Home Work 14

14-1 More mirrors. Object $O$ stands on the central axis of a spherical or plane mirror. For this situation, each problem in the Table refers to (a) the type of mirror, (b) the focal distance $f$, (c) the radius of curvature $r$, (d) the object distance $p$, (e) the image distance $i$, and (f) the lateral magnification $m$. (All distances are in centimeters.) It also refers to whether (g) the image is real (R) or virtual (V), (h) inverted (I) or noninverted (NI) from $O$, and (i) on the same side of the mirror as object $O$ or on the opposite side. Fill in the missing information. Where only a sign is missing, answer with the sign.

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
<th>(i)</th>
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<tbody>
<tr>
<td></td>
<td>Type</td>
<td>$f$</td>
<td>$r$</td>
<td>$p$</td>
<td>$i$</td>
<td>$m$</td>
<td>R/V</td>
<td>I/NI</td>
<td>Side</td>
</tr>
<tr>
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<td>20</td>
<td>+10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td>+24</td>
<td>0.50</td>
<td>I</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>22</td>
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<tr>
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<td>29</td>
<td>Convex</td>
<td>40</td>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Sol

17. (a) The mirror is concave.

(b) $f = +20$ cm (positive, because the mirror is concave).

(c) $r = 2f = 2(+20$ cm$) = +40$ cm.

(d) The object distance $p = +10$ cm, as given in the table.

(e) The image distance is $i = (1/f - 1/p)^{-1} = (1/20$ cm$ - 1/10$ cm$)^{-1} = -20$ cm.

(f) $m = -ilp = -(-20$ cm$/10$ cm$) = +2.0$.

(g) The image is virtual (V).

(h) The image is upright or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.
18. (a) Since the image is inverted, we can scan Figs. 34-8, 34-10, and 34-11 in the textbook and find that the mirror must be concave.

(b) This also implies that we must put a minus sign in front of the “0.50” value given for \( m \). To solve for \( f \), we first find \( i = -pm = +10 \text{ cm} \) from Eq. 34-6 and plug into Eq. 34-4; the result is \( f = +6.7 \text{ cm} \).

(c) Thus, \( r = 2f = +13.4 \text{ cm} \).

(d) \( p = +20 \text{ cm} \), as given in the table.

(e) As shown above, \( i = -pm = +10 \text{ cm} \).

(f) \( m = -0.50 \), with a minus sign.

(g) The image is real (R), since \( i > 0 \).

(h) The image is inverted (I), as noted above.

(i) A real image is formed on the same side as the object.

22. (a) Since \( 0 < m < 1 \), the image is upright but smaller than the object. With that in mind, we examine the various possibilities in Figs. 34-8, 34-10, and 34-11, and note that such an image (for reflections from a single mirror) can only occur if the mirror is convex.

(b) Thus, we must put a minus sign in front of the “20” value given for \( f \), that is, \( f = -20 \text{ cm} \).

(c) Equation 34-3 then gives \( r = 2f = -40 \text{ cm} \).

(d) To solve for \( i \) and \( p \) we must set up Eq. 34-4 and Eq. 34-6 as a simultaneous set and solve for the two unknowns. The results are \( p = +180 \text{ cm} = +1.8 \text{ m} \), and

(e) \( i = -18 \text{ cm} \).

(f) \( m = 0.10 \), as given in the table.

(g) The image is virtual (V) since \( i < 0 \).

(h) The image is upright, or not inverted (NI), as already noted.
(i) A virtual image is formed on the opposite side of the mirror from the object.

23. (a) The magnification is given by \( m = -i/p \). Since \( p > 0 \), a positive value for \( m \) means that the image distance \( (i) \) is negative, implying a virtual image. Looking at the discussion of mirrors in Sections 34-3 and 34-4, we see that a positive magnification of magnitude less than unity is only possible for convex mirrors.

(b) With \( i = -mp \), we may write \( p = f(1-1/m) \). For \( 0 < m < 1 \), a positive value for \( p \) can be obtained only if \( f < 0 \). Thus, with a minus sign, we have \( f = -30 \) cm.

(c) The radius of curvature is \( r = 2f = -60 \) cm.

(d) The object distance is \( p = f(1 - 1/m) = (-30 \text{ cm})(1 -1/0.20) = +120 \text{ cm} = 1.2 \text{ m} \).

(e) The image distance is \( i = -mp = -(0.20)(120 \text{ cm}) = -24 \text{ cm} \).

(f) The magnification is \( m = +0.20 \), as given in the table.

(g) As discussed in (a), the image is virtual (V).

(h) As discussed in (a), the image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.

29. (a) The mirror is convex, as given.

(b) Since the mirror is convex, the radius of curvature is negative, so \( r = -40 \) cm. Then, the focal length is \( f = r/2 = (-40 \text{ cm})/2 = -20 \text{ cm} \).

(c) The radius of curvature is \( r = -40 \) cm.

(d) The fact that the mirror is convex also means that we need to insert a minus sign in front of the “4.0” value given for \( i \), since the image in this case must be virtual. Equation 34-4 leads to

\[
p = \frac{if}{i-f} = \frac{(-4.0 \text{ cm})(-20 \text{ cm})}{-4.0 \text{ cm} - (-20 \text{ cm})} = 5.0 \text{ cm}
\]
(e) As noted above, \( i = -4.0 \text{ cm} \).

(f) The magnification is \( m = \frac{-i}{p} = \frac{-(-4.0 \text{ cm})}{(5.0 \text{ cm})} = +0.80 \).

(g) The image is virtual (V) since \( i < 0 \).

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.

14-2 Spherical refracting surfaces. An object \( O \) stands on the central axis of a spherical refracting surface. For this situation, each problem in the Table refers to the index of refraction \( n_1 \) where the object is located, (a) the index of refraction \( n_2 \) on the other side of the refracting surface, (b) the object distance \( p \), (c) the radius of curvature \( r \) of the surface, and (d) the image distance \( i \). (All distances are in centimeters.) Fill in the missing information, including whether the image is (e) real (R) or virtual (V) and (f) on the same side of the surface as object \( O \) or on the opposite side.

<table>
<thead>
<tr>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( p )</th>
<th>( r )</th>
<th>( i )</th>
<th>R/V</th>
<th>Side</th>
</tr>
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<tbody>
<tr>
<td>33</td>
<td>1.0</td>
<td>1.5</td>
<td>+10</td>
<td>-13</td>
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<td>34</td>
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<td></td>
<td>+100</td>
<td>-30</td>
<td>+600</td>
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<td>+70</td>
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<td>1.0</td>
<td></td>
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<td>-7.5</td>
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</tr>
<tr>
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<td>1.0</td>
<td>+10</td>
<td></td>
<td>-6.0</td>
<td></td>
</tr>
</tbody>
</table>

**Sol**

33. In addition to \( n_1 =1.0 \), we are given (a) \( n_2 = 1.5 \), (b) \( p = +10 \text{ cm} \), and (d) \( i = -13 \text{ cm} \).

(c) Equation 34-8 yields

\[
r = (n_2 - n_1) \left( \frac{n_1 + n_2}{p} \right)^{-1} = (1.5 - 1.0) \left( \frac{1.0}{10 \text{ cm}} + \frac{1.5}{-13 \text{ cm}} \right)^{-1} = -32.5 \text{ cm} \approx -33 \text{ cm} .
\]
(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(e).

34. In addition to \( n_1 = 1.5 \), we are given (b) \( p = +110 \), (c) \( r = -30 \) cm, and (d) \( i = +600 \) cm.

(a) We manipulate Eq. 34-8 to separate the indices:

\[
 n_2 \left( \frac{1}{r} - \frac{1}{i} \right) = \left( \frac{n_1}{p} + \frac{n_1}{r} \right) \Rightarrow n_2 \left( \frac{1}{r} - \frac{1}{i} \right) = \left( \frac{1.5}{110} - \frac{1.5}{-30} \right) \Rightarrow n_2 \left( -0.036 \right) = -0.036
\]

which implies \( n_2 = 1.0 \).

(e) The image is real (R) and inverted.

(f) The object and its image are on the opposite side. The ray diagram would be similar to Fig. 34-12(b) in the textbook.

35. In addition to \( n_1 = 1.5 \), we are also given (a) \( n_2 = 1.0 \), (b) \( p = +70 \) cm, and (c) \( r = +30 \) cm. Notice that \( n_2 < n_1 \).

(d) We manipulate Eq. 34-8 to find the image distance:

\[
i = n_2 \left( \frac{n_2 - n_1}{r} - \frac{n_1}{p} \right)^{-1} = 1.0 \left( \frac{1.0 - 1.5}{30 \text{ cm}} - \frac{1.5}{70 \text{ cm}} \right)^{-1} = -26 \text{ cm}.
\]

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side.

The ray diagram for this problem is similar to the one shown in Fig. 34-12(f). Here refraction always directs the ray away from the central axis; the images are always virtual, regardless of the object distance.

36. In addition to \( n_1 = 1.5 \), we are given (a) \( n_2 = 1.0 \), (c) \( r = -30 \) cm and (d) \( i = -7.5 \) cm.
(b) We manipulate Eq. 34-8 to find \( p \):

\[
p = \frac{n_1}{n_2 - n_1} \left( \frac{r}{i} \right) = \frac{1.5}{1.0 - 1.5} \left( \frac{10}{-30} \right) \left( \frac{1.0}{-7.5} \right) = 10 \text{ cm.}
\]

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(d) in the textbook.

37. In addition to \( n_1 = 1.5 \), we are given (a) \( n_2 = 1.0 \), (b) \( p = +10 \text{ cm} \), and (d) \( i = -6.0 \text{ cm} \).

(c) We manipulate Eq. 34-8 to find \( r \):

\[
r = \left( n_2 - n_1 \right) \left( \frac{n_2}{p} + \frac{n_1}{i} \right)^{-1} = \left( 1.0 - 1.5 \right) \left( \frac{1.5}{10} + \frac{1.0}{-6.0} \right)^{-1} = 30 \text{ cm.}
\]

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(f) in the textbook, but with the object and the image located closer to the surface.

14-3 Thin lenses. Object \( O \) stands on the central axis of a thin symmetric lens. For this situation, each problem in the Table gives object distance \( p \) (centimeters), the type of lens (C stands for converging and D for diverging), and then the distance (centimeters, without proper sign) between a focal point and the lens. Find (a) the image distance \( i \) and (b) the lateral magnification \( m \) of the object, including signs. Also, determine whether the image is (c) real (R) or virtual (V), (d) inverted (I) from object \( O \) or noninverted (NI), and (e) on the same side of the lens as object \( O \) or on the opposite side.

<table>
<thead>
<tr>
<th>( p )</th>
<th>Lens</th>
<th>( i )</th>
<th>( m )</th>
<th>R/V</th>
<th>I/NI</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 53 )</td>
<td>+8.0</td>
<td>D, 12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 54 )</td>
<td>+10</td>
<td>D, 6.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 55 )</td>
<td>+22</td>
<td>D, 14</td>
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<tr>
<td>( 56 )</td>
<td>+12</td>
<td>D, 31</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 57 )</td>
<td>+45</td>
<td>C, 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
53. For a diverging (D) lens, the focal length value is negative. The object distance $p$, the image distance $i$, and the focal length $f$ are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}. $$

The value of $i$ is positive for real images, and negative for virtual images. The corresponding lateral magnification is $m = -i / p$. The value of $m$ is positive for upright (not inverted) images, and is negative for inverted images.

For this lens, we have $f = -12$ cm and $p = +8.0$ cm.

(a) The image distance is $i = \frac{pf}{p-f} = \frac{(8.0 \text{ cm})(-12 \text{ cm})}{8.0 \text{ cm} - (-12) \text{ cm}} = -4.8 \text{ cm}$.

(b) The magnification is $m = -i / p = -(-4.8 \text{ cm}) / (8.0 \text{ cm}) = +0.60$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

54. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -6$ cm).

(a) Equation 34-9 gives $i = \frac{pf}{p-f} = -3.8 \text{ cm}$.

(b) Equation 34-7 gives $m = -i / p = +0.38$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).
55. We recall that for a diverging (D) lens, the focal length value should be negative \((f = -14 \text{ cm})\).

(a) The image distance is \(i = \frac{pf}{p - f} = \frac{(22 \text{ cm})(-14 \text{ cm})}{22 \text{ cm} - (-14 \text{ cm})} = -8.6 \text{ cm}\).

(b) The magnification is \(m = -\frac{i}{p} = -\frac{(-8.6 \text{ cm})}{(22 \text{ cm})} = +0.39\).

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

56. We recall that for a diverging (D) lens, the focal length value should be negative \((f = -31 \text{ cm})\).

(a) Equation 34-9 gives \(i = \frac{pf}{p - f} = -8.7 \text{ cm}\).

(b) Equation 34-7 gives \(m = -\frac{i}{p} = +0.72\).

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(c)).

57. We recall that for a converging (C) lens, the focal length value should be positive \((f = +20 \text{ cm})\).

(a) The image distance is \(i = \frac{pf}{p - f} = \frac{(45 \text{ cm})(20 \text{ cm})}{45 \text{ cm} - 20 \text{ cm}} = +36 \text{ cm}\).
(b) The magnification is \( m = \frac{-i}{p} = \frac{+36 \text{ cm}}{45 \text{ cm}} = -0.80 \).

(c) The fact that the image distance is a positive value means the image is real (R).

(d) A negative value of magnification means the image is inverted (I).

(e) The image is on the opposite side of the object.

The ray diagram for this problem is similar to the one shown in Fig. 34-16(a). The lens is converging, forming a real, inverted image on the opposite side of the object.

14-4 (a) A luminous point is moving at speed \( v_o \) toward a spherical mirror with radius of curvature \( r \), along the central axis of the mirror. Show that the image of this point is moving at speed

\[
v_i = \frac{1}{2} \left( \frac{r}{p} \right) v_o \]

where \( p \) is the distance of the luminous point from the mirror at any given time. Now assume the mirror is concave, with \( r = 15 \text{ cm} \), and let \( v_o = 5.0 \text{ cm/s} \). Find \( v_i \) when (b) \( p = 30 \text{ cm} \) (far outside the focal point), (c) \( p = 8.0 \text{ cm} \) (just outside the focal point), and (d) \( p = 10 \text{ mm} \) (very near the mirror).

**Sol**

(a) From Eqs. 34-3 and 34-4, we obtain

\[
i = \frac{pf}{p - f} = \frac{pr}{2p - r}.
\]

Differentiating both sides with respect to time and using \( v_o = -dp/dt \), we find

\[
v_i = \frac{di}{dt} = \frac{d}{dt} \left( \frac{pr}{2p - r} \right) = -r v_o \left( \frac{2p - r}{2p - r} \right)^2 + 2v_o pr \left( \frac{r}{2p - r} \right)^2 v_o.
\]

(b) If \( p = 30 \text{ cm} \), we obtain \( v_i = \left[ \frac{15 \text{ cm}}{2(30 \text{ cm}) - 15 \text{ cm}} \right]^2 \left( 5.0 \text{ cm/s} \right) = 0.56 \text{ cm/s} \).

(c) If \( p = 8.0 \text{ cm} \), we obtain \( v_i = \left[ \frac{15 \text{ cm}}{2(8.0 \text{ cm}) - 15 \text{ cm}} \right]^2 \left( 5.0 \text{ cm/s} \right) = 1.1 \times 10^3 \text{ cm/s} \).
(d) If \( p = 1.0 \) cm, we obtain
\[
v_i = \left[ \frac{15 \text{cm}}{2(1.0 \text{cm}) - 15 \text{cm}} \right]^2 (5.0 \text{cm/s}) = 6.7 \text{cm/s}.
\]

14-5 The formula \( 1/p + 1/i = 1/f \) is called the Gaussian form of the thin-lens formula. Another form of this formula, the Newtonian form, is obtained by considering the distance \( x \) from the object to the first focal point and the distance \( x' \) from the second focal point to the image. Show that \( xx' = f^2 \) is the Newtonian form of the thin-lens formula.

**Sol**

For a thin lens,
\[
(1/p) + (1/i) = (1/f),
\]
where \( p \) is the object distance, \( i \) is the image distance, and \( f \) is the focal length. We solve for \( i \):
\[
i = \frac{fp}{p - f}.
\]

Let \( p = f + x \), where \( x \) is positive if the object is outside the focal point and negative if it is inside. Then,
\[
i = \frac{f(f + x)}{x}.
\]

Now let \( i = f + x' \), where \( x' \) is positive if the image is outside the focal point and negative if it is inside. Then,
\[
x' = i - f = \frac{f(f + x)}{x} - f = \frac{f^2}{x}
\]
and \( xx' = f^2 \).