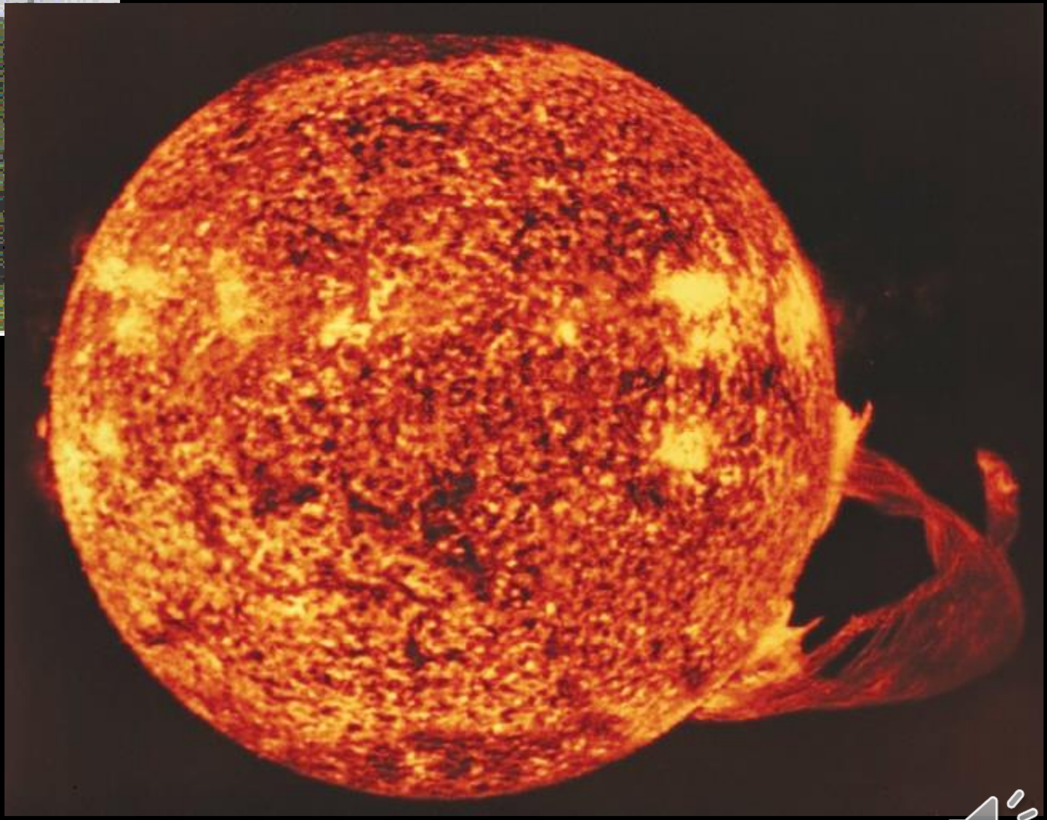
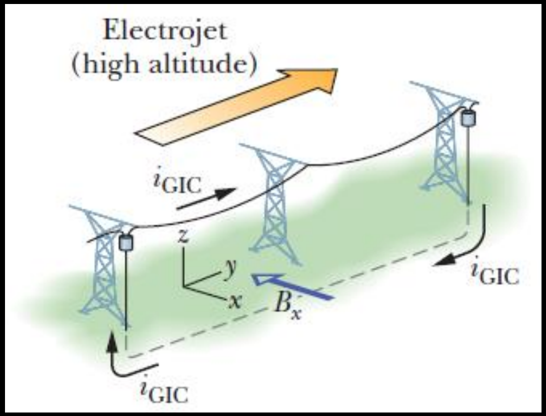
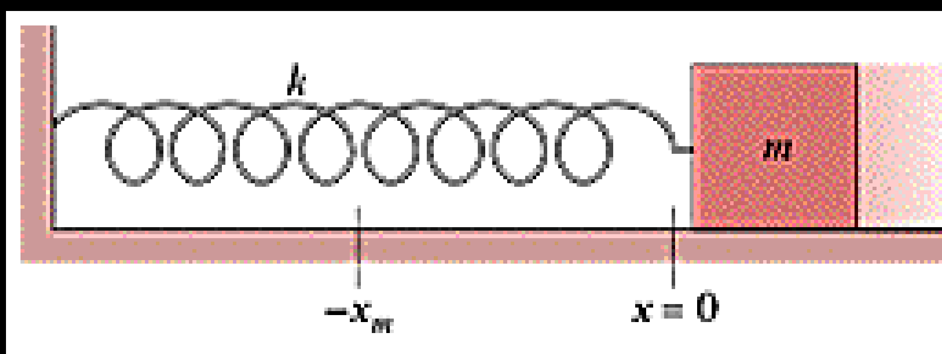
11 Electromagnetic Oscillations and Alternating Current

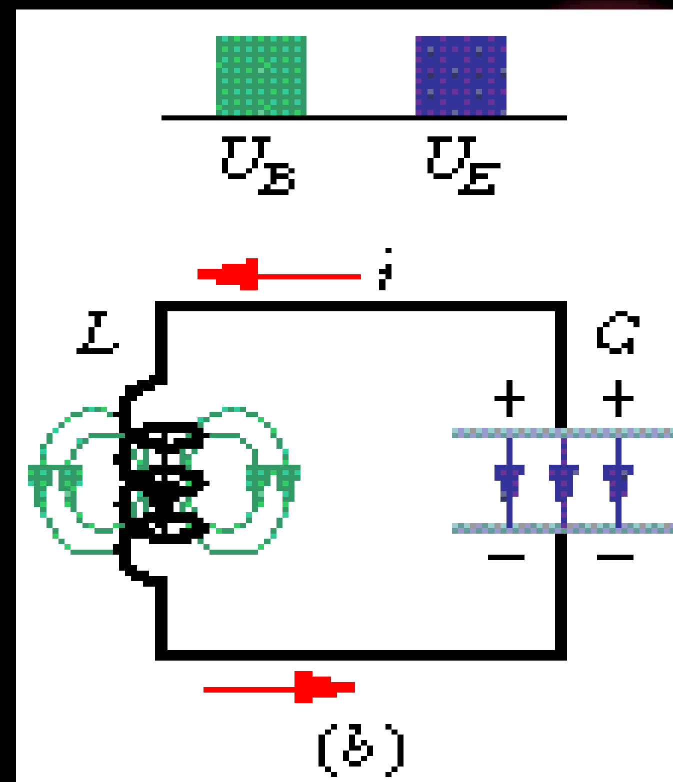




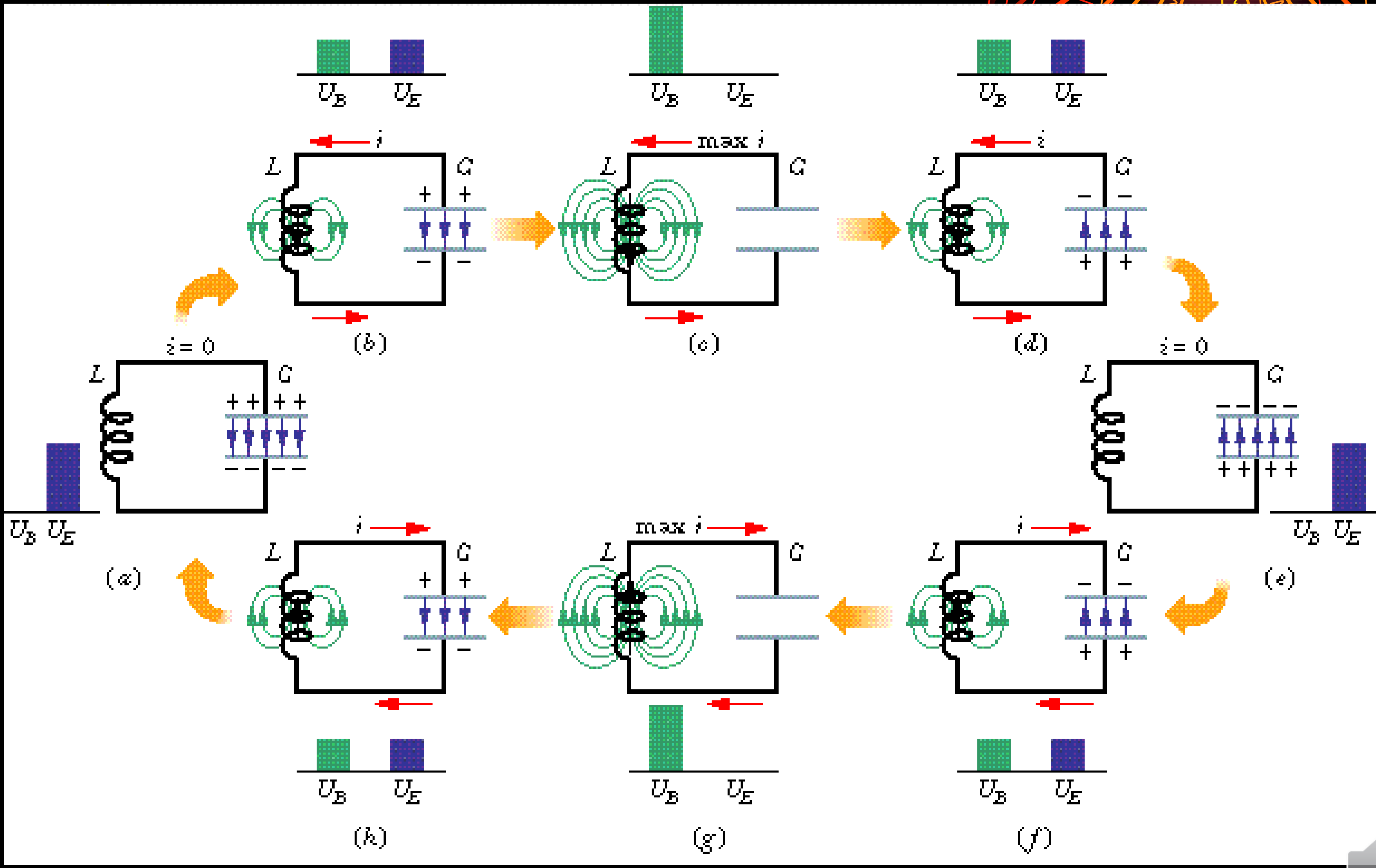
11-1 New Physics - Old Mathematics

- The spring-block system and the LC circuit





11-2 LC Oscillations, Qualitatively



11-3 The Electrical-Mechanical Analogy

$$U_P = \frac{kx^2}{2},\ U_K = \frac{mv^2}{2} \rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$U_E = \frac{q^2}{2C},\ U_B = \frac{Li^2}{2} \rightarrow \omega = \sqrt{\frac{1}{LC}}$$

11-4 LC Oscillations, Quantitatively
⇩ The Block - Spring Oscillator

$$U = U_b + U_s = \frac{mv^2}{2} + \frac{kx^2}{2}$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{mv^2}{2} + \frac{kx^2}{2}\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

$$m\frac{d^2x}{dt^2} + kx = 0 \qquad \omega = \sqrt{\frac{k}{m}}$$

$$x = X\cos(\omega t + \phi) \qquad \text{(displacement)}$$

⇩ The LC Oscillator

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad \omega = \sqrt{\frac{1}{LC}}$$

$$q = Q\cos(\omega t + \phi) \qquad (\text{charge})$$

- **Second-order Differential Equations**

$$L\frac{d^2q}{dt^2}+\frac{1}{C}q=0,\ \ q=Q\cos(\omega t+\phi)$$

- **First-order Differential Equations**

$$R\frac{dq}{dt}+\frac{1}{C}q=\mathcal{E},\ \ q=C\mathcal{E}(1-e^{-t/RC})$$

$$L\frac{di}{dt}+Ri=\mathcal{E},\ i=\frac{\mathcal{E}}{R}(1-e^{-Rt/L})$$

• By Substitution

$$q = Q\cos(\omega t + \phi) \qquad \text{(charge)}$$

$$i = \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi) \qquad \text{(current)}$$

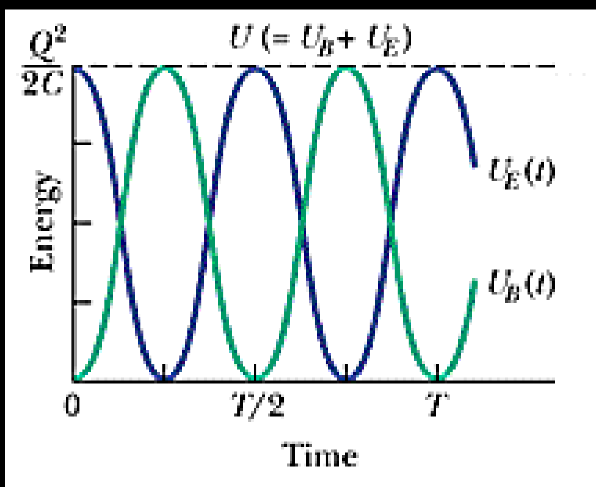
$$i = -I\sin(\omega t + \phi) \qquad I = \omega Q$$

$$\frac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t + \phi)$$

$$-L\omega^2 Q\cos(\omega t + \phi) + \frac{1}{C}Q\cos(\omega t + \phi) = 0$$

$$\rightarrow \omega = \sqrt{1/LC}$$

- # The Stored Electric and Magnetic Energy



$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{Li^2}{2} = \frac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi)$$

$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi)$$

11-5 Damped Oscillation in an RLC Circuit

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

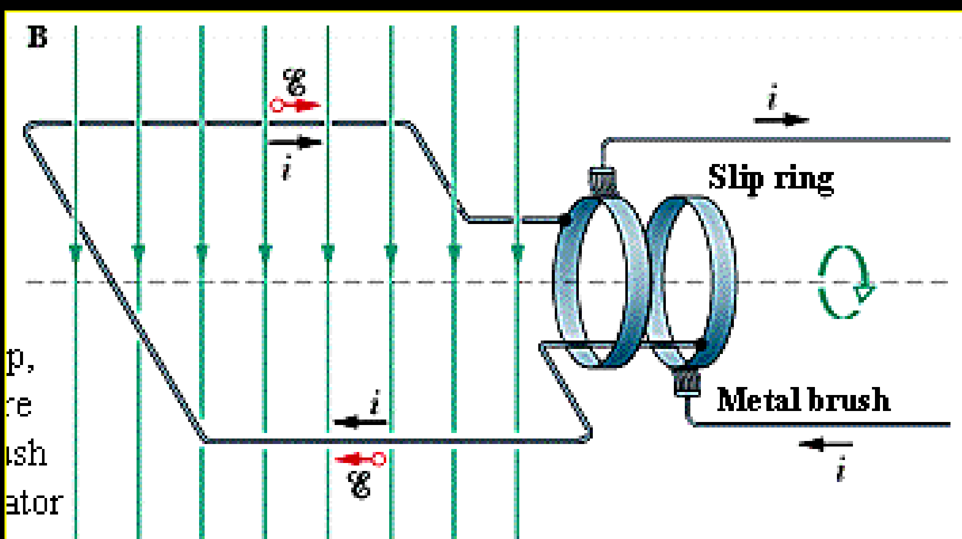
$$\frac{dU}{dt} = -i^2 R = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi) \qquad \omega' = \sqrt{\omega^2 - (R/2L)^2}$$

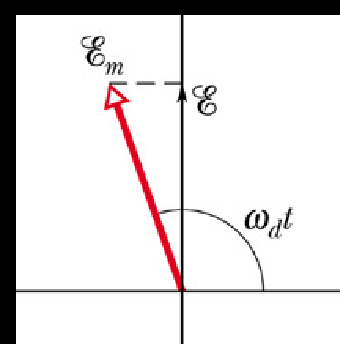
11-6 Alternating Current

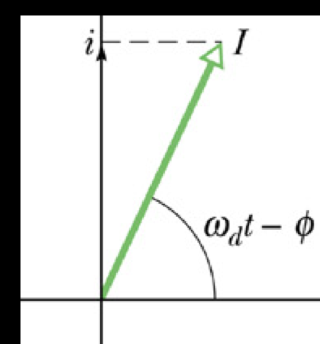
- A sinusoidally oscillating emf $\mathcal{E}$ and a sinusoidally oscillating current i



$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$

$$i = I \sin(\omega_d t - \phi)$$



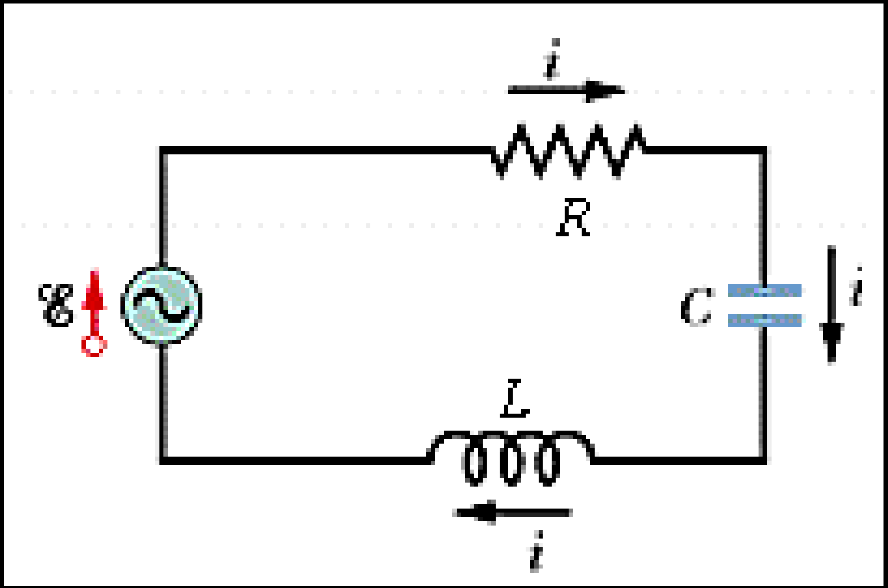


11-7 Forced Oscillations

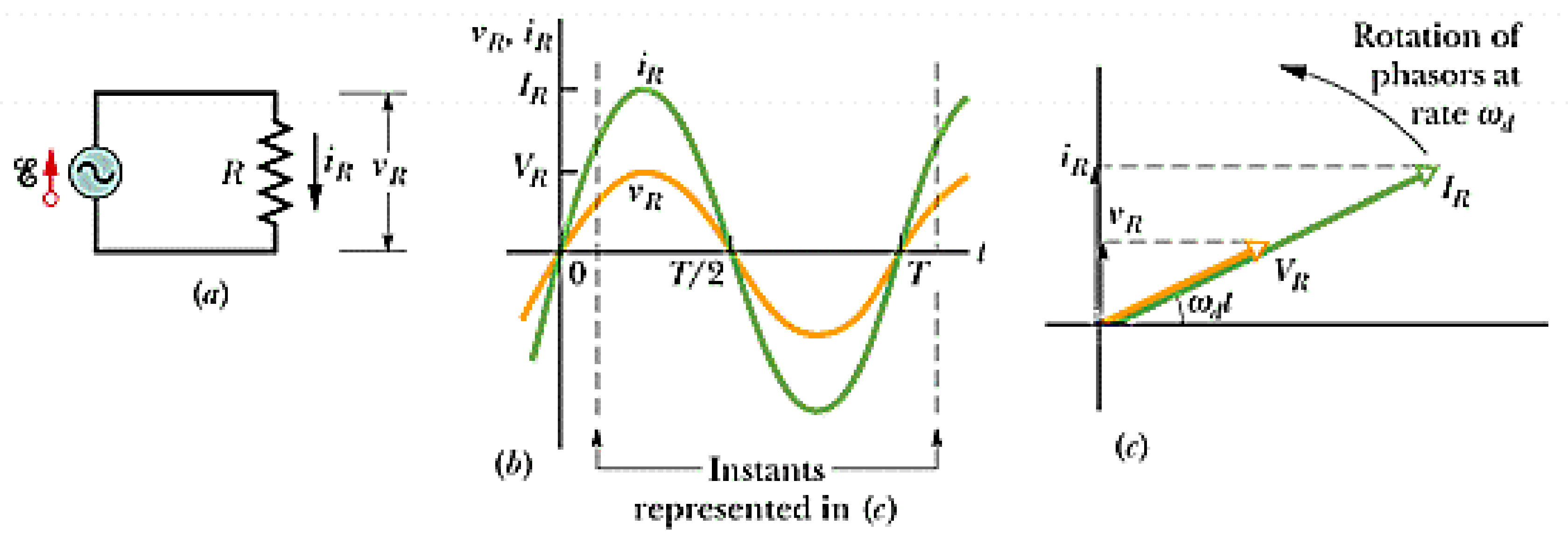
- ω natural angular frequency
- ω_d driving angular frequency
- Forced (driven) oscillations always occur at ω_d
- Resonance: when $\omega = \omega_d$,the amplitude I of the current is maximum

11-8 Three Simple Circuits

- A series RLC circuit



A Resistive Load



$$v_R = \mathcal{E}_m \sin \omega_d t = V_R \sin \omega_d t$$

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t = I_R \sin(\omega_d t - \phi)$$

$$v_R = \mathcal{E}_m \sin \omega_d t = V_R \sin \omega_d t$$

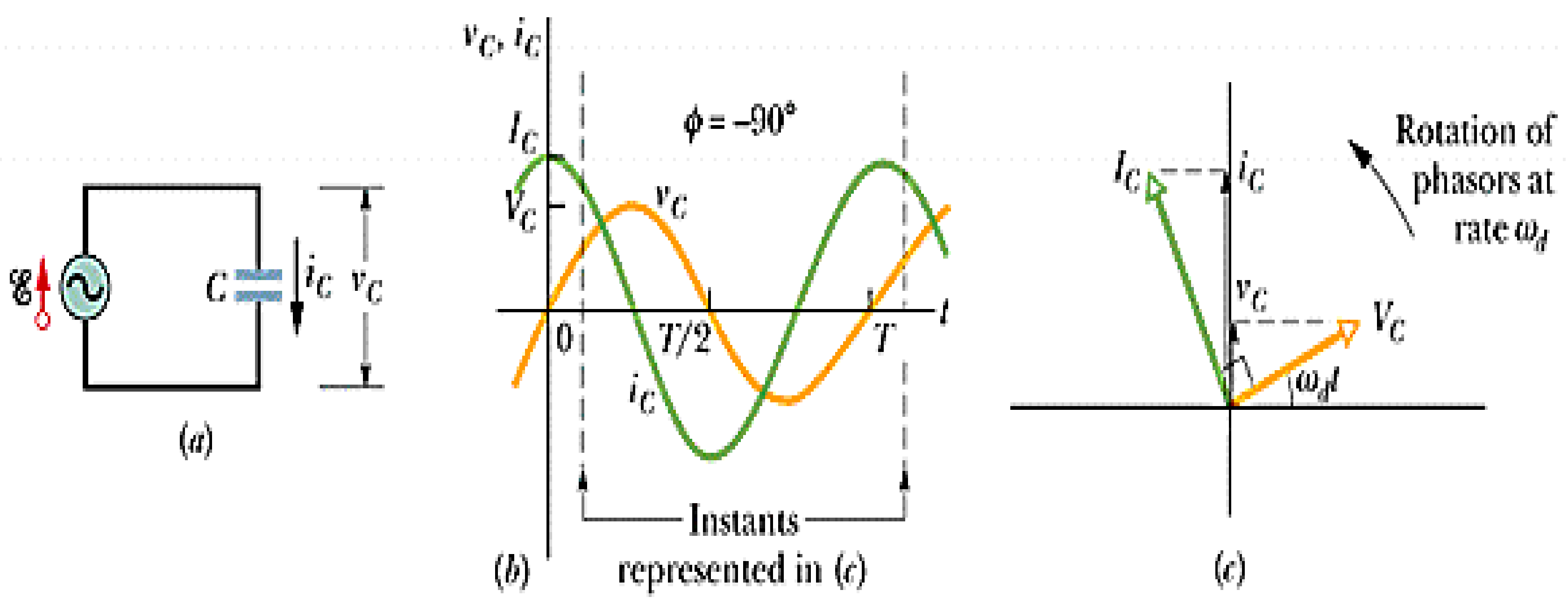
$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t = I_R \sin(\omega_d t - \phi)$$

$$\phi = 0^\circ \quad \text{for a purely resistive load}$$

$$V_R = I_R R$$

- Phasor - a vector rotating around a origin with ω_d

A capacitive load



$$v_C = V_C \sin \omega_d t \quad q_C = C v_C = C V_C \sin \omega_d t$$

$$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos \omega_d t$$

$$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos\omega_d t$$

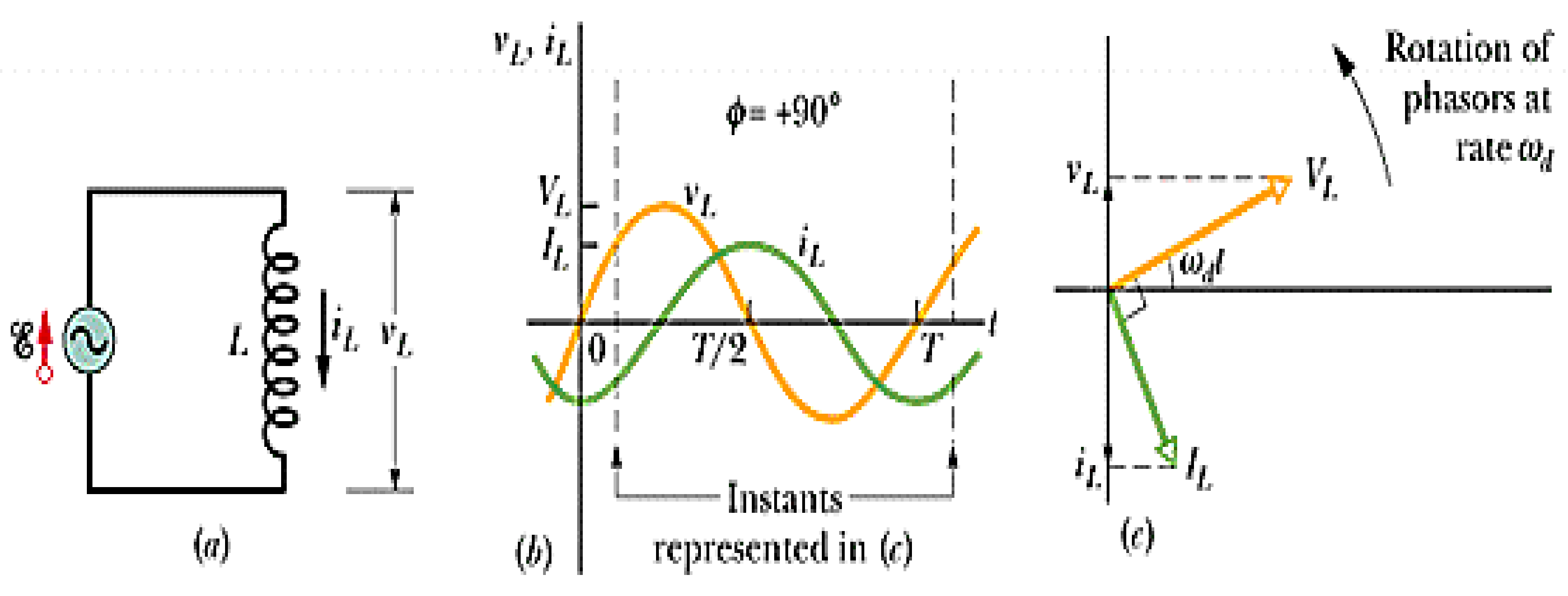
$$X_C \equiv \frac{1}{\omega_d C} \quad \text{(capacitive reactance)}$$

$$\cos\omega_d t = \sin(\omega_d t + 90^\circ)$$

$$i_C = \frac{V_C}{X_C}\sin(\omega_d t + 90^\circ) = I_C \sin(\omega_d t - \phi)$$

$$\phi = -90^\circ \qquad V_C = I_C X_C$$

An inductive load



$$v_L = V_L \sin \omega_d t \qquad v_L = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_L}{L}\sin \omega_d t$$

$$i_L = \int di_L = \frac{V_L}{L}\int \sin\omega_d t dt \quad = -(\frac{V_L}{\omega L})\cos\omega_d t$$

$$X_L \equiv \omega_d L \quad \text{(inductive reactance)}$$

$$-\cos\omega_d t = \sin(\omega_d t - 90^\circ)$$

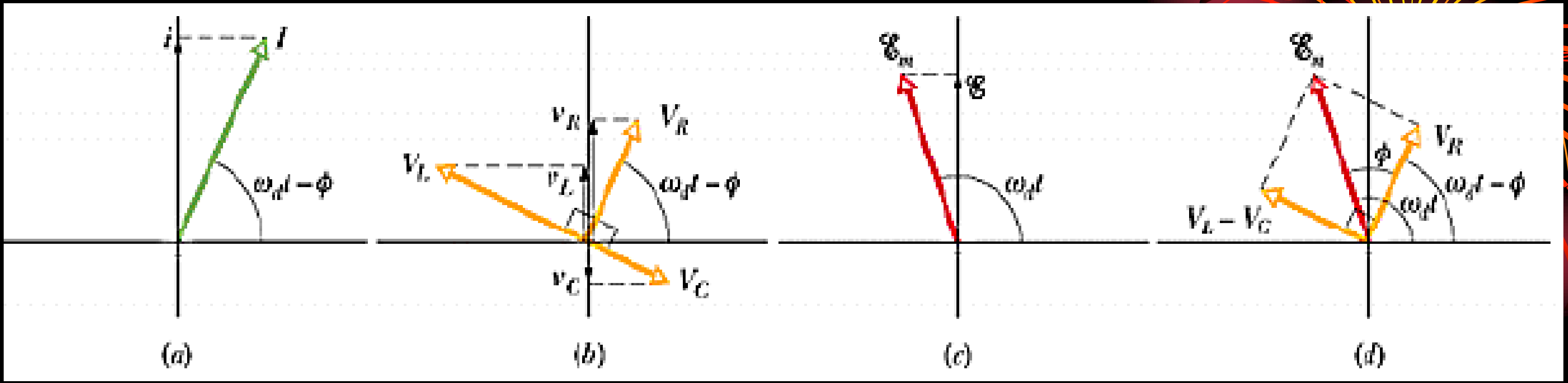
$$i_L = \frac{V_L}{X_L}\sin(\omega_d t - 90^\circ) = I_L \sin(\omega_d t - \phi)$$

$$\phi = +90^\circ \qquad V_L = I_L X_L$$

11-9 The Series RLC Circuits

TABLE 33-2 PHASE AND AMPLITUDE RELATIONS FOR ALTERNATING CURRENTS AND VOLTAGES

CIRCUIT ELEMENT	SYMBOL	RESISTANCE OR REACTANCE	PHASE OF THE CURRENT	PHASE ANGLE ϕ	AMPLITUDE RELATION
Resistor	R	R	In phase with v_R	0	$V_R = I_R R$
Capacitor	C	$X_C = 1/\omega_d C$	Leads v_C y 90	- 90	$V_C = I_C X_C$
Inductor	L	$X_L = \omega_d L$	Lags v_L y 90	+ 90	$V_L = I_L X_L$



$$\mathcal{E} = v_R + v_C + v_L$$

$$\mathcal{E}_m^2 = V_R^2 + (V_L - V_C)^2$$

$$= (IR)^2 + (IX_L - IX_C)^2$$

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} \equiv \frac{\mathcal{E}_m}{Z}$$

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} \equiv \frac{\mathcal{E}_m}{Z}$$

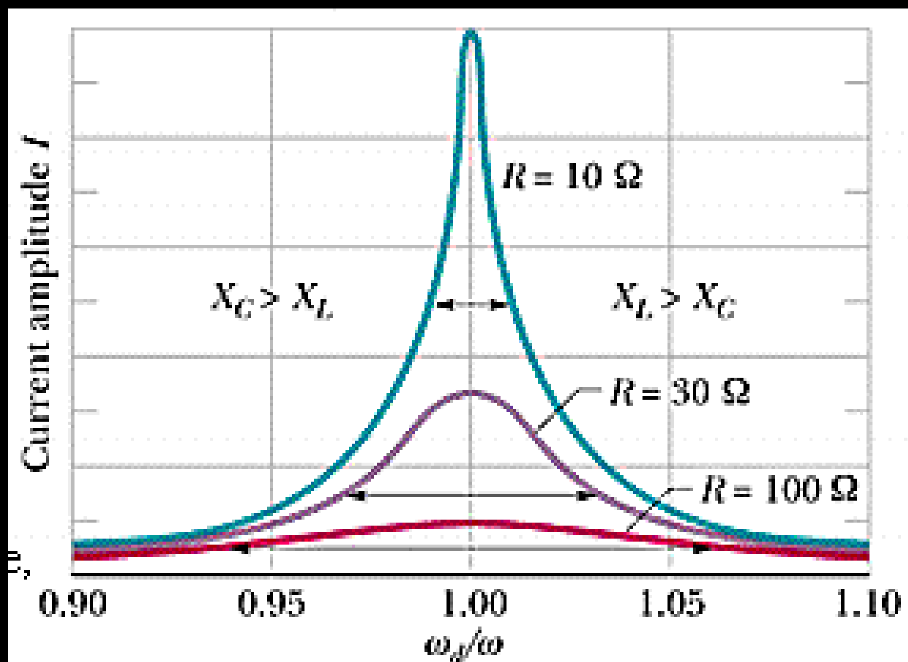
$$Z = \text{impedance}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

- $X_L > X_C$: inductive load
- $X_L < X_C$: capacitive load
- $X_L = X_C$: resonance

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

$$X_L = X_C \rightarrow \omega_d L = \frac{1}{\omega_d C} \rightarrow \omega_d = \frac{1}{\sqrt{LC}}$$

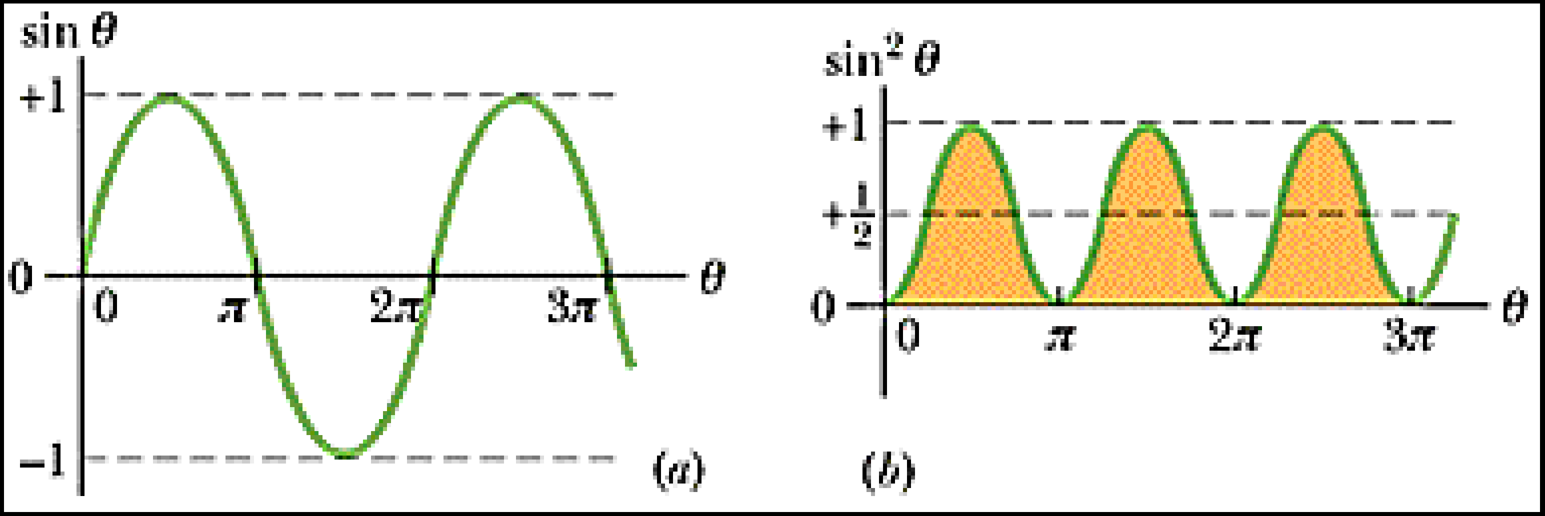


11-10 Power in Alterning Circuit

$$P = i^2 R = [I\sin(\omega_d t - \phi)]^2 R$$

$$= I^2 R \sin^2(\omega_d t - \phi)$$

$$P_{av} = I^2 R/2 = (I/\sqrt{2})^2 R \equiv I_{rms}^2 R$$



The power factor cosφ

$$P_{av} = I_{rms}^2 R \quad I_{rms} = \mathcal{E}_{rms} / Z$$

$$P_{av} = (\mathcal{E}_{rms} / Z) \underline{I_{rms} R} = \mathcal{E}_{rms} I_{rms} R / Z$$

$$R / Z = IR / IZ = V_R / \mathcal{E}_m = \cos\phi$$

$$\rightarrow P_{av} = \mathcal{E}_{rms} I_{rms} \cos\phi \quad \text{(power factor)}$$

11-11 Transformers

- A practical example - Quebec →Montreal

$$P_{av} = \mathcal{E}I$$

$$= (7.35\times10^5\,\mathrm{V})(500\mathrm{A}) = 368\mathrm{MW}$$

$$P_{av,1} = I^2R = (500\mathrm{A})^2$$

$$\times(0.22\Omega/m\times1000km) = 55.0\mathrm{MW}$$

$$P_{av,2} = I^2R = (1000\mathrm{A})^2$$

$$\times(0.22\Omega/m\times1000km) = 220\mathrm{MW}$$

The transformer

$$\mathcal{E}_{turn} = d\Phi_B / dt = V_P / N_P = V_S / N_S$$

$$\rightarrow V_S = V_P (N_S / N_P)$$

$$I_S = I_P (N_P / N_S) \qquad (I_P V_P = I_S V_S)$$

$$I_S = V_S / R = V_P (N_S / N_P) / R$$

$$\rightarrow I_P = V_P (N_S / N_P)^2 / R$$

$$\rightarrow R_{eq} = (N_P / N_S)^2 R$$