6 電流與電阻

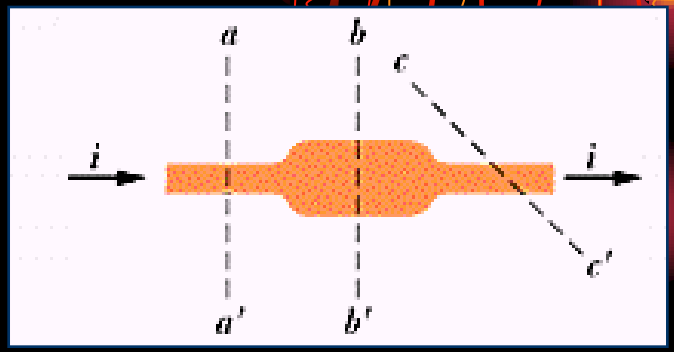
The Hindenburg disaster

Contents

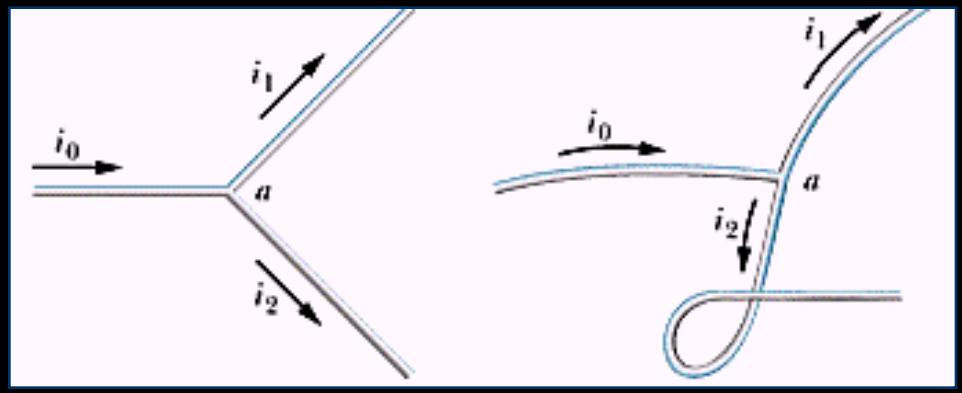
6-1 電流

$$i = \frac{dq}{dt}$$

$$(\text{ampere; A} = \text{C/s})$$



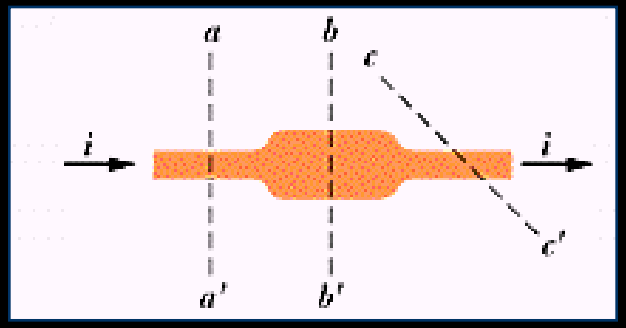
$$i_0 = i_1 + i_2$$

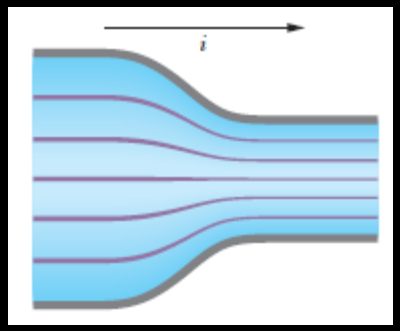


- 電荷守恆

  克希荷夫電流(節點)定理

電流密度



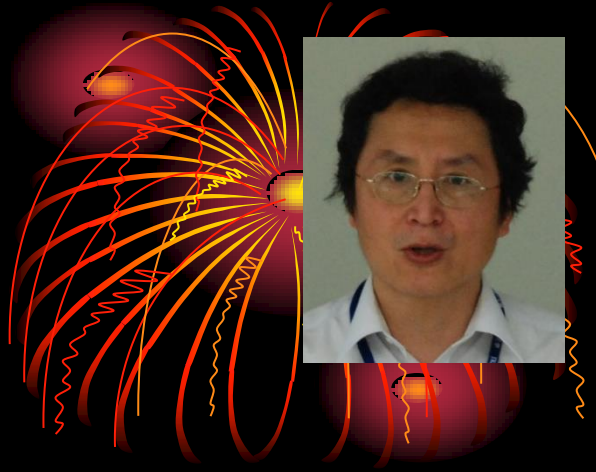


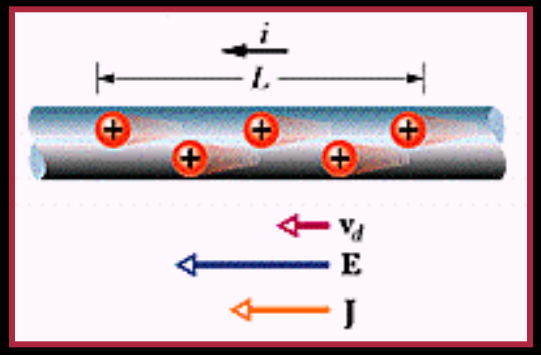
$$i = \int \vec{J} \cdot d\vec{A}$$

$$i = \int J dA = J \int dA = JA \rightarrow J = \frac{i}{A}$$

J: A/m^2

電流密度與漂移速度





$$q = (nAL)e, t = \frac{L}{v_d}$$

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$v_d = \frac{i}{nAe} = \frac{J}{ne}, \vec{J} = (ne)\vec{v}_d$$

铜的漂移速度

$d_{Al}=2.5mm$, $d_{cu}=1.8mm$

$i=17mA$, $\rho=9\times10^{3}kg/m^{3}$, $M=64\times10^{-3}kg/mol$

$$A_{Al}=\pi(d/2)^2=4.91\times10^{-6}m^2$$

$$J_{Al}=i/A_{Al}=3.5\times10^{3}A/m^2$$

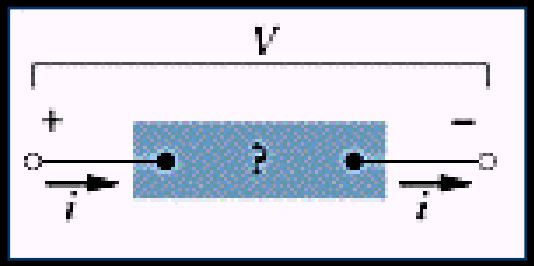
$$J_{Cu}=i/A_{Cu}=6.7\times10^{3}A/m^2$$

$$\frac{n}{N_A}=\frac{\rho}{M}\text{ or }\frac{\text{atoms/m}^3}{\text{atoms/mol}}=\frac{\text{mass/m}^3}{\text{mass/mol}}$$

$$n=\frac{N_A\rho}{M}=8.47\times10^{28}\text{electrons/m}^3$$

$$v_d=\frac{J}{ne}=4.9\times10^{-7}\text{m/s}=1.88\text{mm/h}$$

6-2 電阻與電阻率



- Resistance
- Resistivity
- Conductivity

$$R = \frac{V}{i} \quad (\text{ohm} = \Omega)$$

$$\rho = \frac{E}{J} \quad (\Omega \cdot m)$$

$$\sigma = \frac{1}{\rho} \quad (\Omega \cdot m)^{-1}$$

$$\vec{E} = \rho \vec{J}$$

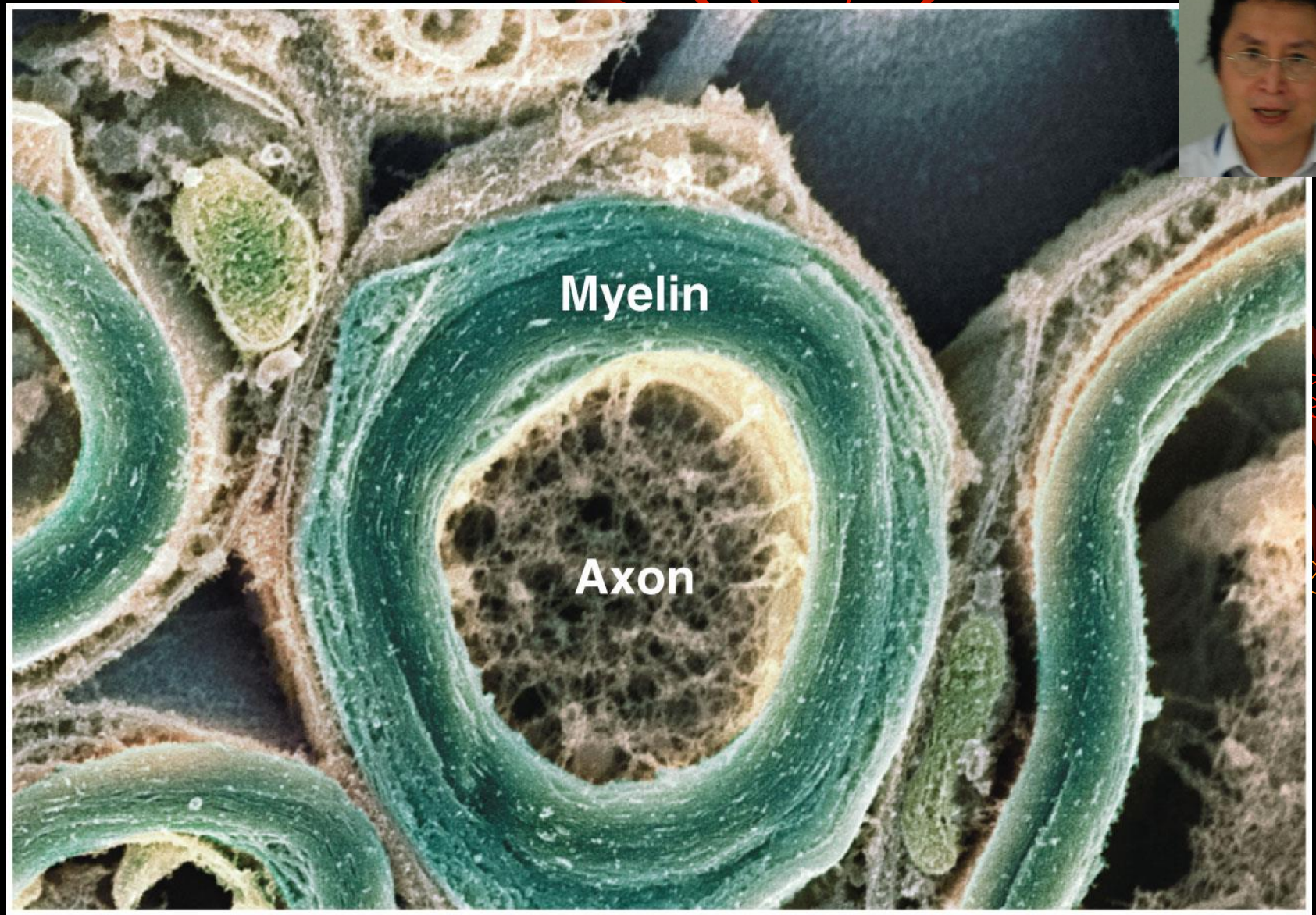
$$\vec{J} = \sigma \vec{E}$$

電阻率

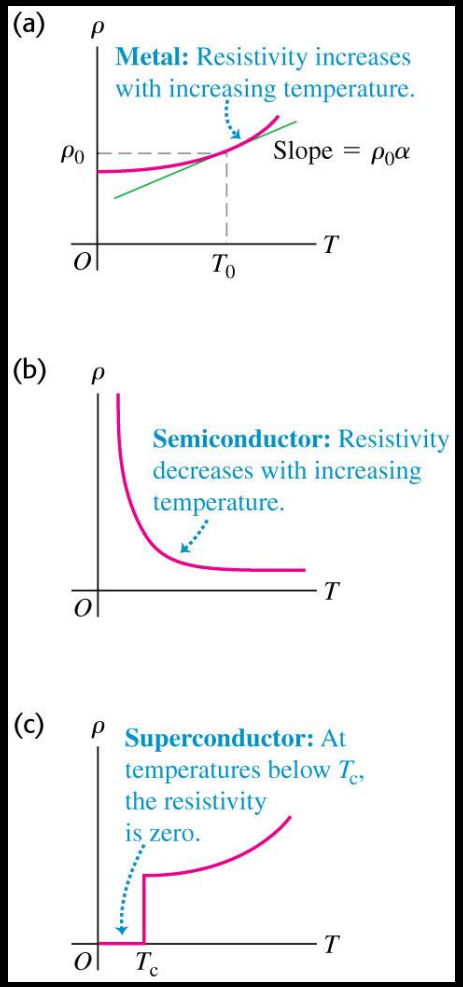
TABLE 27-1 RESISTIVITIES OF SOME MATERIALS AT ROOM TEMPERATURE (20° C)

MATERIAL	RESISTIVITY, ρ ($\Omega \cdot m$)	TEMPERATURE COEFFICIENT OF RESISTIVITY, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
Manganin[a]	48.2×10^{-8}	0.002×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	10^{10}-10^{14}	
Fused quartz	$\sim 10^{16}$	

Figure 25.4



Resistivity and temperature



- Resistivity depends on temperature. See Figure 25.6 at the left.
- Table 25.2 shows some temperature coefficients of resistivity.

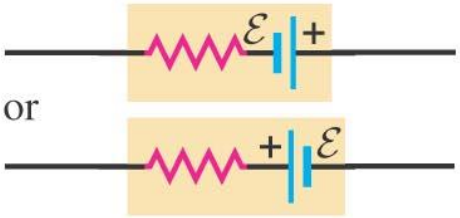
Table 25.2 Temperature Coefficients of Resistivity
(Approximate Values Near Room Temperature)

Material	$\alpha\,[(°C)^{-1}]$	Material	$\alpha\,[(°C)^{-1}]$
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

Symbols for circuit diagrams

- **Table 25.4 shows the usual symbols used in circuit diagrams.**

Table 25.4 Symbols for Circuit Diagrams

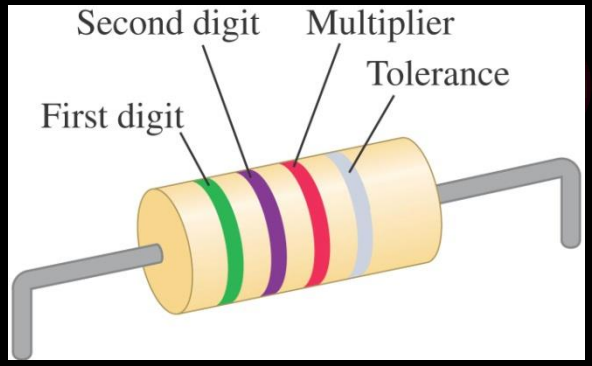
	Conductor with negligible resistance
	Resistor
	Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)
	Source of emf with internal resistance r (r can be placed on either side)
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current through it)

Resistors are color-coded for easy identification

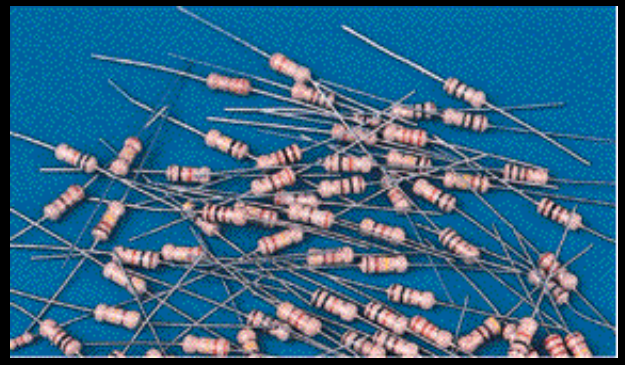
- This resistor has a resistance of 5.7 kΩ with a tolerance of ±10%.

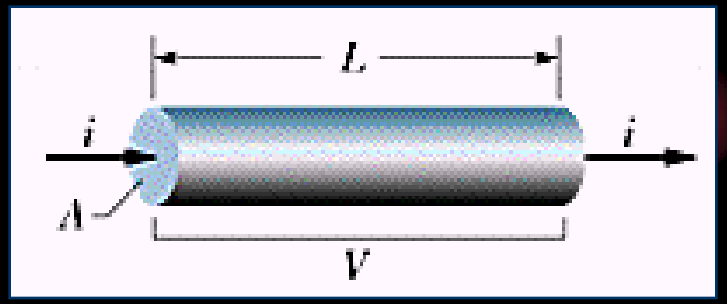
Table 25.3 Color Codes for Resistors

Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9



電阻的計算

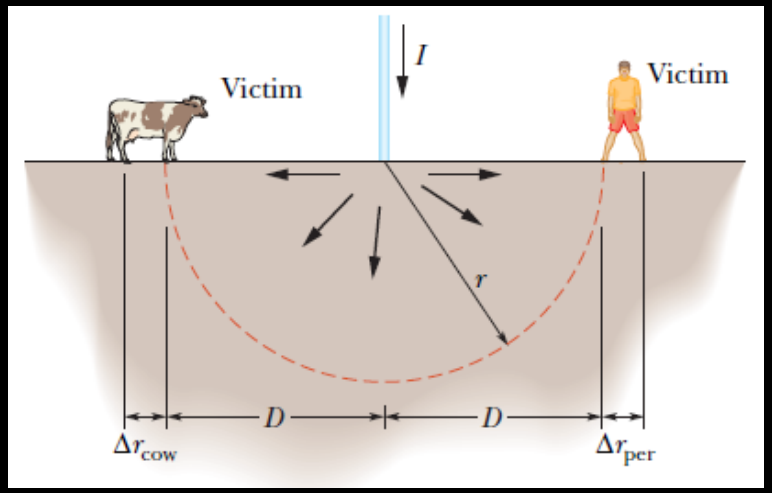




$$E = V / L,\ J = i / A$$

$$\rho = \frac{E}{J} = \frac{V / L}{i / A} = R\frac{A}{L} \rightarrow R = \rho\frac{L}{A}$$

Ex. Current from a lightning strike



$$J = \frac{I}{2\pi r^2},$$

$$E = \rho_{gr} J = \frac{\rho_{gr} I}{2\pi r^2}.$$

$$\Delta V = -\int_D^{D+\Delta r} E\, dr.$$

$$\Delta V = -\int_D^{D+\Delta r} \frac{\rho_{gr} I}{2\pi r^2}\, dr = -\frac{\rho_{gr} I}{2\pi}\left[-\frac{1}{r}\right]_D^{D+\Delta r}$$

$$= \frac{\rho_{gr} I}{2\pi}\left(\frac{1}{D+\Delta r} - \frac{1}{D}\right)$$

$$= -\frac{\rho_{gr} I}{2\pi}\frac{\Delta r}{D(D+\Delta r)}.$$

$$i = \frac{V}{R} = \frac{\rho_{gr} I}{2\pi}\frac{\Delta r}{D(D+\Delta r)}\frac{1}{R}.$$

$$i_p = \frac{(100\ \Omega\cdot\text{m})(100\ \text{kA})}{2\pi} \times \frac{0.50\ \text{m}}{(60\ \text{m})(60.0\ \text{m} + 0.50\ \text{m})}\frac{1}{4.00\ \text{k}\Omega}$$

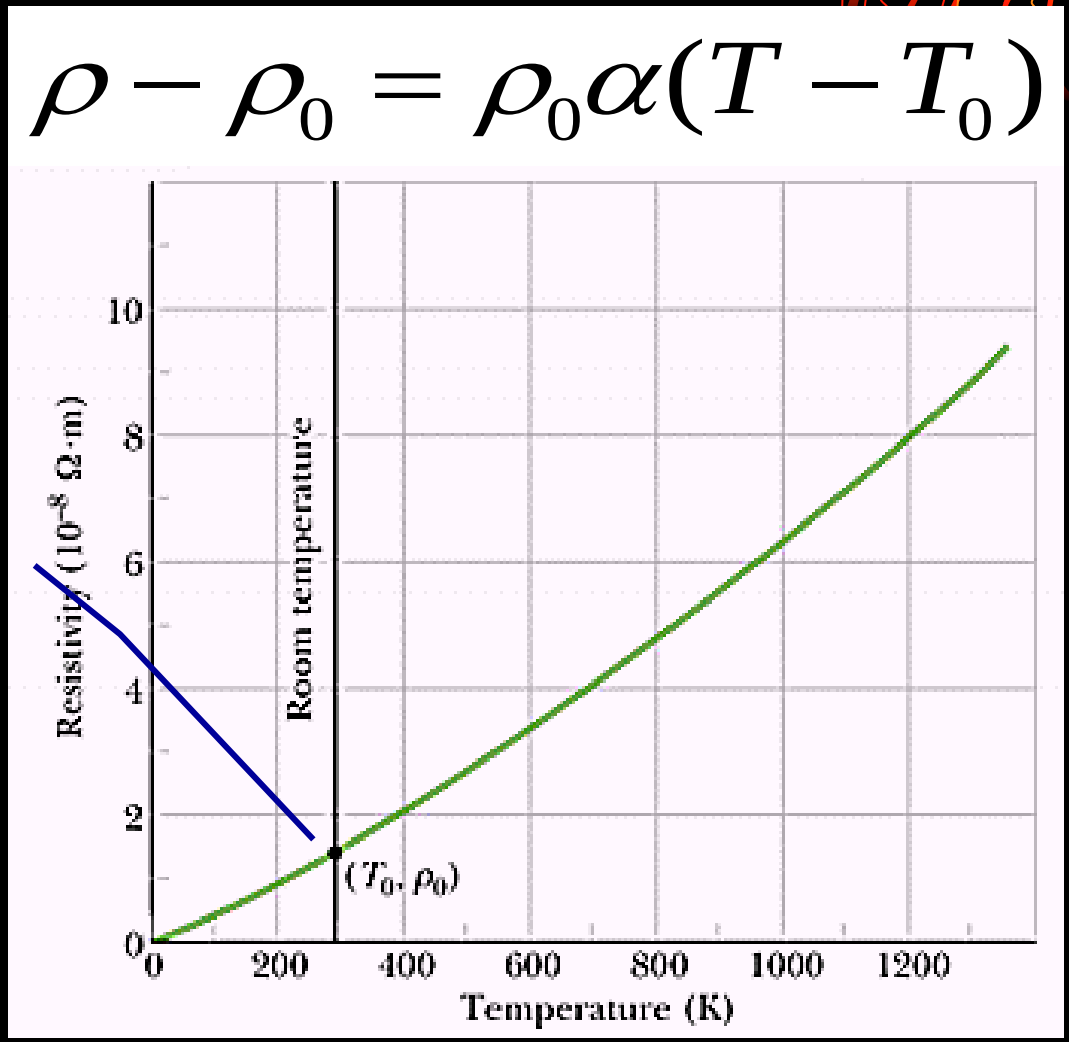
$$= 0.0548\ \text{A} = 54.8\ \text{mA}.$$

電阻率的溫度效應

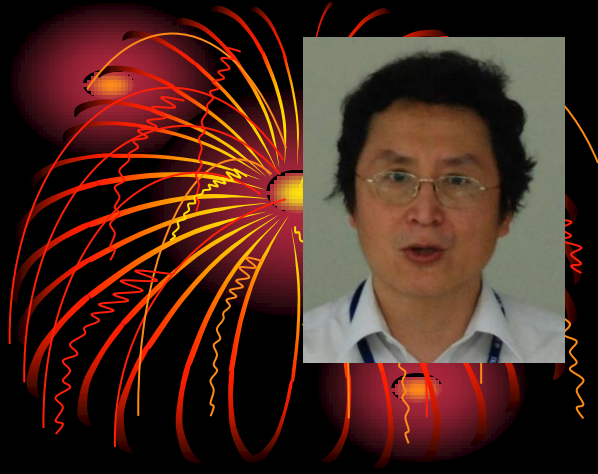
$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

Cu: 293K

1.69×10^{-8}



The effect of temperature

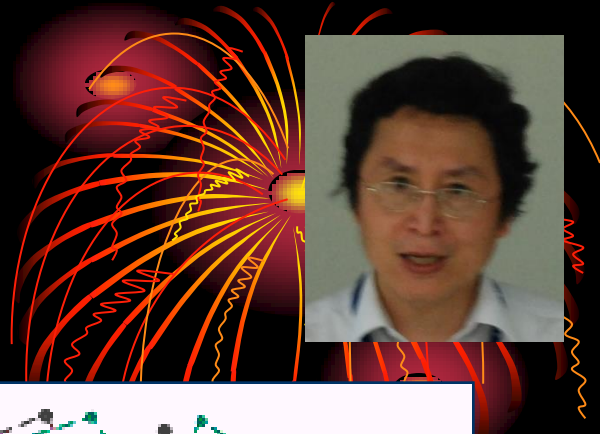


$$a = F/m = eE/m, v_d = a\tau = eE\tau/m$$

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m},\ E = \left(\frac{m}{e^2 n\tau}\right)J, E = \rho J$$

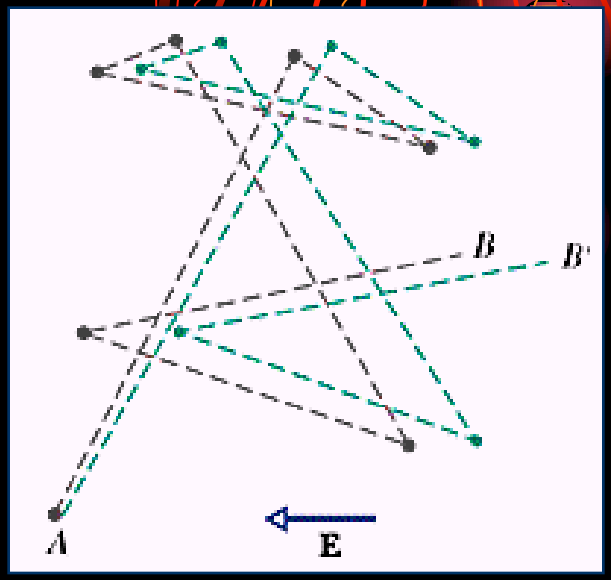
$$\rightarrow \rho = \frac{m}{e^2 n\tau}$$

電阻的微觀意義



$$v_{eff} \approx 1.6 \times 10^{6} m/s$$

$$v_d \approx (10^{-13}) v_{eff}$$



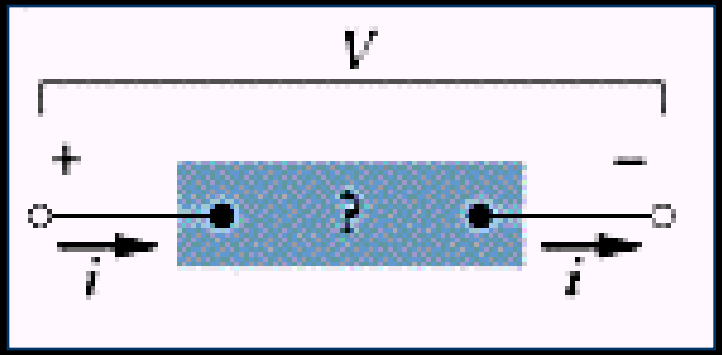
$$\rho = \frac{m}{e^2 n \tau}$$

$$\tau = 2.5 \times 10^{-14} s$$

$$\lambda = \tau v_{eff} \approx 40nm$$

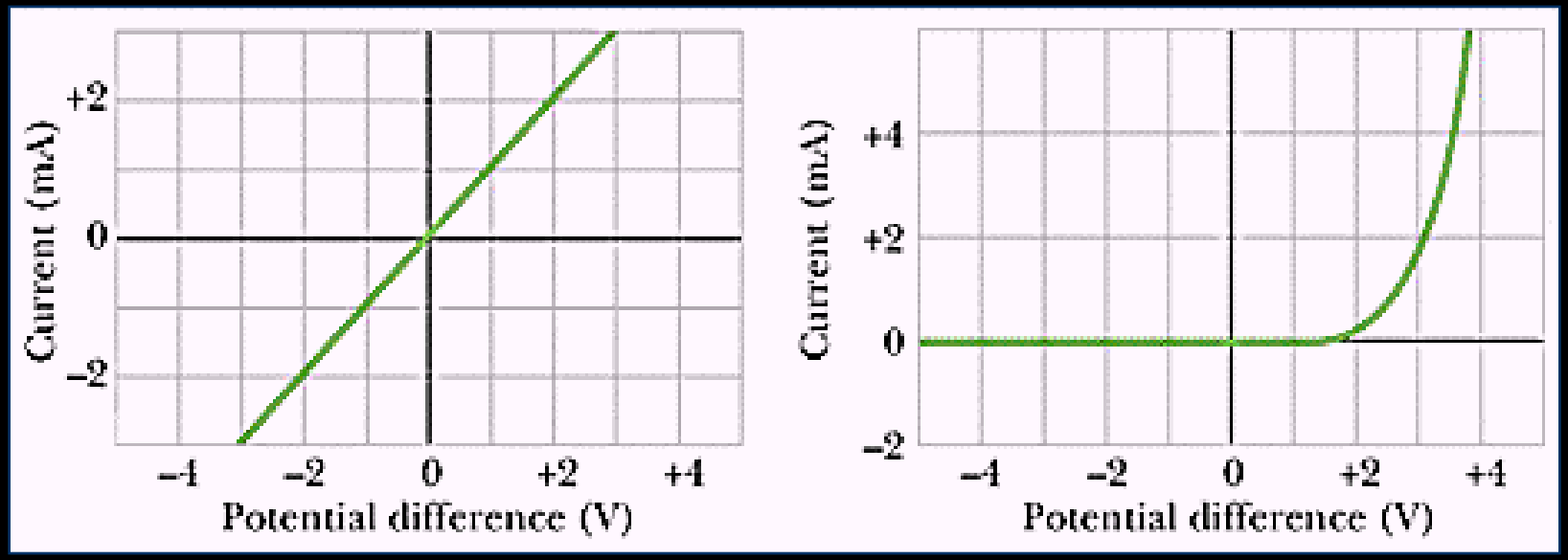
6-3 Ohm's Law

- 流過一電路元件之電流與該元件之電位差成正比
- 電阻率與電場的大小及方向無關



$$i = \frac{V}{R} \qquad \vec{E} = \rho \vec{J}$$

I-V curves

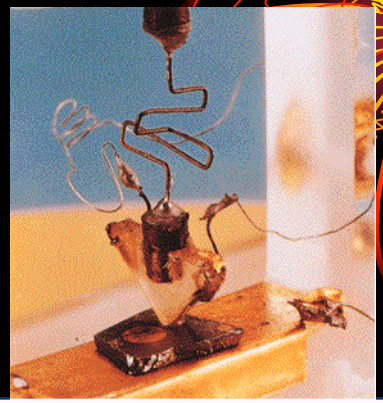


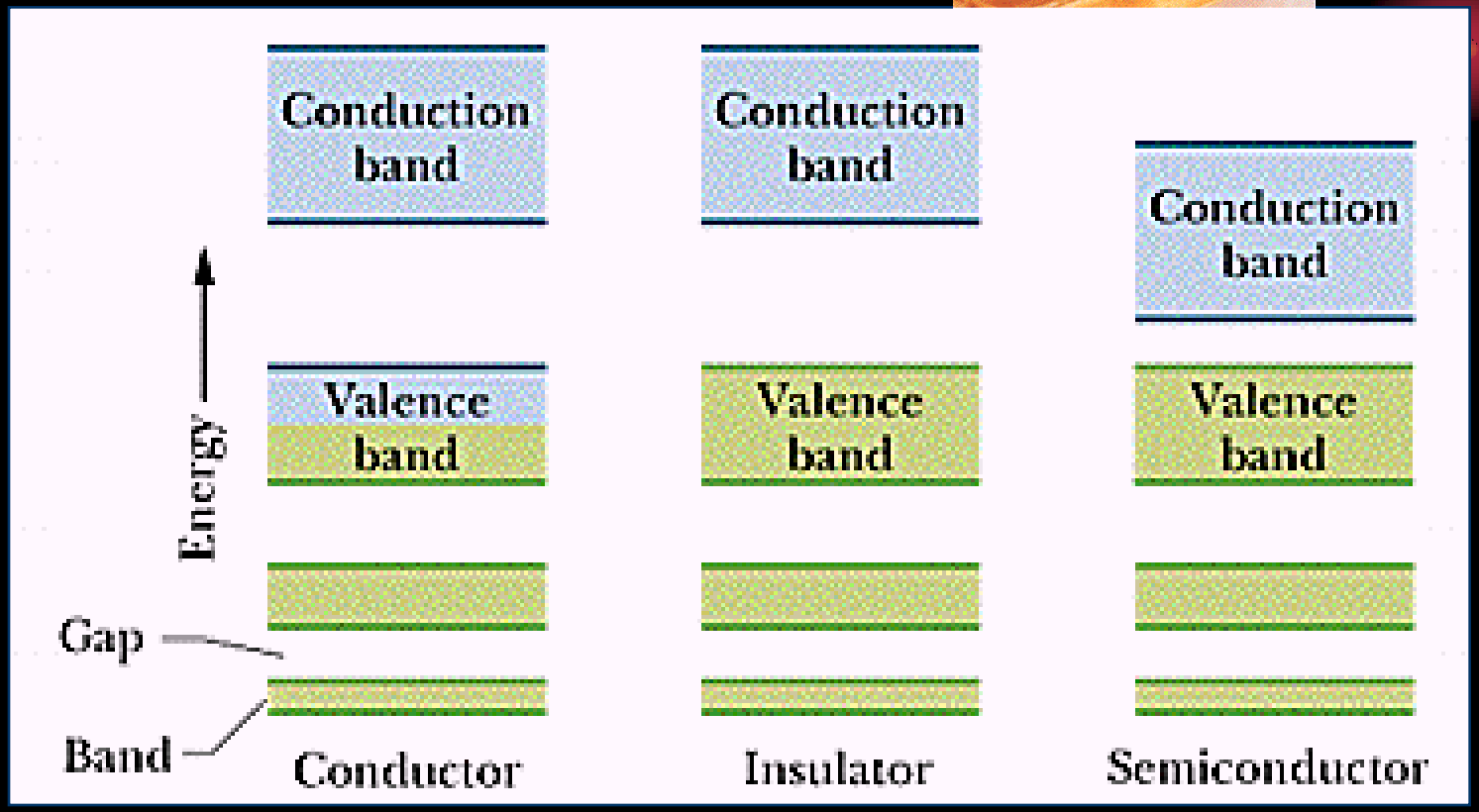
conductor

pn junction diode

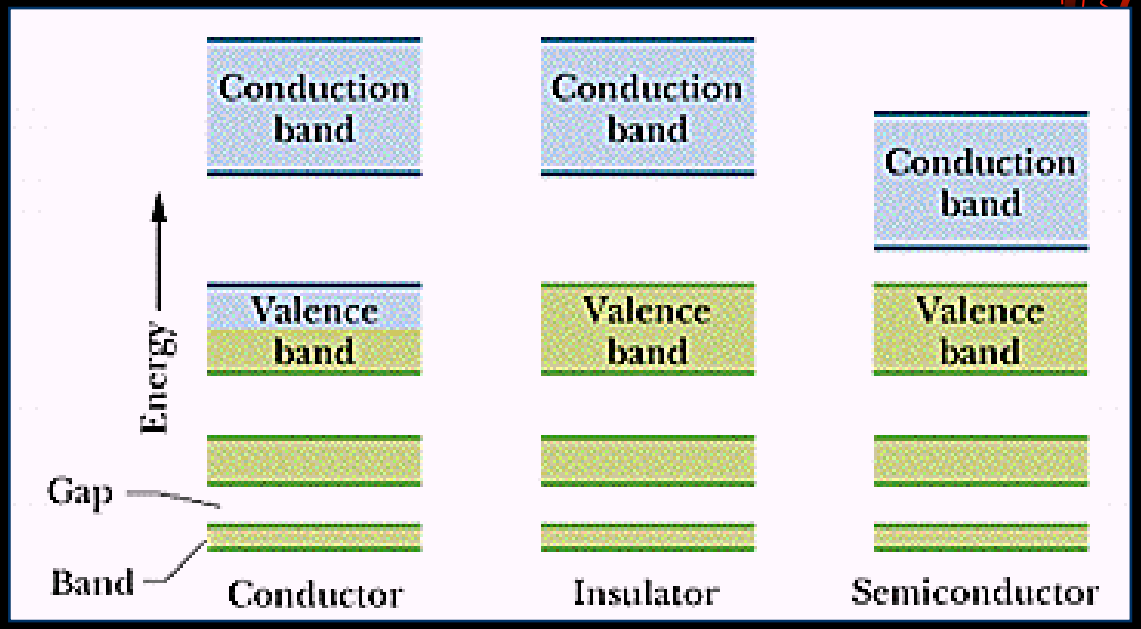
6-4 Semiconductors

半導體





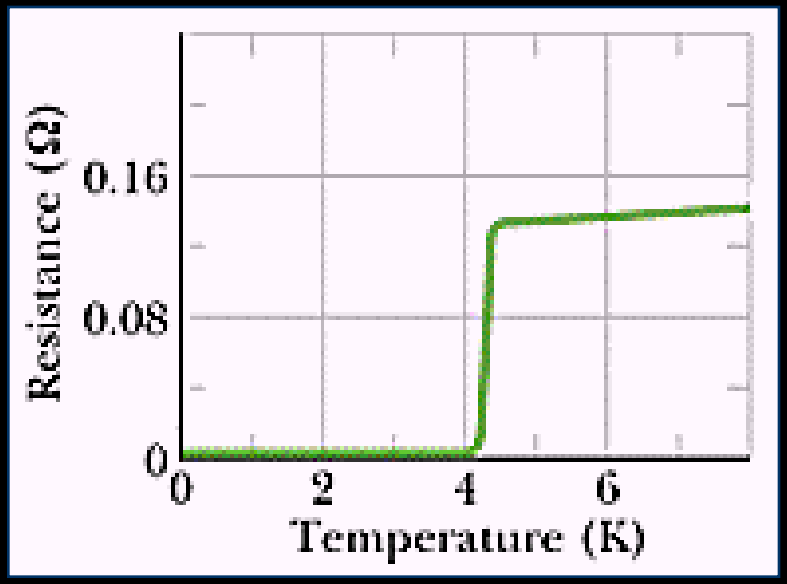
電阻率的溫度效應：導體與半導體的差異

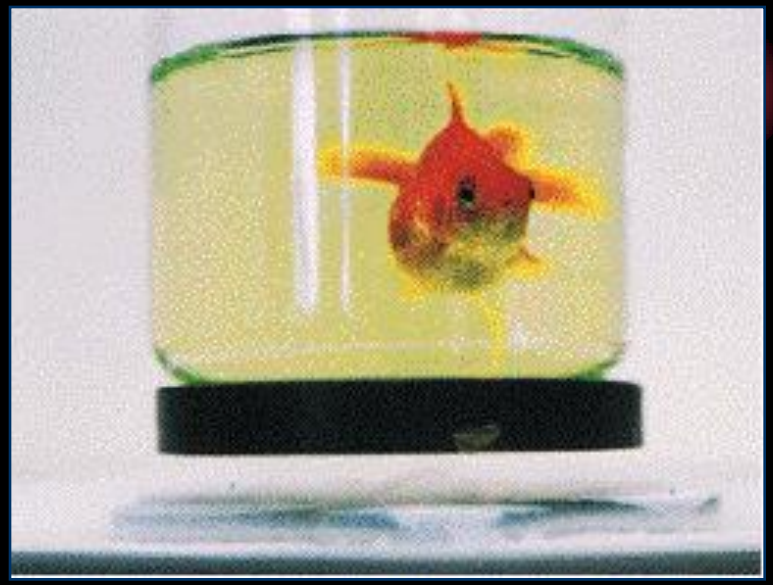


$$\rho = \frac{m}{e^2 n \tau}$$

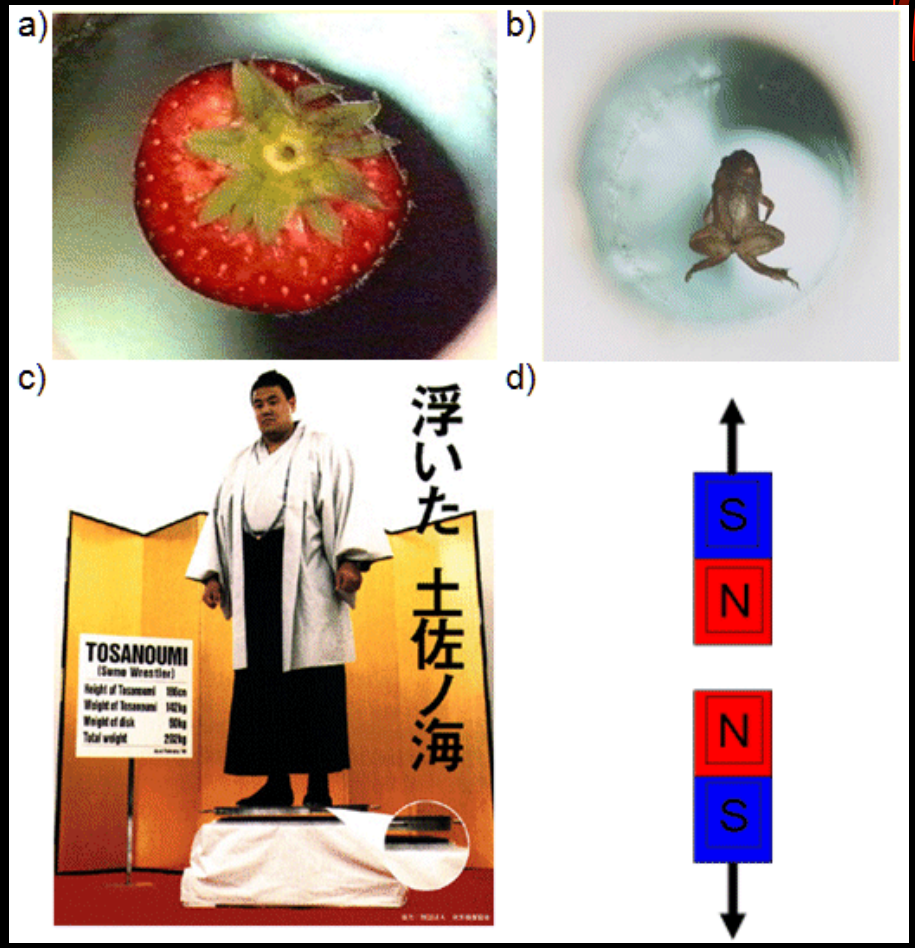
Superconductors

超導體





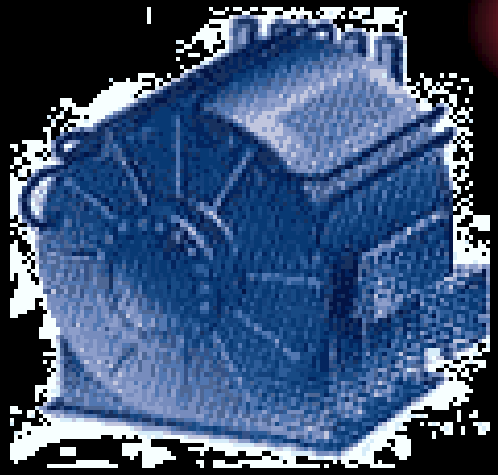
More Maglev - magnetic levitation



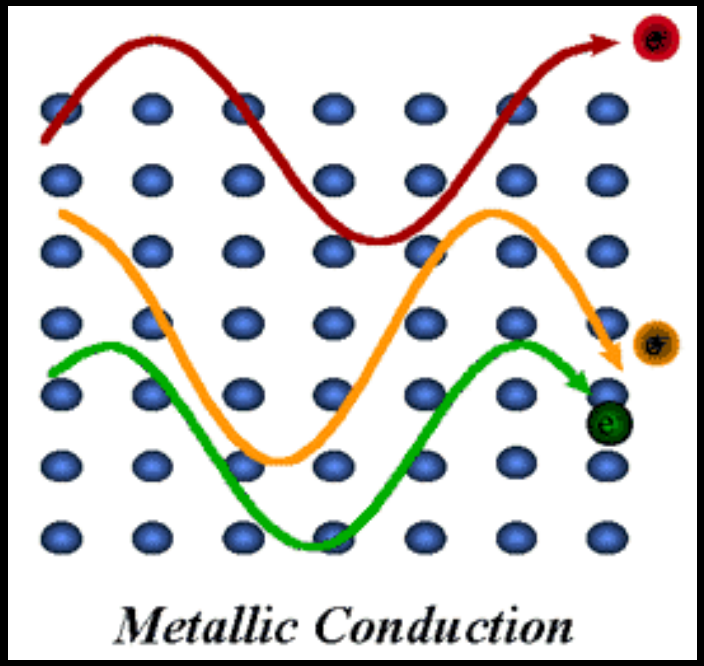
http://web.mit.edu/physics/hudson/ResearchBackground.htm

Maglev、MRI and Tevatron

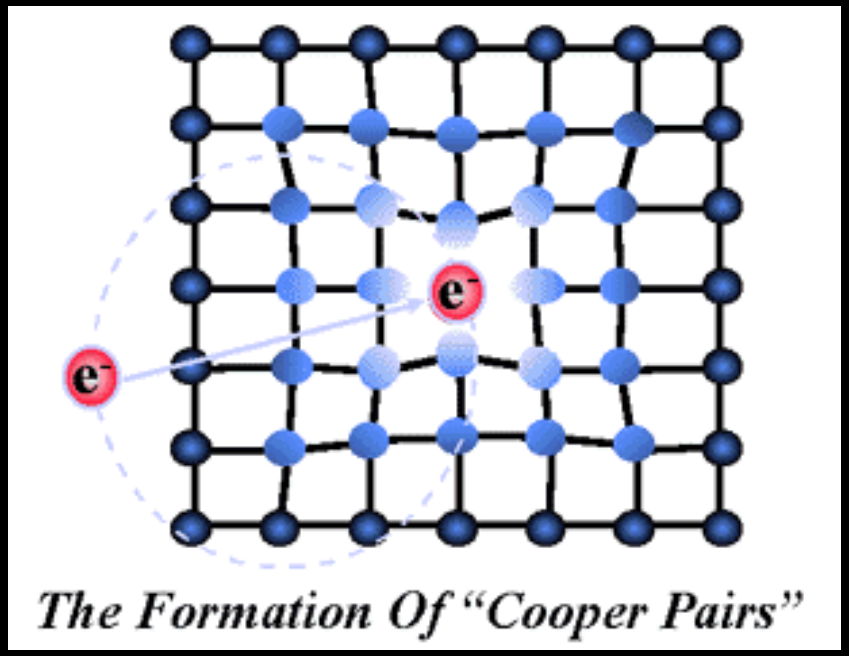
Power line and motor



BCS Theory-The Cooper pairs



Metallic Conduction



The Formation Of "Cooper Pairs"

http://www.chm.bris.ac.uk/webprojects2000/igrant/hightctheory.html