Home Work 15

15-1 Two-lens systems. In the Figure below, stick figure O (the object) stands on the common central axis of two thin, symmetric lenses, which are mounted in the boxed regions. Lens 1 is mounted within the boxed region closer to O, which is at object distance \( p_1 \). Lens 2 is mounted within the farther boxed region, at distance \( d \). Each problem in the Table refers to a different combination of lenses and different values for distances, which are given in centimeters. The type of lens is indicated by C for converging and D for diverging; the number after C or D is the distance between a lens and either of its focal points (the proper sign of the focal distance is not indicated).

<table>
<thead>
<tr>
<th></th>
<th>( p_1 )</th>
<th>Lens 1</th>
<th>( d )</th>
<th>Lens 2</th>
<th>( i_2 )</th>
<th>( M )</th>
<th>R/V</th>
<th>I/NI</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>83</td>
<td>+20</td>
<td>C, 9.0</td>
<td>8.0</td>
<td>C, 5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>84</td>
<td>+15</td>
<td>C, 12</td>
<td>67</td>
<td>C, 10</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>85</td>
<td>+4.0</td>
<td>C, 6.0</td>
<td>8.0</td>
<td>D, 6.0</td>
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<td></td>
</tr>
<tr>
<td>86</td>
<td>+12</td>
<td>C, 8.0</td>
<td>30</td>
<td>D, 8.0</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>87</td>
<td>+20</td>
<td>D, 12</td>
<td>10</td>
<td>D, 8.0</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Sol

83. To analyze two-lens systems, we first ignore lens 2, and apply the standard procedure used for a single-lens system. The object distance \( p_1 \), the image distance \( i_1 \), and the focal length \( f_1 \) are related by:

\[
\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{i_1}.
\]

Next, we ignore the lens 1 but treat the image formed by lens 1 as the object for lens 2. The object distance \( p_2 \) is the distance between lens 2 and the location of the first image. The location of the final image, \( i_2 \), is obtained by solving

\[
\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{i_2}
\]

where \( f_2 \) is the focal length of lens 2.

(a) Since lens 1 is converging, \( f_1 = +9 \text{ cm} \), and we find the image distance to be

\[
i_1 = \frac{p_1f_1}{p_1 - f_1} = \frac{(20 \text{ cm})(9 \text{ cm})}{20 \text{ cm} - 9 \text{ cm}} = 16.4 \text{ cm}.
\]

This serves as an “object” for lens 2 (which has \( f_2 = +5 \text{ cm} \)) with an object distance given by \( p_2 = d - i_1 = -8.4 \text{ cm} \). The negative sign means that the “object” is behind lens 2. Solving the lens
equation, we obtain

\[ i_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-8.4 \text{ cm})(5.0 \text{ cm})}{-8.4 \text{ cm} - 5.0 \text{ cm}} = 3.13 \text{ cm}. \]

(b) The overall magnification is

\[ M = m_1 m_2 = \frac{(-i_1 / p_1)(-i_2 / p_2)}{p_1, p_2} = -0.31. \]

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

Since this result involves a negative value for \(p_2\) (and perhaps other “non-intuitive” features), we offer a few words of explanation: lens 1 is converging the rays toward an image (that never gets a chance to form due to the intervening presence of lens 2) that would be real and inverted (and 8.4 cm beyond lens 2’s location). Lens 2, in a sense, just causes these rays to converge a little more rapidly, and causes the image to form a little closer (to the lens system) than if lens 2 were not present.

84. (a) The image from lens 1 (which has \( f_1 = +12 \text{ cm}\)) is at \( i_1 = +36 \text{ cm}\) (by Eq. 34-9). This serves as an “object” for lens 2 (which has \( f_2 = +10 \text{ cm}\)) with \( p_2 = d - i_1 = 31 \text{ cm}\). Then Eq. 34-9 (applied to lens 2) yields \( i_2 = 15 \text{ cm}\).

(b) Equation 34-11 yields

\[ M = m_1 m_2 = \frac{(-i_1 / p_1)(-i_2 / p_2)}{p_1, p_2} = +0.97. \]

(c) The fact that the (final) image distance is positive means the image is virtual (R).

(d) The fact that the magnification is a positive value means the image is non inverted (NI).

(e) The image is on the opposite side as the object (relative to lens 2).

85. (a) The image from lens 1 (which has \( f_1 = +6 \text{ cm}\)) is at \( i_1 = -12 \text{ cm}\) (by Eq. 34-9). This serves as an “object” for lens 2 (which has \( f_2 = -6 \text{ cm}\)) with \( p_2 = d - i_1 = 20 \text{ cm}\). Then Eq. 34-9 (applied to lens 2) yields \( i_2 = -4.6 \text{ cm}\).

(b) Equation 34-11 yields \( M = +0.69 \).

(c) The fact that the (final) image distance is negative means the image is virtual (V).
(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

86. (a) The image from lens 1 (which has \( f_1 = +8 \text{ cm} \)) is at \( i_1 = +40 \text{ cm} \) (by Eq. 34-9). This serves as an “object” for lens 2 (which has \( f_2 = -8 \text{ cm} \)) with \( p_2 = d - i_1 = -10 \text{ cm} \). Then Eq. 34-9 (applied to lens 2) yields \( i_2 = -40 \text{ cm} \).

(b) Equation 34-11 yields \( M = +16 \).

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a positive value means the image is not inverted (NI).

(e) The image is on the opposite side as the object (relative to lens 2).

87. (a) The image from lens 1 (which has \( f_1 = -12 \text{ cm} \)) is at \( i_1 = -7.5 \text{ cm} \) (by Eq. 34-9). This serves as an “object” for lens 2 (which has \( f_2 = -8 \text{ cm} \)) with

\[
p_2 = d - i_1 = 17.5 \text{ cm}.
\]

Then Eq. 34-9 (applied to lens 2) yields \( i_2 = -5.5 \text{ cm} \).

(b) Equation 34-11 yields \( M = +0.12 \).

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

15-2 If the angular magnification of an astronomical telescope is 36 and the diameter of the objective is 75 mm, what is the minimum diameter of the eyepiece required to collect all the light entering the objective from a distant point source on the telescope axis?

**Sol**

The minimum diameter of the eyepiece is given by
In a compound microscope, the focal length of the objective is 4.00 cm, and that of the eyepiece is 8.00 cm. The distance between the lenses is 25.0 cm. (a) What is the tube length \( s \)? (b) If image \( I \) is to be just inside focal point, how far from the objective should the object be? What then are (c) the lateral magnification \( m \) of the objective, (d) the angular magnification \( m_\theta \) of the eyepiece, and (e) the overall magnification \( M \) of the microscope?

**Sol**

(a) If \( L \) is the distance between the lenses, then according to Fig. 34-20, the tube length is

\[
s = L - f_{ob} - f_{ey} = 25.0 \text{ cm} - 4.00 \text{ cm} - 8.00 \text{ cm} = 13.0 \text{ cm}.
\]

(b) We solve \( (1/p) + (1/i) = (1/f_{ob}) \) for \( p \). The image distance is

\[
i = f_{ob} + s = 4.00 \text{ cm} + 13.0 \text{ cm} = 17.0 \text{ cm},
\]

so

\[
p = \frac{if_{ob}}{i - f_{ob}} = \frac{(17.0 \text{ cm})(4.00 \text{ cm})}{17.0 \text{ cm} - 4.00 \text{ cm}} = 5.23 \text{ cm}.
\]

(c) The magnification of the objective is

\[
m = -\frac{i}{p} = -\frac{17.0 \text{ cm}}{5.23 \text{ cm}} = -3.25.
\]

(d) The angular magnification of the eyepiece is

\[
m_\theta = \frac{25 \text{ cm}}{f_{ey}} = \frac{25 \text{ cm}}{8.00 \text{ cm}} = 3.13.
\]

(e) The overall magnification of the microscope is

\[
M = mm_\theta = (-3.25)(3.13) = -10.2.
\]

An object is 10.0 mm from the objective of a certain compound microscope. The lenses are 300 mm apart, and the intermediate image is 50.0 mm from the eyepiece. What overall magnification is produced by the instrument?

**Sol**

We refer to Fig. 34-20. For the intermediate image, \( p = 12 \text{ mm} \) and
\[ i = (f_{ob} + s + f_{ey}) - f_{ey} = 300 \text{ m} - 50 \text{ mm} = 250 \text{ mm}, \]

so

\[
\frac{1}{f_{ob}} = \frac{1}{i} + \frac{1}{p} = \frac{1}{250 \text{ mm}} + \frac{1}{12 \text{ mm}} \Rightarrow f_{ob} = 11.5 \text{ mm},
\]

and

\[ s = (f_{ob} + s + f_{ey}) - f_{ob} - f_{ey} = 300 \text{ mm} - 11.5 \text{ mm} - 50 \text{ mm} = 239 \text{ mm}. \]

Then from Eq. 34-14,

\[ M = -\frac{s}{f_{ob}} \frac{25 \text{ cm}}{f_{ey}} = -\left( \frac{239 \text{ mm}}{11.5 \text{ mm}} \right) \left( \frac{250 \text{ mm}}{50 \text{ mm}} \right) = -104. \]

15-5 In the Figure below, an object is placed in front of a converging lens at a distance equal to twice the focal length \( f_1 \) of the lens. On the other side of the lens is a concave mirror of focal length \( f_2 \) separated from the lens by a distance \( 2(f_1 + f_2) \). Light from the object passes rightward through the lens, reflects from the mirror, passes leftward through the lens, and forms a final image of the object. What are (a) the distance between the lens and that final image and (b) the overall lateral magnification \( M \) of the object? Is the image (c) real or virtual (if it is virtual, it requires someone looking through the lens toward the mirror), (d) to the left or right of the lens, and (e) inverted or noninverted relative to the object?

![Diagram of optical system](image)

Sol

(a) First, the lens forms a real image of the object located at a distance

\[ i_1 = \frac{1}{f_1} \frac{1}{p_1} \Rightarrow i_1 = \frac{1}{2f_1} \Rightarrow p_1 = 2f_1 \]

\[ p_2 = 2(f_1 + f_2) - 2f_1 = 2f_2 \]

in front of the mirror. The subsequent image formed by the mirror is located at a distance
\[
i_2 = \frac{i_2}{p_2} \cdot \frac{1}{-k} = \frac{i_2}{p_2} \cdot \frac{1}{2f_2} = 2f_2
\]
to the left of the mirror, or at
\[
p' = 2(f_1 + f_2) - 2f_2 = 2f_1
\]
to the right of the lens. The final image formed by the lens is at a distance \( i'_1 \) to the left of the lens, where
\[
i'_1 = \frac{i'_1}{p'_1} = \frac{1}{2f_1} = 2f_1,
\]
This turns out to be the same as the location of the original object.

(b) The lateral magnification is
\[
m = \frac{i}{p} = \frac{i}{p} = \frac{2f_1}{2f_1} = \frac{2f_2}{2f_2} = \frac{2f_1}{2f_1} = -1.0.
\]

(c) The final image is real (R).

(d) It is at a distance \( i'_1 \) to the left of the lens,

(e) and inverted (I), as shown in the figure below.

15-6 Someone with a near point \( P_n \) of 25 cm views a thimble through a simple magnifying lens of focal length 10 cm by placing the lens near his eye. What is the angular magnification of the thimble if it is positioned so that its image appears at (a) \( P_n \) and (b) infinity?

Sol

(a) Without the magnifier, \( \theta = h/P_n \) (see Fig. 34-19). With the magnifier, letting
\[ i = -|i| = -P_n, \]
we obtain
\[ \frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}. \]

Consequently,
\[ m_o = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25 \text{ cm}}{f}. \]

With \( f = 10 \text{ cm}, \) \( m_o = 1 + \frac{25 \text{ cm}}{10 \text{ cm}} = 3.5. \)

(b) In the case where the image appears at infinity, let \( i = -|i| \to -\infty, \) so that
\[ 1/p + 1/i = 1/p = 1/f, \]
we have
\[ m_o = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25 \text{ cm}}{f}. \]

With \( f = 10 \text{ cm}, \)
\[ m_o = \frac{25 \text{ cm}}{10 \text{ cm}} = 2.5. \]