [f(x+Δx)-f(x) / Δx] g(x)
(f(x)g(x))' = f(x)g'(x) + f'(x)g(x)