Home Work 16

16-1 A thin film of acetone ($n = 1.25$) coats a thick glass plate ($n = 1.50$). White light is incident normal to the film. In the reflections, fully destructive interference occurs at $600 \text{ nm}$ and fully constructive interference at $700 \text{ nm}$. Calculate the thickness of the acetone film.

Sol: The situation is analogous to that treated in Sample Problem 35-6, in the sense that the incident light is in a low index medium, the thin film of acetone has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. This is the same as Eq. 35-36 which was developed for the opposite situation (constructive interference) regarding a thin film surrounded on both sides by air (a very different context than the one in this problem). By analogy, we expect Eq. 35-37 to apply in this problem to reflection maxima. A more careful analysis such as that given in §35-7 bears this out. Thus, using Eq. 35-37 with $n_2 = 1.25$ and $\lambda = 700 \text{ nm}$ yields

$$L = 0, 280 \text{ nm}, 560 \text{ nm}, 840 \text{ nm}, 1120 \text{ nm}, \ldots$$

for the first several $m$ values. And the equation shown above (equivalent to Eq. 35-36) gives, with $\lambda = 600 \text{ nm}$,

$$L = 120 \text{ nm}, 360 \text{ nm}, 600 \text{ nm}, 840 \text{ nm}, 1080 \text{ nm}, \ldots$$

for the first several $m$ values. The lowest number these lists have in common is $L = 840 \text{ nm}$.

16-2 A thin flake of mica ($n = 1.58$) is used to cover one slit of a double-slit interference arrangement. The central point on the viewing screen is now occupied by what had been the seventh bright side fringe ($m = 7$). If $\lambda = 550 \text{ nm}$, what is the thickness of the mica?

Sol: Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness $x$ is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$2\varphi_m - 2\varphi = 2\varphi - \frac{1}{2}\pi$$

where $\lambda_m$ is the wavelength in the mica and $n$ is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for $\lambda_m$. Since the waves are now in phase at the screen,

$$2\frac{\varphi}{\varphi} = 14\frac{\varphi}{\varphi}$$

or
16-3 Figure 2 shows an optical fiber in which a central plastic core of index of refraction \( n_1 = 1.58 \) is surrounded by a plastic sheath of index of refraction \( n_2 = 1.53 \). Light can travel along different paths within the central core, leading to different travel times through the fiber. This causes an initially short pulse of light to spread as it travels along the fiber, resulting in information loss. Consider light that travels directly along the central axis of the fiber and light that is repeatedly reflected at the critical angle along the core-sheath interface, reflecting from side to side as it travels down the central core. If the fiber length is 300 m, what is the difference in the travel times along these two routes?

**Sol** A light ray traveling directly along the central axis reaches the end in time 

\[
t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c}.
\]

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is \( v_1 \cos \theta' \), so the time of travel for this ray is 

\[
t_{\text{zig zag}} = \frac{L}{v_1 \cos \theta'} = \frac{n_1 L}{c \sqrt{1 - (\sin \theta / n_1)^2}}
\]

using results from the previous solution. Plugging in \( \sin \theta = \sqrt{n_1^2 - n_2^2} \) and simplifying, we obtain 

\[
t_{\text{zig zag}} = \frac{n_1 L}{c b_2 / n_1} = \frac{n_1^2 L}{n_2 c^2}.
\]

The difference is 

\[
\Delta t = t_{\text{zig zag}} - t_{\text{direct}} = \frac{n_1^2 L}{n_2 c} - \frac{n_1 L}{c} = \frac{n_1 L}{c} \left( \frac{n_1^2}{n_2} - 1 \right).
\]

With \( n_1 = 1.58, n_2 = 1.53, \) and \( L = 300 \text{ m} \), we obtain

\[
\Delta t = \frac{1.58 \cdot 300}{1.53} - \frac{1.58}{1} \left( \frac{1.58^2}{1.53} - 1 \right)
\]

\[
\Delta t = 736.5 - 1.58 \left( \frac{2.4964}{1.53} - 1 \right)
\]

\[
\Delta t = 736.5 - 1.58 \left( 1.6274 - 1 \right)
\]

\[
\Delta t = 736.5 - 1.58 \cdot 0.6274
\]

\[
\Delta t = 736.5 - 0.9991
\]

\[
\Delta t = 735.5009 \text{ s}
\]
\[ \Delta t = \frac{n_1 L}{c} \left( \frac{n_1}{n_2} - 1 \right) = \frac{(1.58)(300 \text{ m})}{3.0 \times 10^5 \text{ m/s}} \left( \frac{1.58}{1.53} - 1 \right) = 5.16 \times 10^{-8} \text{ s} = 51.6 \text{ ns} . \]

16-4 In Figure 2, an airtight chamber of length \( d = 5.0 \text{ cm} \) is placed in one of the arms of a Michelson interferometer. (The glass window on each end of the chamber has negligible thickness.) Light of wavelength \( \lambda = 500 \text{ nm} \) is used. Evacuating the air from the chamber causes a shift of 60 bright fringes. From these data and to six significant figures, find the index of refraction of air at atmospheric pressure.

**Sol:** Let \( \phi_1 \) be the phase difference of the waves in the two arms when the tube has air in it, and let \( \phi_2 \) be the phase difference when the tube is evacuated. These are different because the wavelength in air is different from the wavelength in vacuum. If \( \lambda \) is the wavelength in vacuum, then the wavelength in air is \( \lambda/n \), where \( n \) is the index of refraction of air. This means
\[ \phi_1 - \phi_2 = 2L \left[ \frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda} \right] = \frac{4\pi (n-1)L}{\lambda} \]

where \( L \) is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror. Each shift by one fringe corresponds to a change in phase of \( 2\pi \text{ rad} \), so if the interference pattern shifts by \( N \) fringes as the tube is evacuated,
\[ \frac{4\pi (n-1)}{\lambda} = 2N\pi \]

and
\[ n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60 \times 300 \times 10^{-9} \text{ m}}{2 \times 30 \times 10^{-3} \text{ m}} = 1.00030 . \]

16-5 Figure 3a shows a lens with radius of curvature \( R \) lying on a flat glass plate and illuminated from above by light with wavelength \( \lambda \). Figure 3b (a photograph taken from above the lens) shows that circular interference fringes (called Newton’s rings) appear, associated with the variable thickness \( d \) of the air film between the lens and the plate. Find the radii \( r \) of the interference maxima assuming \( r/R, < < 1 \).

**Sol:** Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a \( \pi \text{ rad} \) phase change while the wave reflected from the upper surface does not. At a place where the
thickness of the wedge is \(d\), the condition for a maximum in intensity is
\[
2d = b + \frac{1}{2}g
\]
where \(\lambda\) is the wavelength in air and \(m\) is an integer. Therefore,
\[
d = (2m + 1)\lambda/4.
\]
As the geometry of Fig. 35-47 shows, \(d = R - \sqrt{R^2 - r^2}\), where \(R\) is the radius of curvature of the lens and \(r\) is the radius of a Newton’s ring. Thus, \(b + 1\frac{g}{h} = R - \sqrt{R^2 - r^2}\). First, we rearrange the terms so the equation becomes
\[
\sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4}.
\]
Next, we square both sides, rearrange to solve for \(r^2\), then take the square root. We get
\[
r = \sqrt{\frac{b + 1\frac{g}{h}}{2} - \frac{b + 1\frac{g}{h}}{16}}.
\]
If \(R\) is much larger than a wavelength, the first term dominates the second and
\[
r = \sqrt{\frac{b + 1\frac{g}{h}}{2}}.
\]
16-6 A Newton’s rings apparatus is to be used to determine the radius of curvature of a lens (see Figure 3 and Problem 75). The radii of the \(n\)th and \((n + 20)\)th bright rings are measured and found to be 0.162 and 0.368 cm, respectively, in light of wavelength 546 nm. Calculate the radius of curvature of the lower surface of the lens.

**Sol:** We solve for \(m\) using the formula \(r = \sqrt{b + 1\frac{g}{h}}\) obtained in Problem 35-75 and find
\[
m = r^2/R\lambda - 1/2.
\]
Now, when \(m\) is changed to \(m + 20\), \(r\) becomes \(r’\), so
\[
m + 20 = r’^2/R\lambda - 1/2.
\]
Taking the difference between the two equations above, we eliminate \(m\) and find
\[
R = \frac{r^2 - r’^2}{20} = \frac{0.368 - 0.162}{20} = \frac{0.162 - 10^{-7}}{20} \approx 100\text{\ cm}.
\]