ʃ 電容 Capacitance

National Ignition Facility

Explosions in Airborne Dust

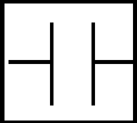
[Flour explosion](#) [sugar](#)

5-1電容與電容器

- 電容（Capacitance）

$$C = \frac{q(\text{電荷})}{V(\text{電位差})}$$

- 平行板電容器



- $C = \dfrac{K\varepsilon_0 A}{d}$ （F 法拉）

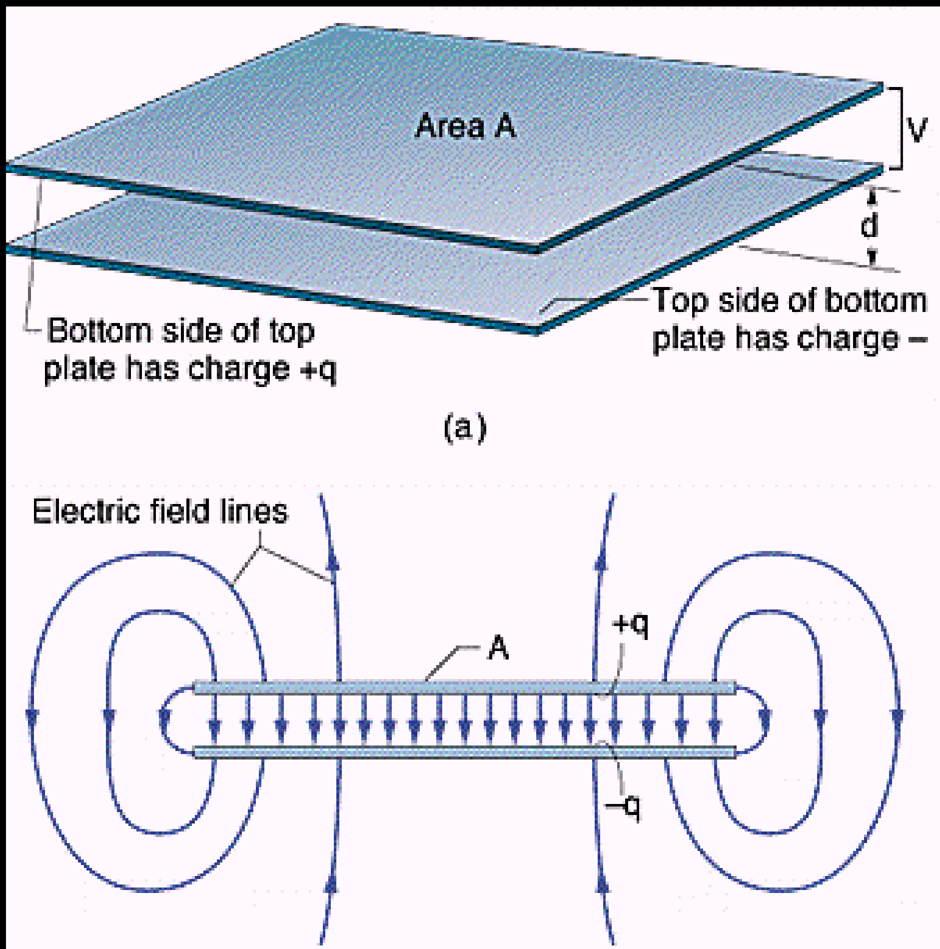
K: 介電常數，真空 = 1

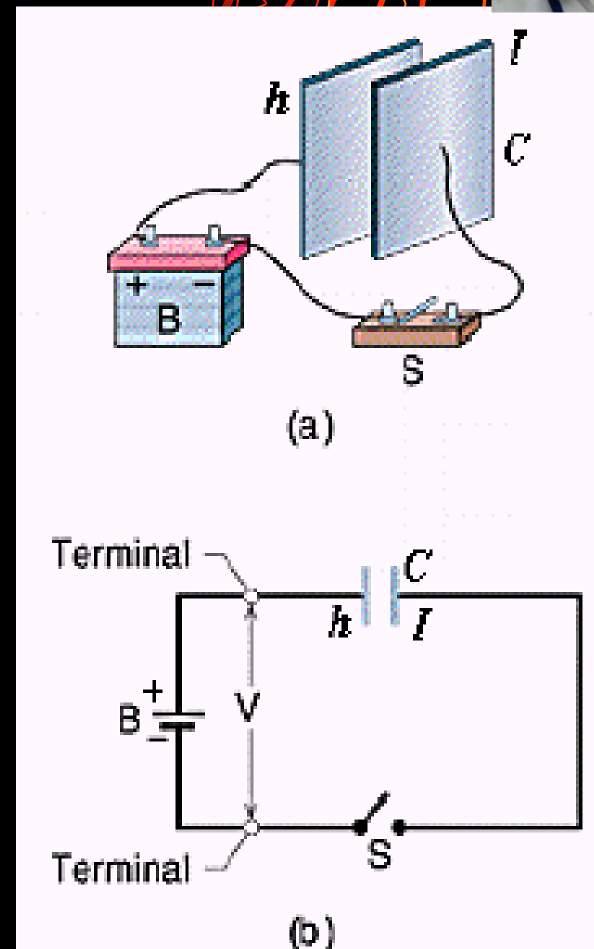
ε_0: 真空電容率 A: 板面積 d : 兩板距離

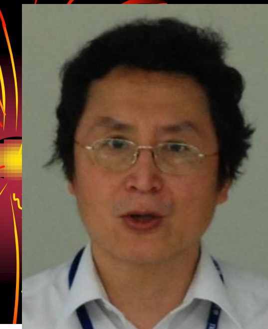
The Uses of Capacitors

- As storehouses of electric potential energy
- As vital elements in tuned circuits (radio and TV) and RAM chips

Charging a capacitor





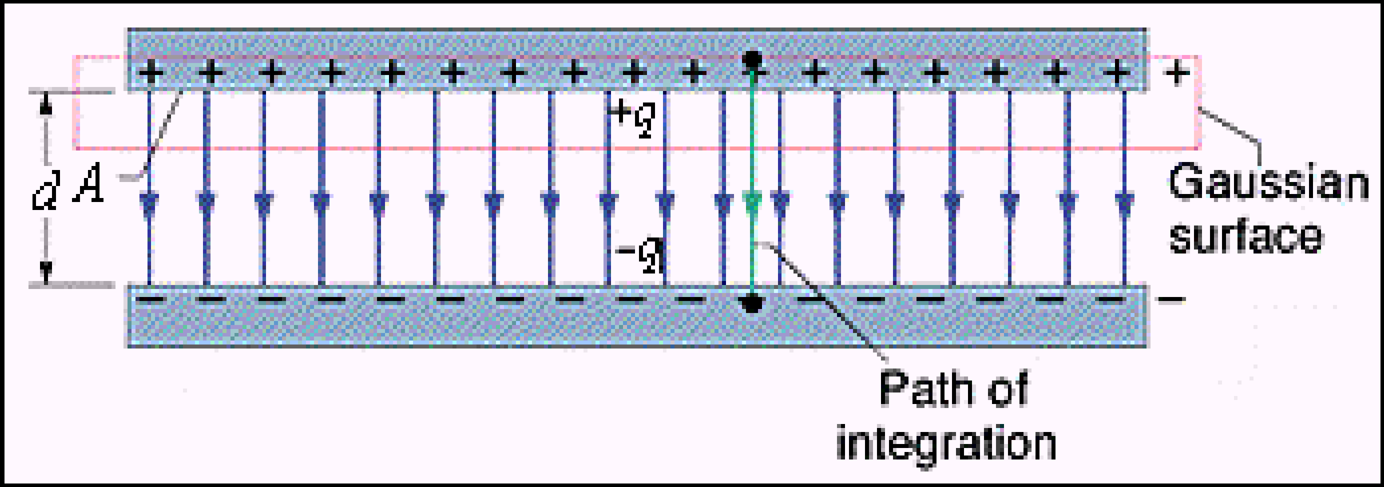


5-2 Calculating the Capacitance

1) Put a charge q on the plates
2) Find E using Gauss's Law
3) Find V between the plates
4) Calculate C by its definition

Calculating the electric field

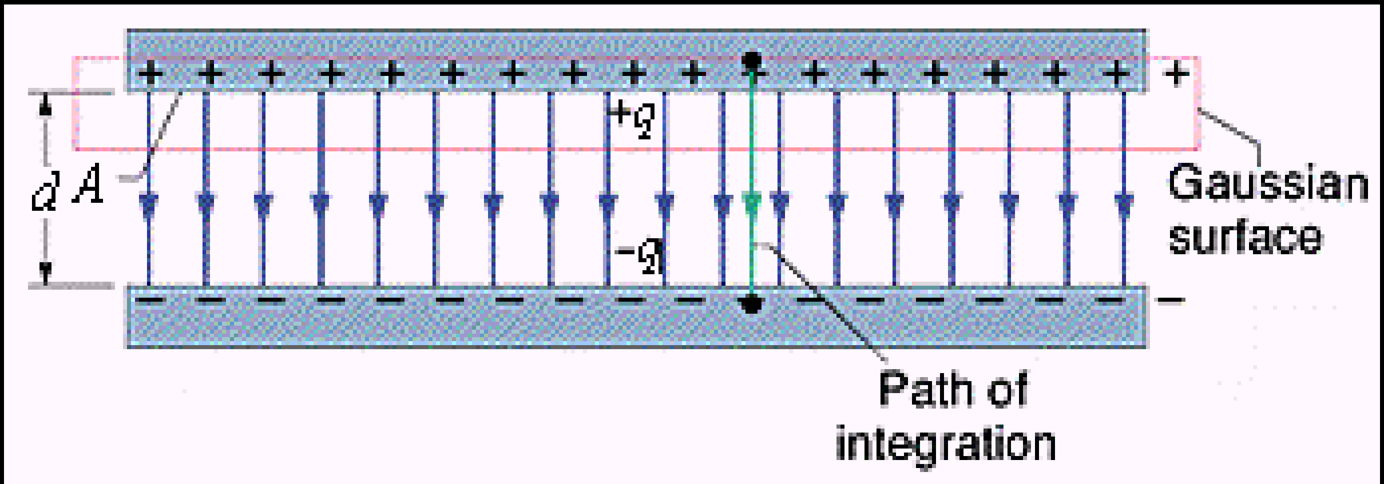
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q \rightarrow q = \varepsilon_0 EA$$



Calculating the potential difference

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$V = \int_+^- E ds \quad (V_f - V_i = -V)$$



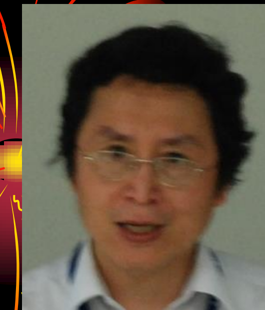
A parallel-plate capacitor

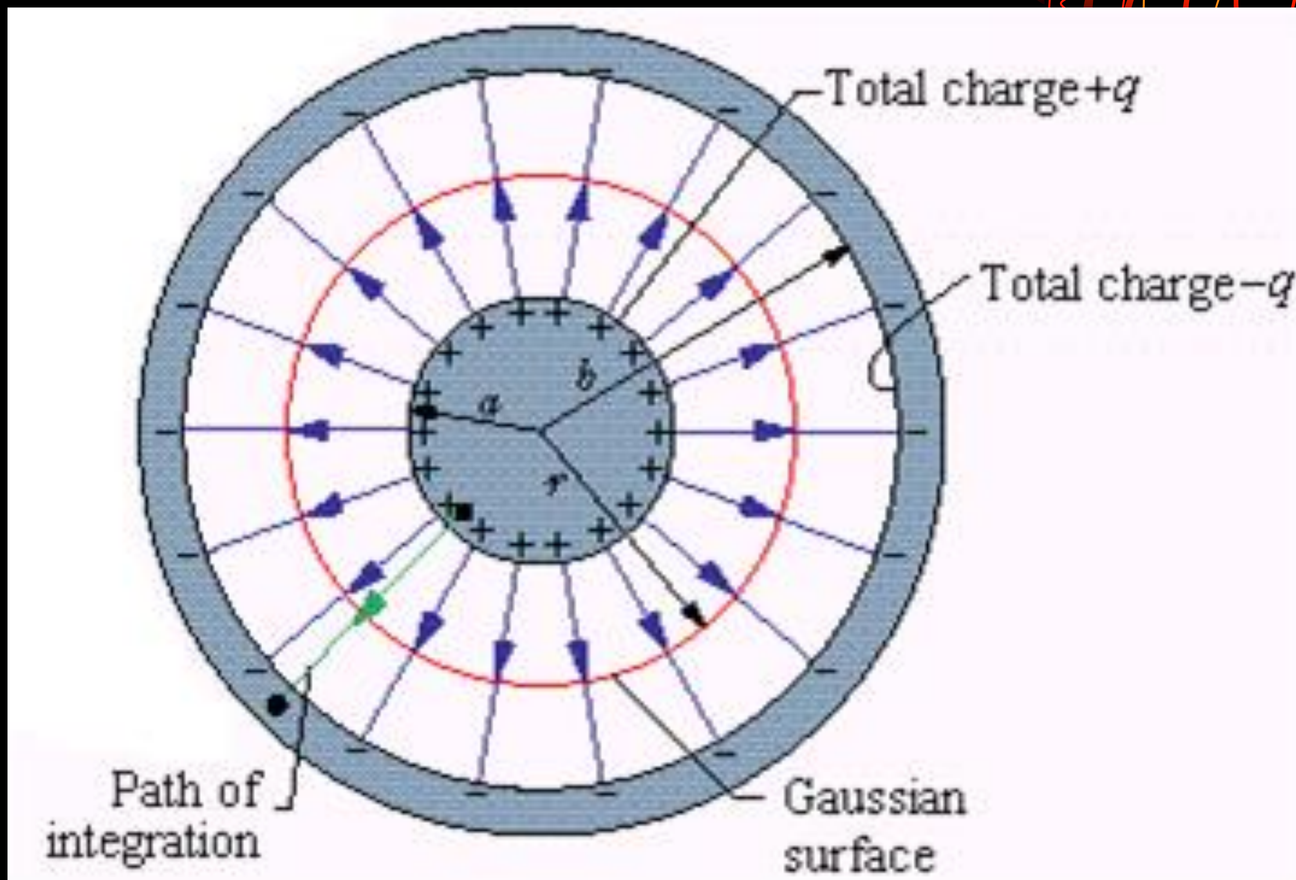
$$q = \varepsilon_0 E A$$

$$V = \int_{+}^{-} E\,ds = E\int_0^d ds = Ed$$

$$C = \frac{q}{V} = \frac{\varepsilon_0 A}{d}$$

A cylindrical capacitor





$$q = \varepsilon_0 E A = \varepsilon_0 E(2\pi r L)$$

$$E = \frac{q}{2\pi\varepsilon_0 L r}$$

$$V = \int_{+}^{-} E\,ds = \frac{q}{2\pi\varepsilon_0 L}\int_a^b \frac{dr}{r}$$

$$= \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \text{ or } \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

A spherical capacitor

$$q = \varepsilon_0 E A = \varepsilon_0 E(4\pi r^2)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$V = \int_{+}^{-} E ds = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab}$$

$$C = \frac{q}{V} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

An Isolated Sphere - the missing plate is a conducting sphere of infinite radius

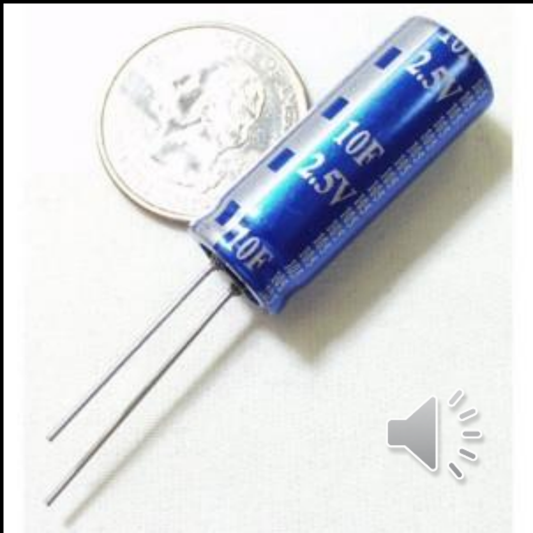
$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b} \rightarrow 4\pi\varepsilon_0 R \, (\mathrm{b} \rightarrow \infty)$$

Ex.1 1.0 F 電容器

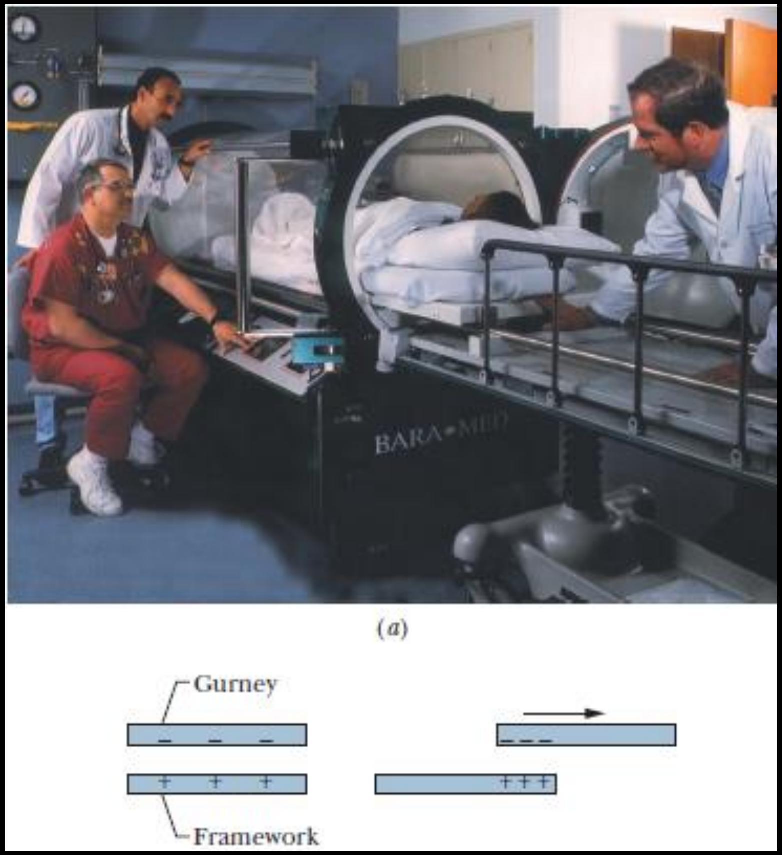
- d = 1.0mm

$$A = \frac{Cd}{\varepsilon_0} = \frac{(1.0F)(1.0\times 10^{-3}m)}{8.85\times 10^{-12}F/m}$$

$$= 1.1\times 10^{8}m^{2} \sim 100km^{2}$$



Ex.2 a hyperbaric chamber



(a)

- Spark : > 2000V
- fire : > 0.20 mJ
- $V_f = 600V$

→

- V_f : > 6000V
- U_f : > 0.45 mJ

$$q = C_i V_i = C_f V_f, \; U = 1/2CV^2$$

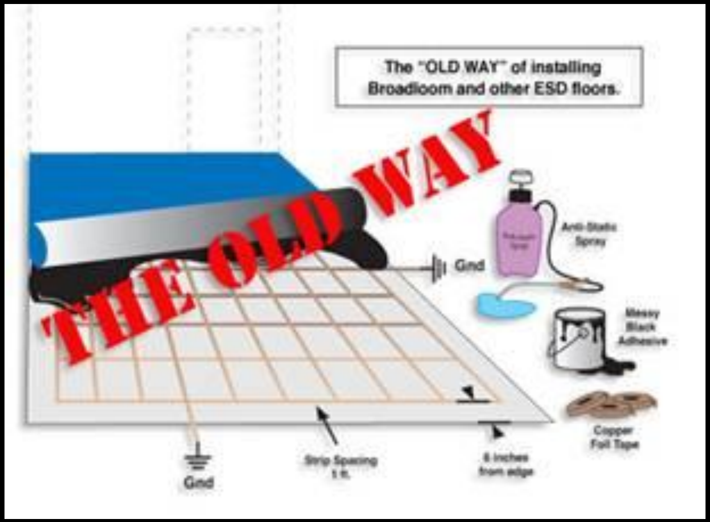
Ex.3 the threshold value U_t (150 m required to ignite airborne grains.

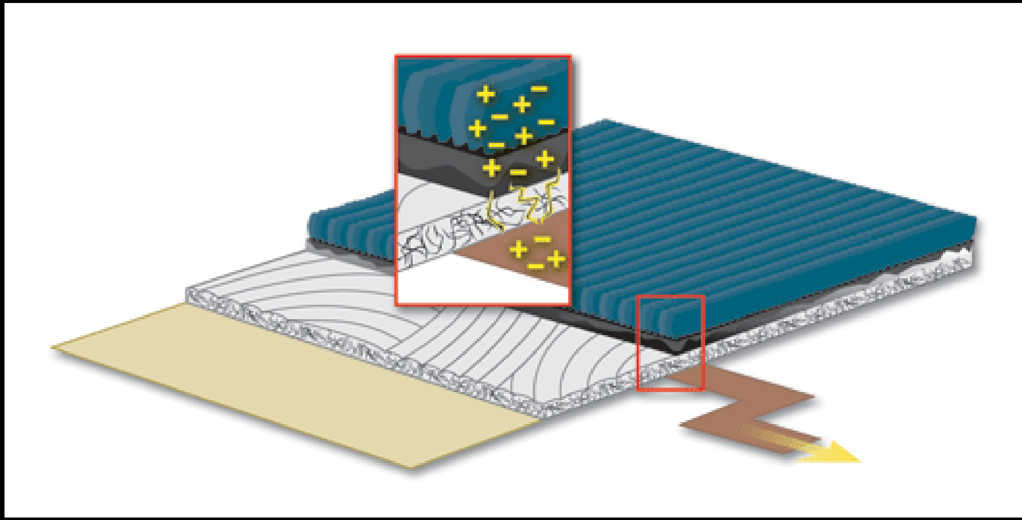
$$C = 4\pi\varepsilon_0 R\ ,\quad U = 1/2CV^2$$

$$\rightarrow V = \sqrt{\frac{2U_t}{4\pi\varepsilon_0 R}} = \sqrt{\frac{2(150\times10^{-3})}{4\pi\varepsilon_0(1.8\text{m})}}$$

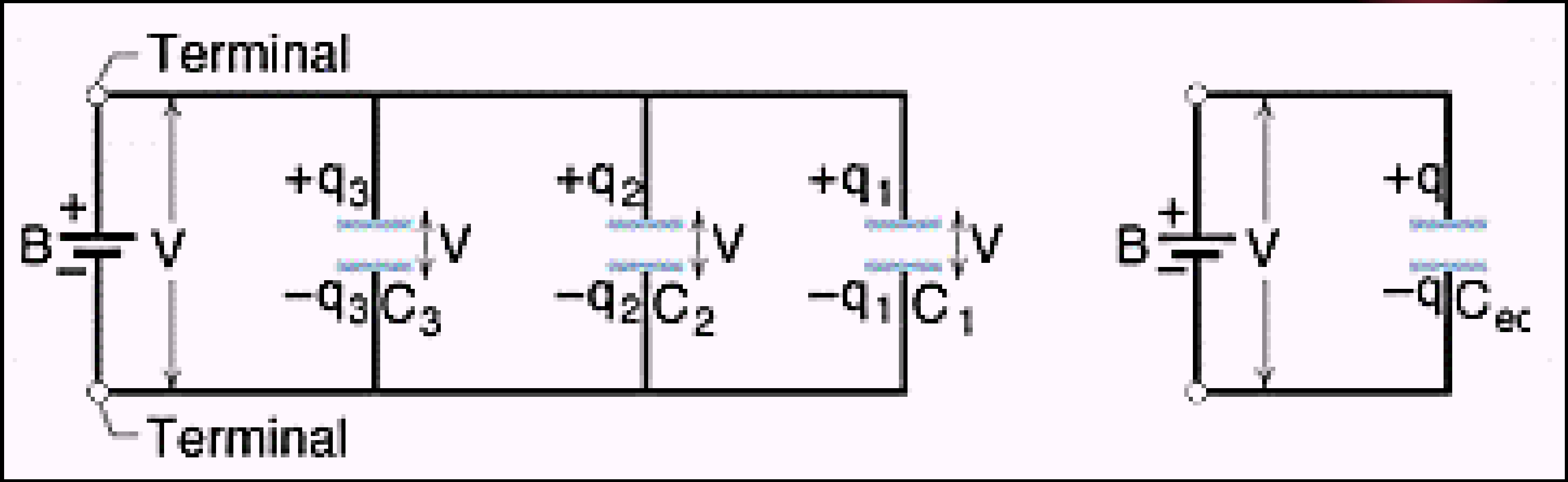
$$= 3.9\times10^4\,\text{V}$$

Conducting floor

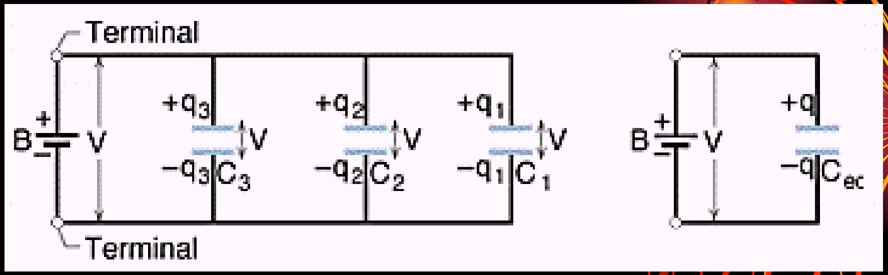




5-3 Capacitors in parallel



等效電容



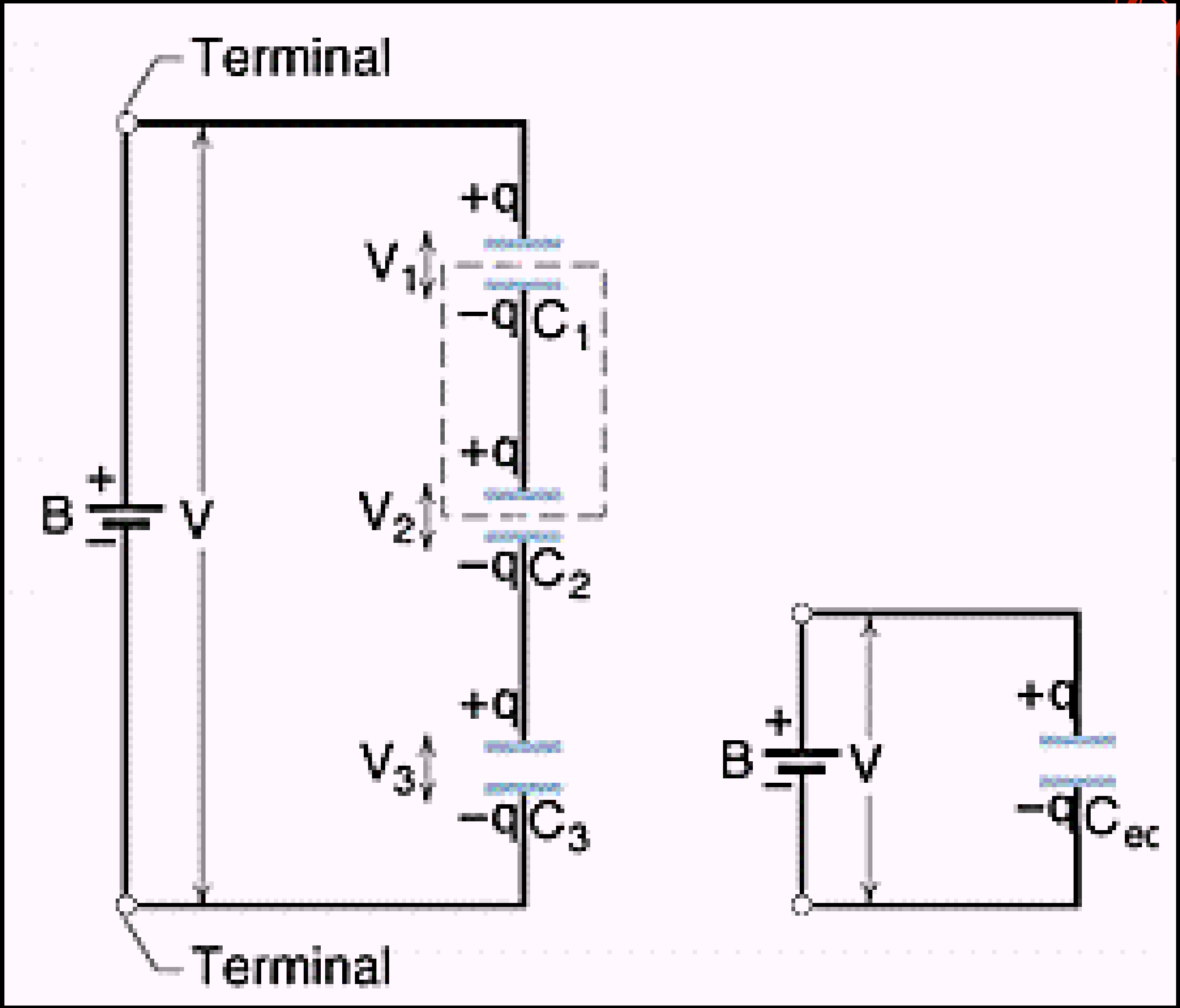
$$q_1 = C_1 V, \; q_2 = C_2 V, \; q_3 = C_3 V$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

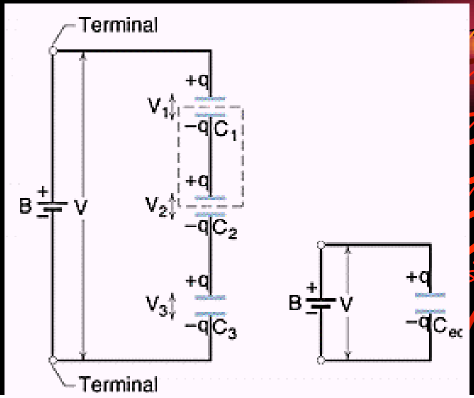
$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$$

$$\rightarrow C_{eq} = \sum_{j=1}^{n} C_j$$

5-4 Capacitors in series



等效電容



$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$$

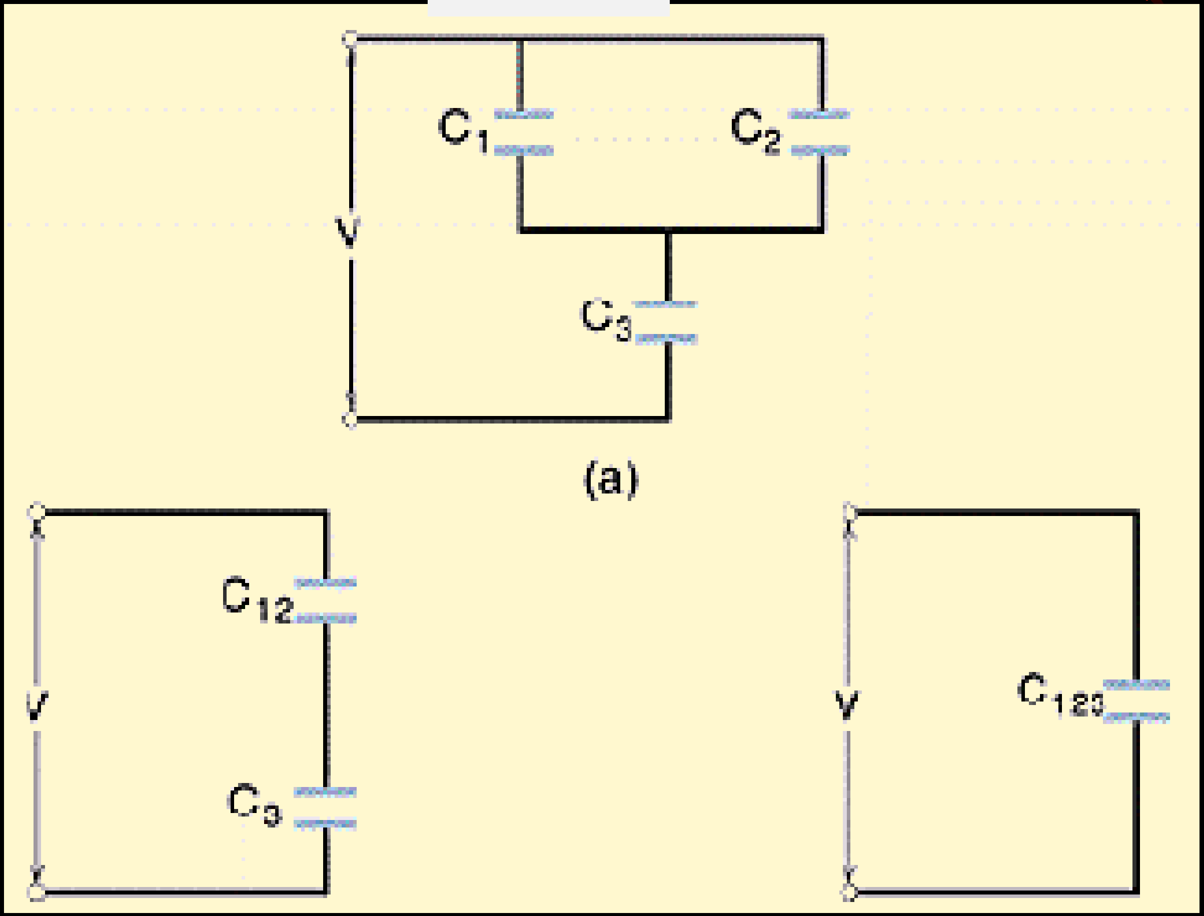
$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

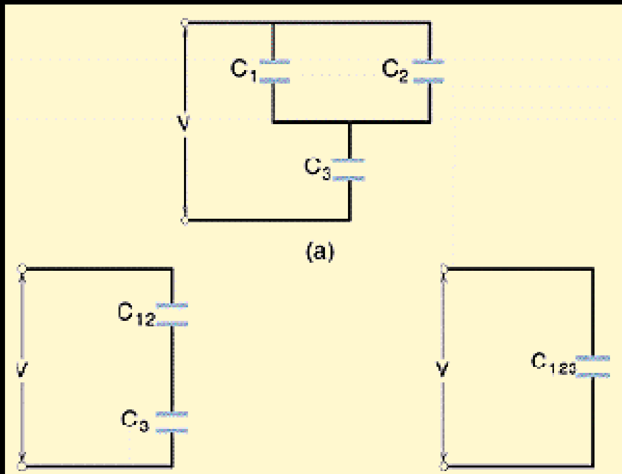
$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Ex.4 Finding eq. capacitor

$q_1 = ?$





$$C_{12} = C_1 + C_2, \quad \frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$q_{123} = C_{123} V \rightarrow V_{12} = \frac{q_{12}}{C_{12}} \quad (q_{12} = q_{123})$$

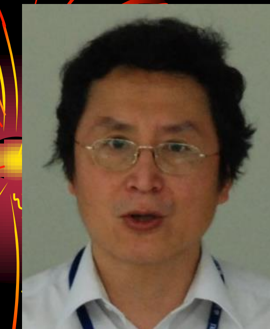
$$\rightarrow q_1 = C_1 V_1 \quad (V_1 = V_{12}) = 31.0 \mu F$$

5-5 Storing Energy in an Electric Field

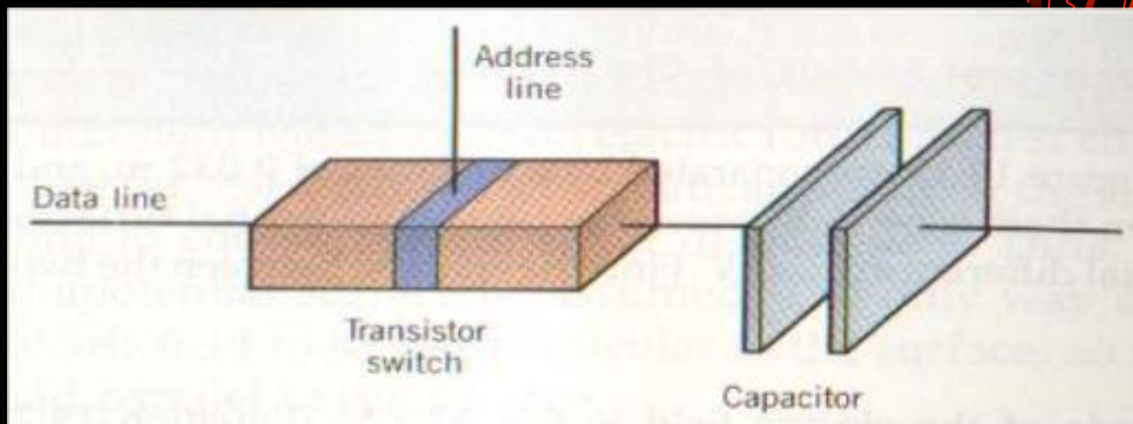
$$dW = V'dq' = \frac{q'}{C}dq'$$

$$W = \int dW = \frac{1}{C}\int_0^q q'dq' = \frac{q^2}{2C}$$

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2$$



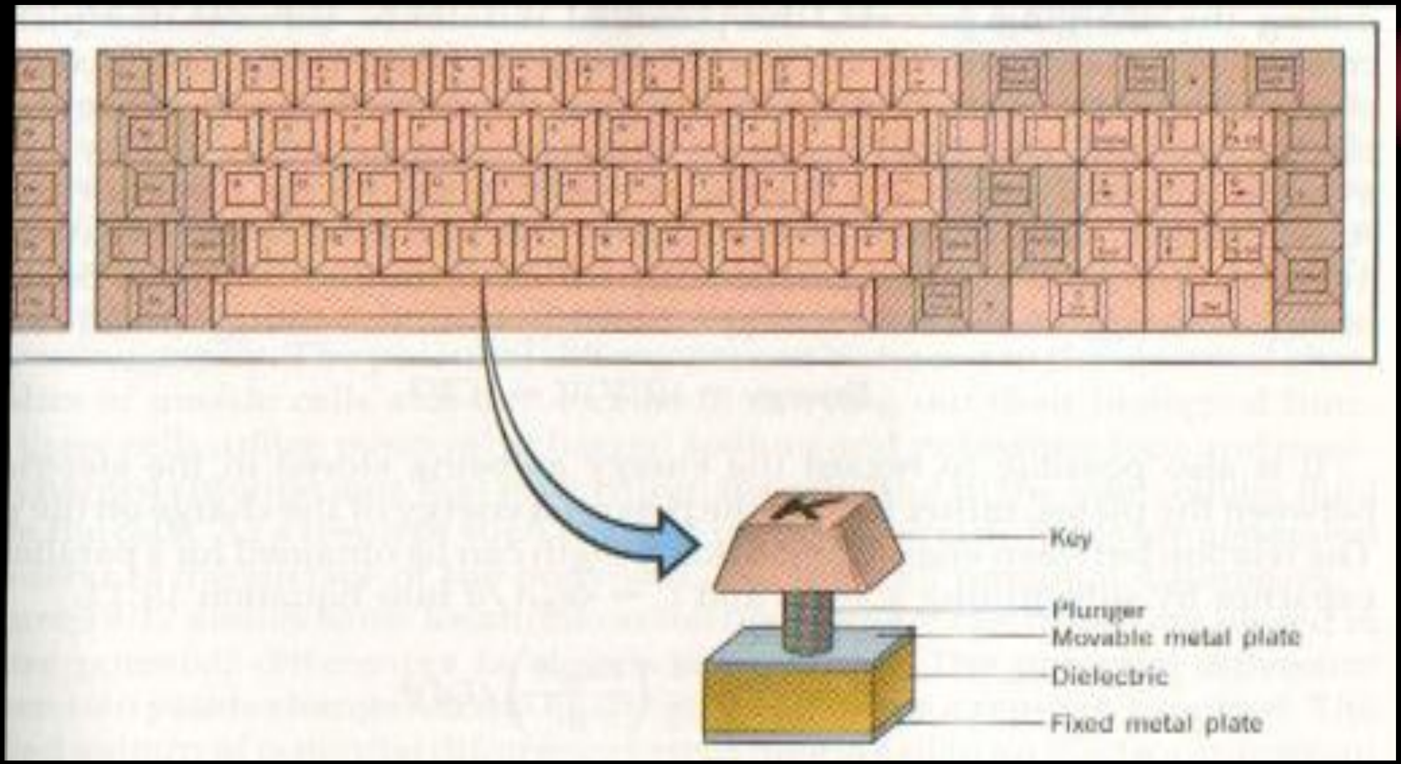
Ex.5 A RAM Chip



- C = 55fF（10^{-15}） V = 5.3 volt

$$n = \frac{q}{e} = \frac{CV}{e} = \frac{\left(55\times10^{-15}F\right)\left(5.3V\right)}{1.60\times10^{-19}C} = 1.8\times10^{6}\,\text{electrons}$$

Ex.6 電容式鍵盤


Key
Plunger
Movable metal plate
Dielectric
Fixed metal plate

Ex.6 (cont)

- A = 9.50 × 10^{-5}m^2 ; k = 3.50
- 按鍵後d由5.00mm減為0.150mm

$$\frac{k\varepsilon_0 A}{d} = 19.6PF\left(10^{-12}F\right)$$

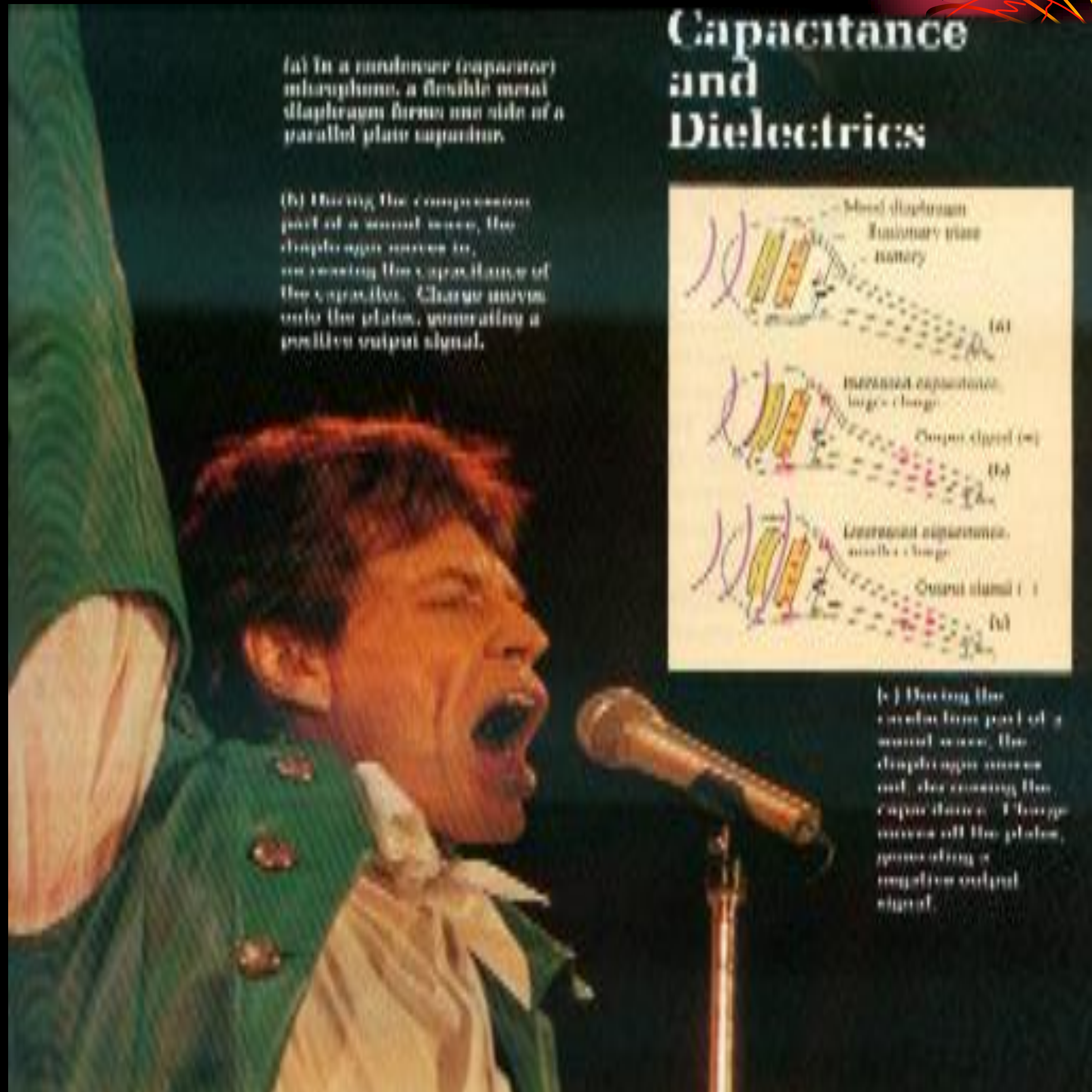
- 按鍵前 C = 0.589PF

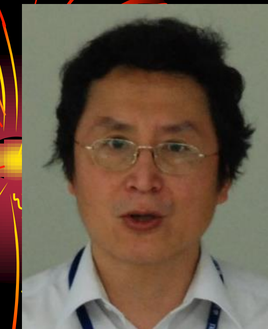
$$\delta C = 19.0PF$$

Touch Screen

Indium Tin Oxide；ITO 氧化銦錫

電容式麥克風



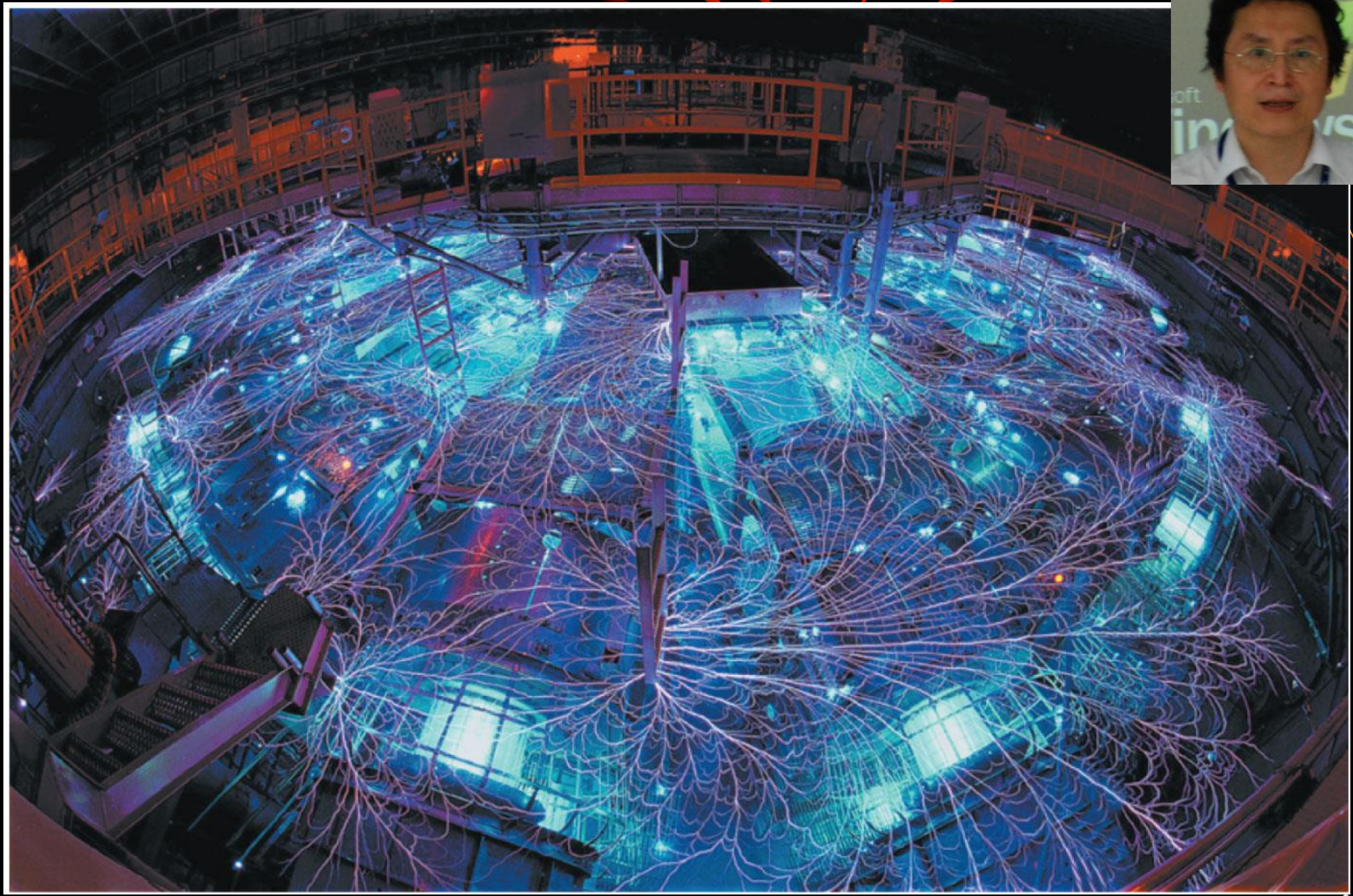


Ex.7 defibrillator（除纖顫機）

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(70\times10^{-6}\,\mathrm{F})(5000\,\mathrm{V})^2 = 875\,\mathrm{J}$$

$$P = \frac{U}{t} = \frac{200\,\mathrm{J}}{2.0\times10^{-3}\,\mathrm{s}} = 100\,\mathrm{kW}$$

Z machine



Z pinch/Θ pinch

$2\times10^{9}\,\mathrm{K}/2.9\times10^{14}\,\mathrm{W}$

5-6 Capacitor with a dielectric

- The capacitance is increased by **κ** times

κ: the dielectric constant

TABLE 26-1 Some Properties of Dielectrics[a]

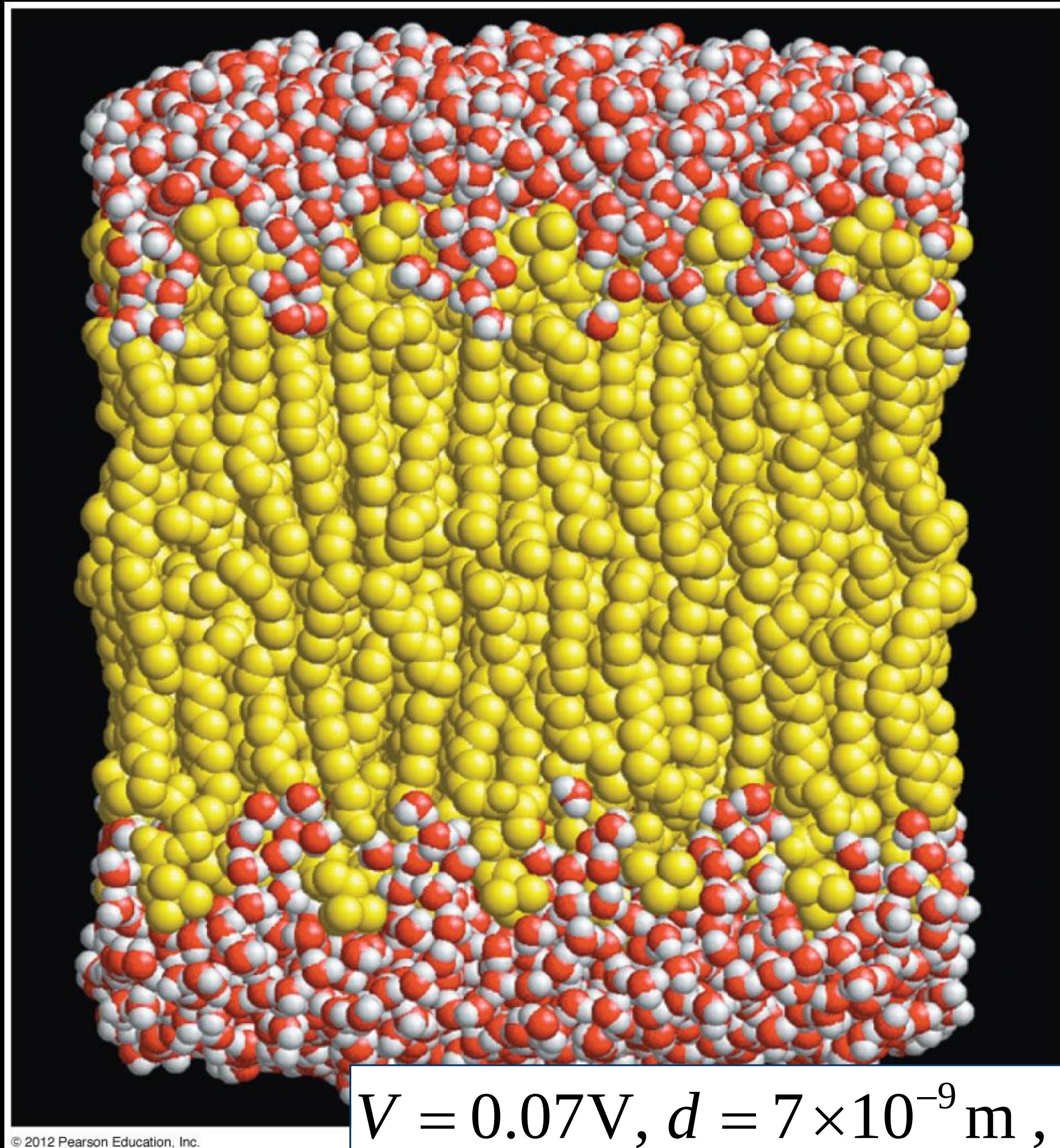
Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	

(Continued from the last slide)

Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

[a]Measured at room temperature, except for the water.

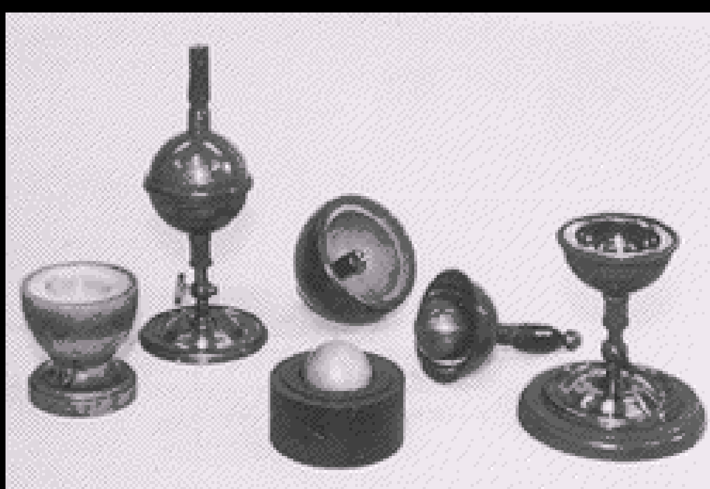


$V = 0.07\text{V}$, $d = 7\times10^{-9}\,\text{m}$, E=10^{7} V/m

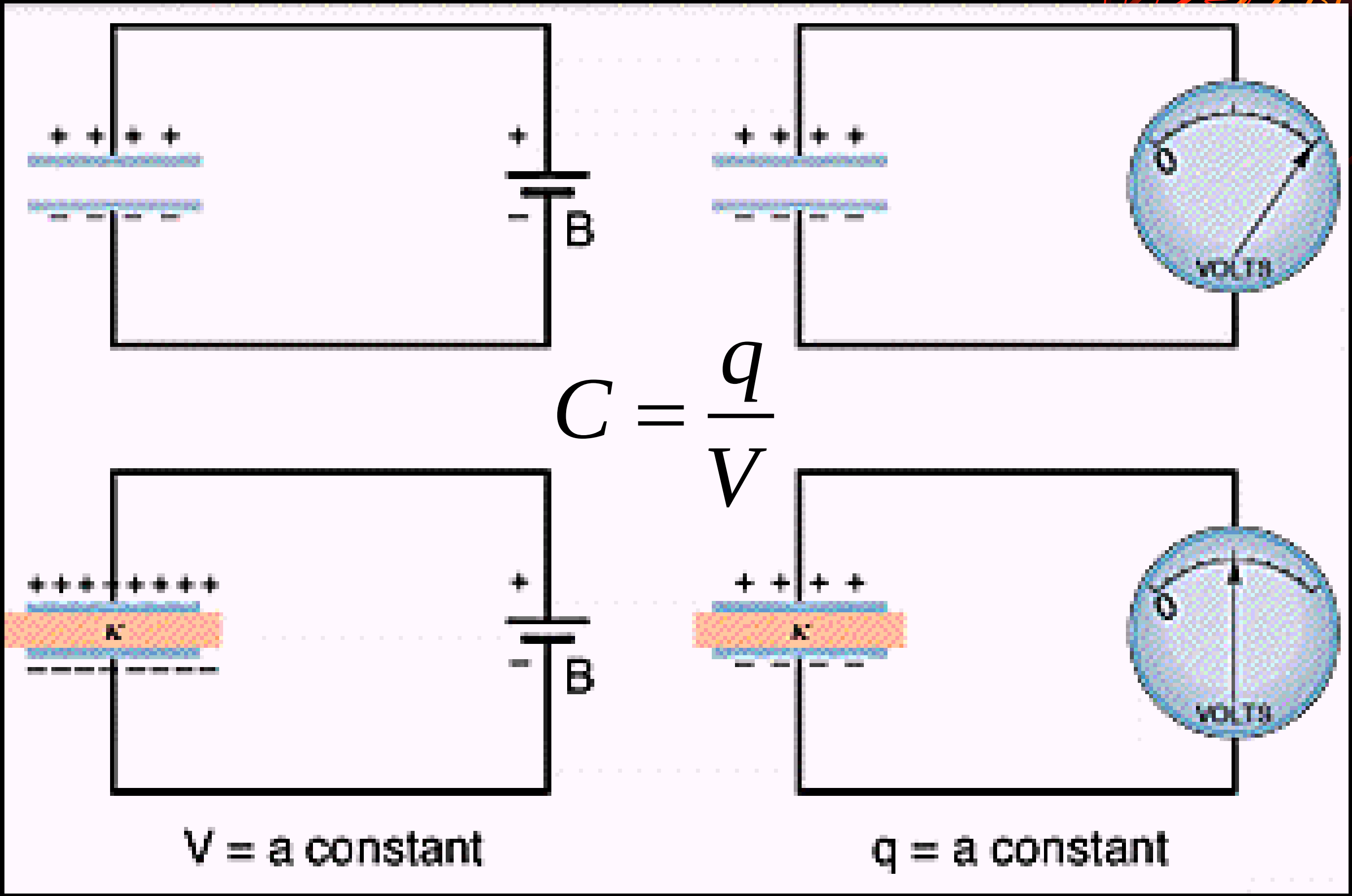
$$C=\frac{\varepsilon_0 A}{d},C=2\pi\varepsilon_0\frac{L}{\ln(b/a)},C=4\pi\varepsilon_0\frac{ab}{b-a}$$

$$C_{air}=\varepsilon_0 L,\quad C=\kappa C_{air}=\kappa\varepsilon_0 L$$

$$E=\frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2},\quad E=\frac{\sigma}{\kappa\varepsilon_0}$$



The effect of dielectrics



$$C = \frac{q}{V}$$

q increased

V reduced

Ex 5-8 A porcelain slab as the dielectric

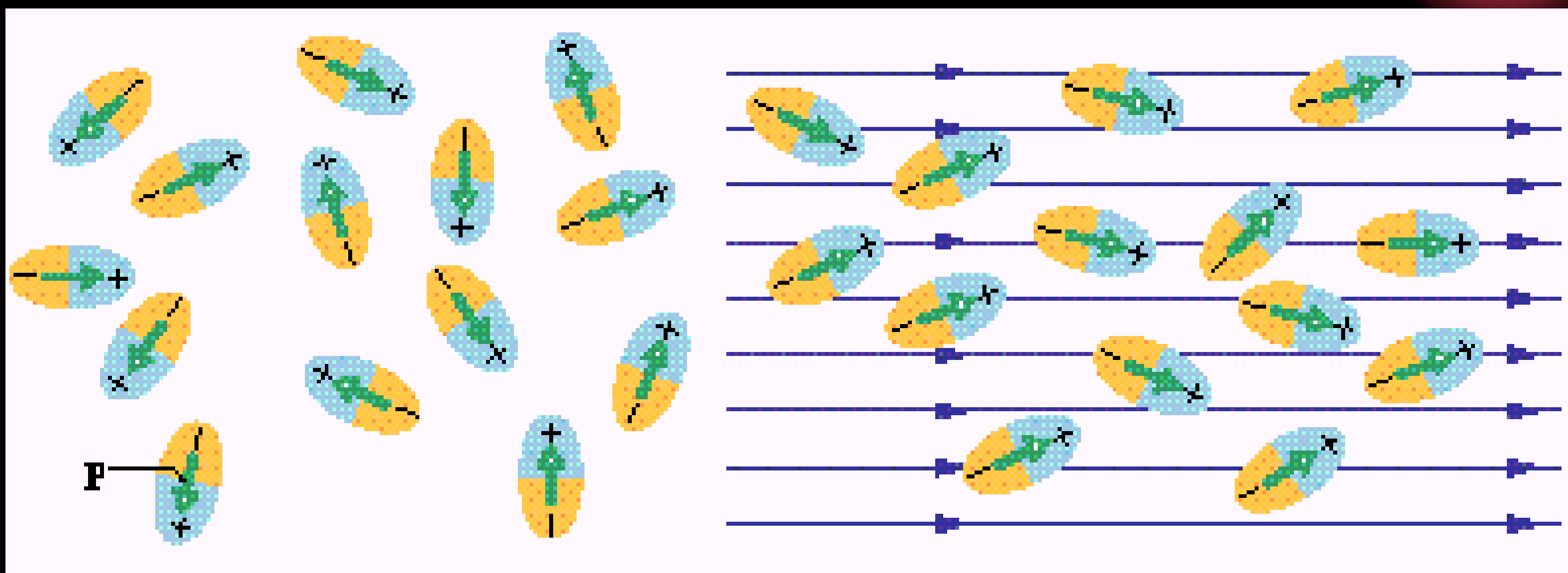
$$U_i = \frac{1}{2}CV^2 = 1055\,pJ = \frac{q^2}{2C}$$

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = 162\,pJ$$

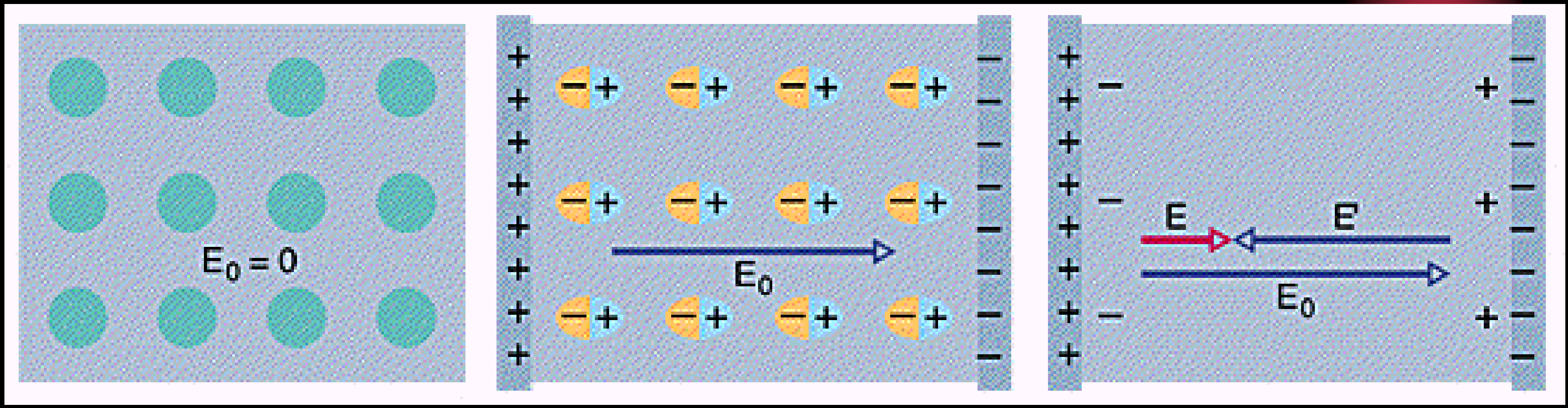
$$W = U_i - U_f = 893\,pJ$$

5-7 Dielectrics: An Atomic View

- Polar and Nonpolar Dielectrics



- The resultant electric field is weakened



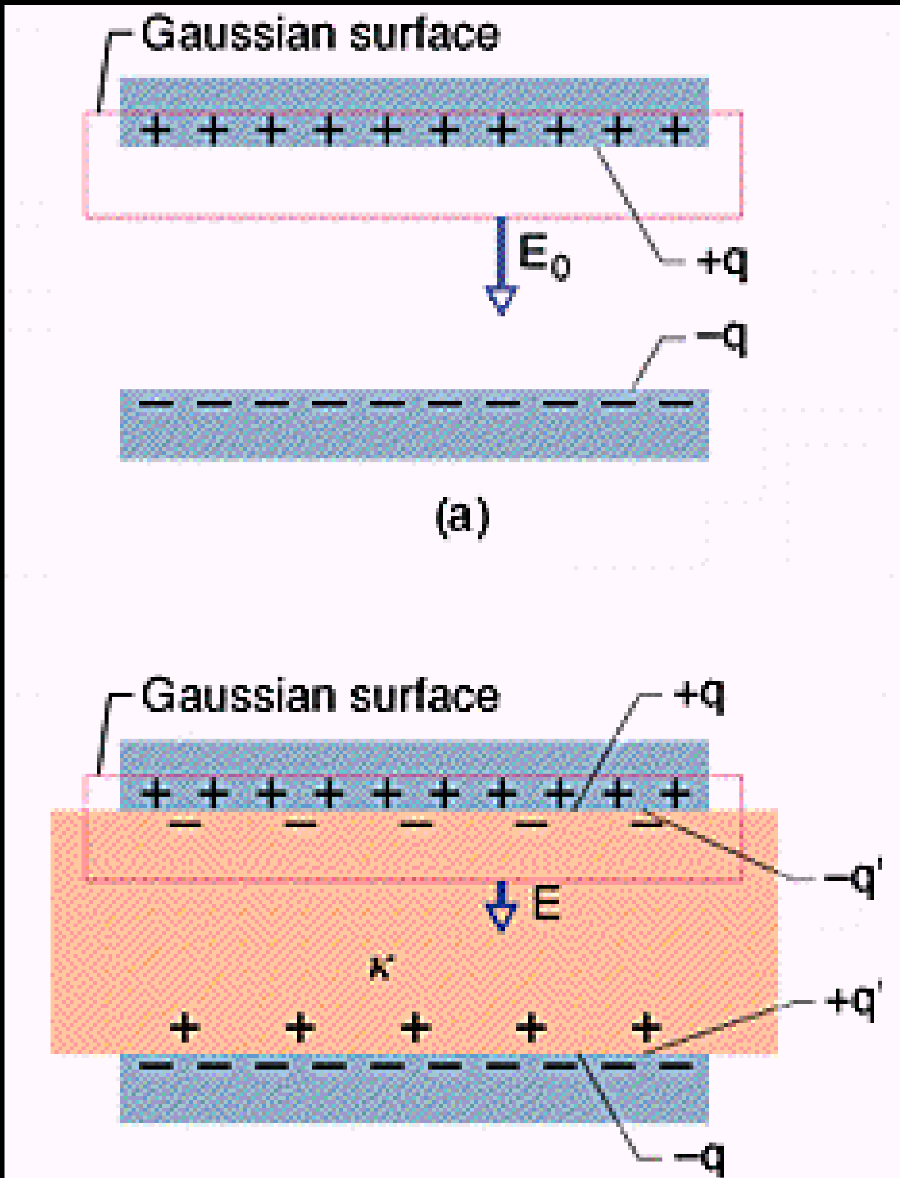
5-8 Dielectrics and Gauss' Law

Without and with Dielectrics

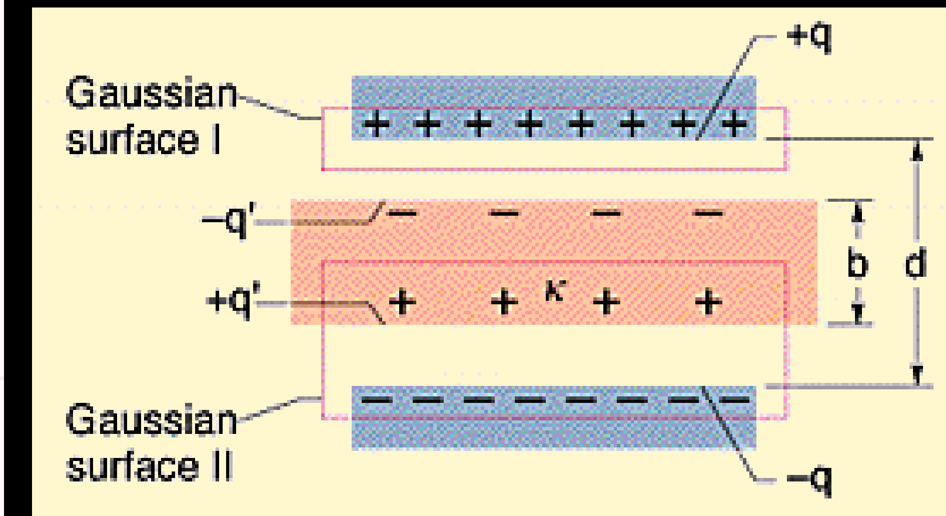
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E_0 A = q \rightarrow E_0 = \frac{q}{\varepsilon_0 A}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E A = q - q' \rightarrow E = \frac{q - q'}{\varepsilon_0 A}$$

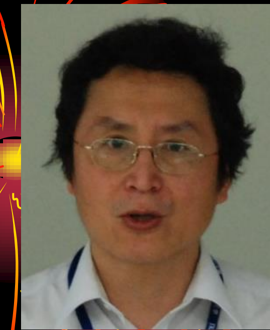
$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \varepsilon_0 A} \rightarrow q - q' = \frac{q}{\kappa}$$



Gaussian surface
+q
E_0
−q
(a)
Gaussian surface
+q
−q'
E
κ
+q'
−q



+q
Gaussian surface I
−q'
+q'
κ
b
d
Gaussian surface II
−q



Gauss' Law revised

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 EA = q - q' = \frac{q}{\kappa}$$

$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = q \quad \text{or} \quad \oint \vec{D} \cdot d\vec{A} = q$$

$\vec{D}$ the electric displacement