1. Figure 1 shows a variable “air gap” capacitor for manual tuning. Alternate plates are connected together; one group of plates is fixed in position, and the other group is capable of rotation. Consider a capacitor of \( n = 8 \) plates of alternating polarity, each plate having area \( A = 1.25 \text{ cm}^2 \) and separated from adjacent plates by distance \( d = 3.40 \text{ mm} \). What is the maximum capacitance of the device?

![Figure 1](image)

**Sol**

For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of \((n-1)\) identical single capacitors connected in parallel. Each capacitor has surface area \( A \) and plate separation \( d \) so its capacitance is given by \( C_0 = \varepsilon_0 A/d \). Thus, the total capacitance of the combination is

\[
C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} = \frac{(8-1)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.25 \times 10^{-4} \text{ m}^2)}{3.40 \times 10^{-3} \text{ m}} = 2.72 \times 10^{-12} \text{ F}
\]

2. Figure 2 displays a 12.0 V battery and 3 uncharged capacitors of capacitances \( C_1 = 4.00 \mu\text{F} \), \( C_2 = 6.00 \mu\text{F} \), and \( C_3 = 3.00 \mu\text{F} \). The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?

![Figure 2](image)

**Sol**

The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

\[
\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.
\]

Thus, \( C_{\text{eq}} = C_2 C_3/(C_2 + C_3) \). The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination, and the potential difference across the equivalent capacitor is given by \( q_2/C_{\text{eq}} \). The potential difference across capacitor 1 is \( q_1/C_1 \), where \( q_1 \) is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so \( q_1/C_1 = q_2/C_{\text{eq}} \). Now some of
the charge originally on capacitor 1 flows to the combination of 2 and 3. If \( q_0 \) is the original charge, conservation of charge yields \( q_1 + q_2 = q_0 = C_1 V_0 \), where \( V_0 \) is the original potential difference across capacitor 1.

(a) Solving the two equations

\[
\frac{q_1}{C_1} = \frac{q_2}{C_{eq}}
\]

\[q_1 + q_2 = C_1 V_0\]

for \( q_1 \) and \( q_2 \), we obtain

\[q_1 = \frac{C_2 V_0}{C_{eq} + C_1} = \frac{C_2 C_3 V_0}{C_2 C_3 + C_1} = \frac{C_1 C_2 C_3 (C_2 + C_3)V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3}.
\]

With \( V_0 = 16.0 \text{ V}, C_1 = 4.00 \mu\text{F}, C_2 = 6.00 \mu\text{F} \) and \( C_3 = 3.00 \mu\text{F} \), we find \( C_{eq} = 2.00 \mu\text{F} \) and \( q_1 = 42.7 \mu\text{C} \).

(b) The charge on capacitors 2 is

\[q_2 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(16.0 \text{ V}) - 42.7 \mu\text{C} = 21.3 \mu\text{C}.
\]

(c) The charge on capacitor 3 is the same as that on capacitor 2:

\[q_3 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(16.0 \text{ V}) - 42.7 \mu\text{C} = 21.3 \mu\text{C}.
\]

3. The parallel plates in a capacitor, with a plate area of 8.50 cm\(^2\) and an air-filled separation of 3.00 mm, are charged by a 6.00 V battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of 8.00 mm. Neglecting fringing, find (a) the potential difference between the plates, (b) the initial stored energy, (c) the final stored energy, and (d) the work required to separate the plates.

Sol
(a) Let \( q \) be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by \( \varepsilon_0 A / d_i \), the charge is \( q = CV = \varepsilon_0 A V_f / d_i \). After the plates are pulled apart, their separation is \( d_f \) and the potential difference is \( V_f \). Then \( q = \varepsilon_0 A V_f / d_f \) and

\[V_f = \frac{d_f}{\varepsilon_0 A} q = \frac{d_f}{\varepsilon_0 A} \frac{\varepsilon_0 A}{d_i} V_i = \frac{d_f}{d_i} V_i.
\]

With \( d_i = 3.00 \times 10^{-3} \text{ m}, V_i = 6.00 \text{ V}, \) and \( d_f = 8.00 \times 10^{-3} \text{ m}, \) we have \( V_f = 16.0 \text{ V} \).

(b) The initial energy stored in the capacitor is
\[ U_i = \frac{1}{2} CV_i^2 = \frac{\varepsilon_0 AV_i^2}{2d_i} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(8.50 \times 10^{-4} \text{ m}^2)(6.00 \text{ V})^2}{2(3.00 \times 10^{-3} \text{ m})} = 4.51 \times 10^{-11} \text{ J}. \]

(c) The final energy stored is

\[ U_f = \frac{1}{2} \varepsilon_0 AV_f^2 = \frac{1}{2} \varepsilon_0 A \left( \frac{d_f}{d_i} V_i \right)^2 = \frac{d_f}{d_i} \left( \frac{\varepsilon_0 AV_i^2}{d_i} \right) = \frac{d_f}{d_i} U_i. \]

With \( d_f / d_i = 8.00 / 3.00 \), we have \( U_f = 1.20 \times 10^{-10} \text{ J} \).

(d) The work done to pull the plates apart is the difference in the energy:

\[ W = U_f - U_i = 7.52 \times 10^{-11} \text{ J}. \]

4. For the arrangement of Fig. 3, suppose that the battery remains connected while the dielectric slab is being introduced. Calculate (a) the capacitance, (b) the charge on the capacitor plates, (c) the electric field in the gap, and (d) the electric field in the slab, after the slab is in place.

![Fig. 3](image)

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(a) The electric field \( E_1 \) in the free space between the two plates is \( E_1 = q / \varepsilon_0 A \) while that inside the slab is \( E_2 = E_1 / \kappa = q / \kappa \varepsilon_0 A \). Thus,

\[ V_0 = E_1 (d - b) \quad E_2 b = \frac{q}{\varepsilon_0 A} (d - b) \frac{b}{\kappa} \]

and the capacitance is

\[ C = \frac{q}{V_0} = \frac{\varepsilon_0 A \kappa}{\kappa (d - b) + b} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(115 \times 10^{-4} \text{ m}^2)(2.61)}{(2.61)(0.0124 \text{ m} - 0.00780 \text{ m}) + (0.00780 \text{ m})} = 13.4 \text{ pF}. \]

(b) \( q = CV = (13.4 \times 10^{-12} \text{ F})(85.5 \text{ V}) = 1.15 \text{ nC}. \)

(c) The magnitude of the electric field in the gap is

\[ E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15 \times 10^{-9} \text{ C}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(115 \times 10^{-4} \text{ m}^2)} = 1.13 \times 10^4 \text{ N/C}. \]
Using Eq. 25-34, we obtain

\[ E_x = \frac{E_1}{\kappa} = \frac{1.13 \times 10^4 \text{ N/C}}{2.61} = 4.33 \times 10^3 \text{ N/C}. \]

5. A slab of copper of thickness \( b = 2.00 \text{ mm} \) is thrust into a parallel-plate capacitor of plate area \( A = 2.40 \text{ cm}^2 \) and plate separation \( d = 5.00 \text{ mm} \), as shown in Fig. 4; the slab is exactly halfway between the plates. (a) What is the capacitance after the slab is introduced? (b) If a charge \( q = 3.40 \mu \text{C} \) is maintained on the plates, what is the ratio of the stored energy before to that after the slab is inserted? (c) How much work is done on the slab as it is inserted? (d) Is the slab sucked in or must it be pushed in?

![Fig. 4](image)

Sol

(a) The length \( d \) is effectively shortened by \( b \) so \( C' = \varepsilon_0 A/(d - b) = 1.06 \text{ pF} \).

(b) The energy before, divided by the energy after inserting the slab is

\[ \frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d - b)}{\varepsilon_0 A/d} = \frac{d}{d - b} = \frac{5.00}{5.00 - 3.00} = 2.50. \]

(c) The work done is

\[ W = \Delta U = U' - U = \frac{q^2}{2} \left( \frac{1}{C'} - \frac{1}{C} \right) = \frac{q^2}{2\varepsilon_0 A} (d - b - d) = -\frac{q^2 b}{2\varepsilon_0 A} = -8.16 \text{ J}. \]

(d) Since \( W < 0 \), the slab is sucked in.