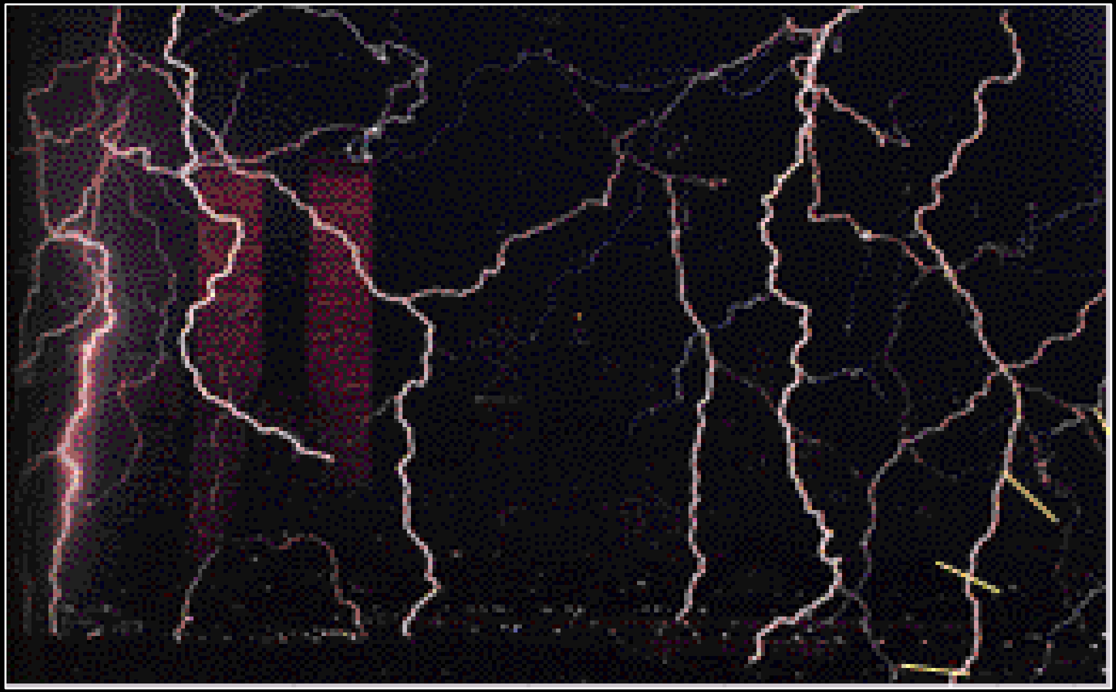
3 Gauss' Law 高斯定律



How wide is a lightning strike ?

Contents

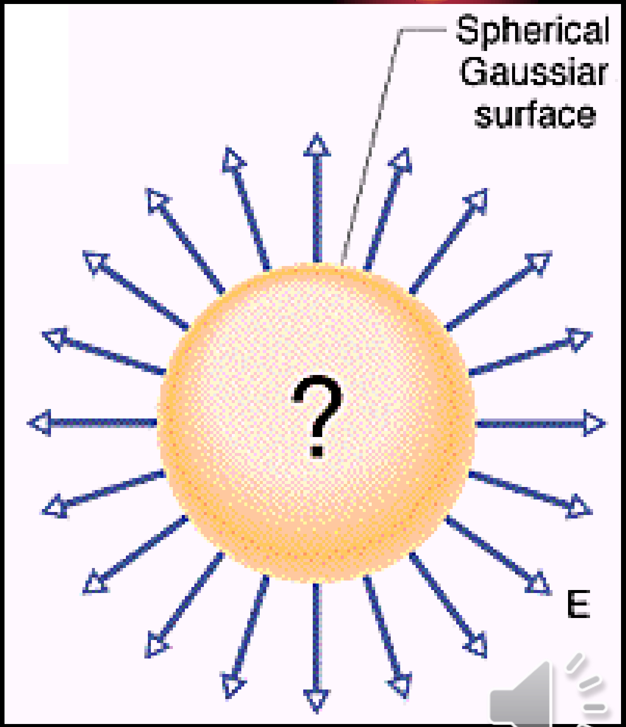
Contents

- 3-7 Applying Gauss' Law: Cylindrical Symmetry
- 3-8 Applying Gauss' Law: Planar Symmetry
- 3-9 Applying Gauss' Law: Spherical Symmetry
- 3-10 Gauss' Law in Differential Form
- 3-11 Vector Calculus

3-1 A New Look at Coulomb's Law

- Using Gauss's law to take advantage of special symmetry situations
- Gaussian surfaces
- 高斯面上各點電場與面內總電荷相關

$$\varepsilon_0 \Phi = q_{enc}$$

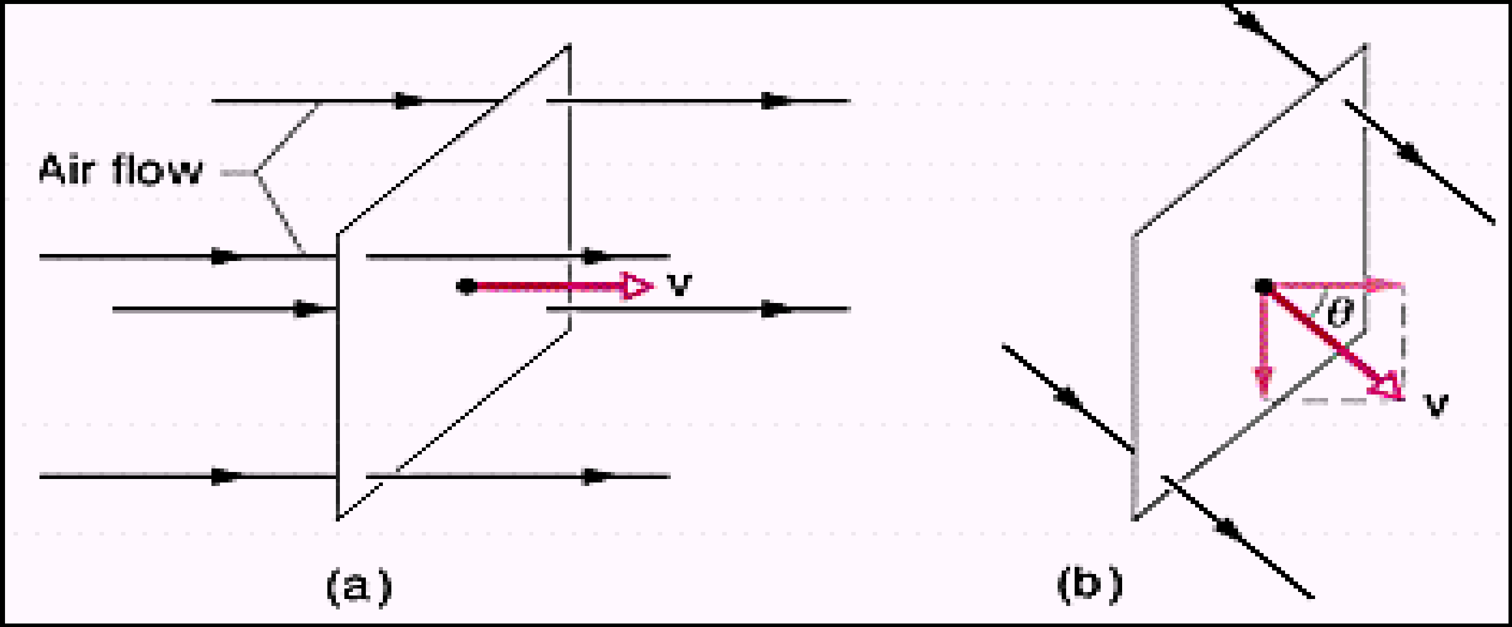


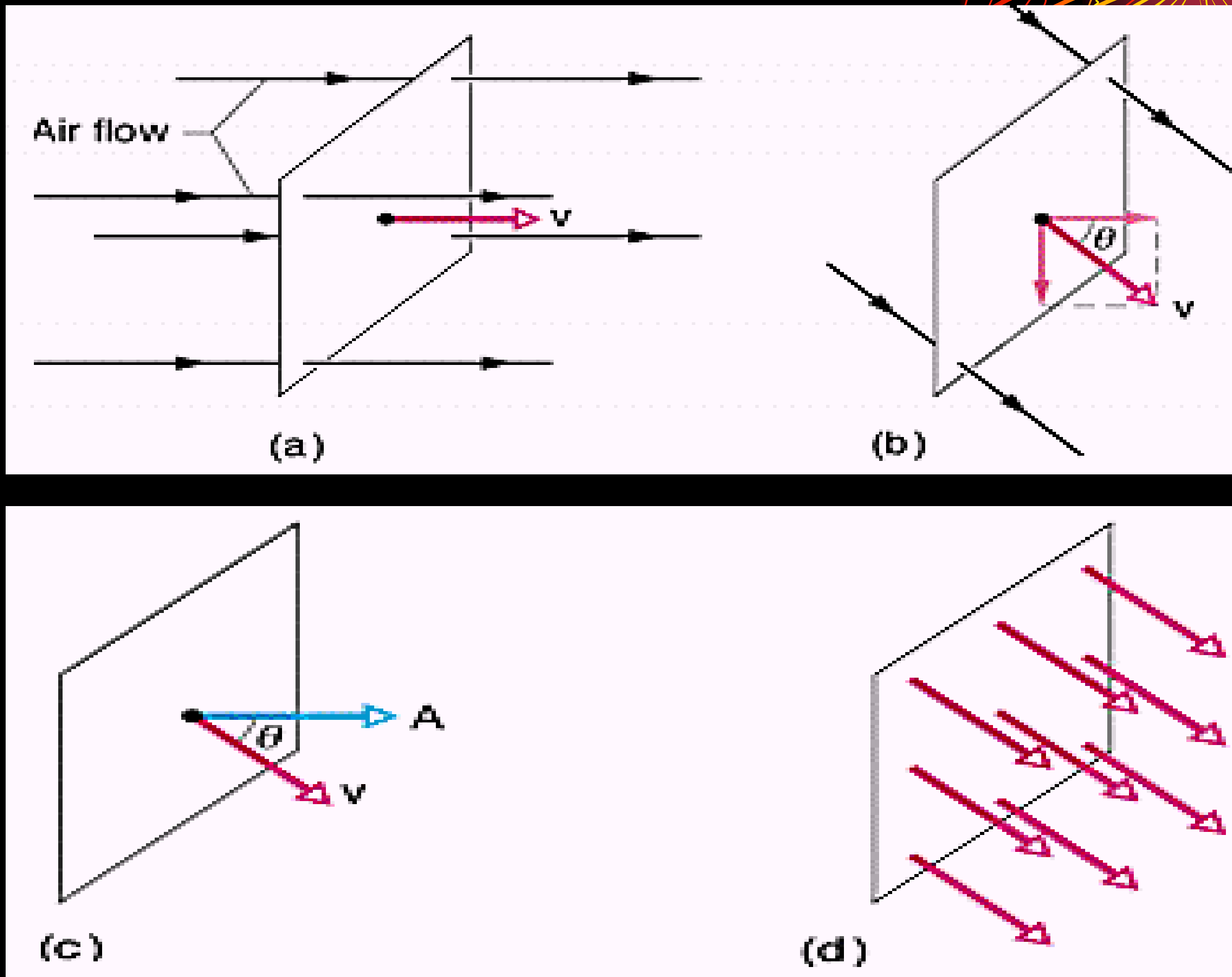
3-2 Flux (通量/流量)

- For a fluid

$m/s \times m^2 = m^3/s$

$$\Phi = (v\cos\theta)A = vA\cos\theta = \vec{v}\cdot\vec{A}$$




Air flow
v
(a)
θ
v
(b)
A
θ
v
(c)
(d)

Basking shark 姥鯊

3-3 Flux of an Electric Field

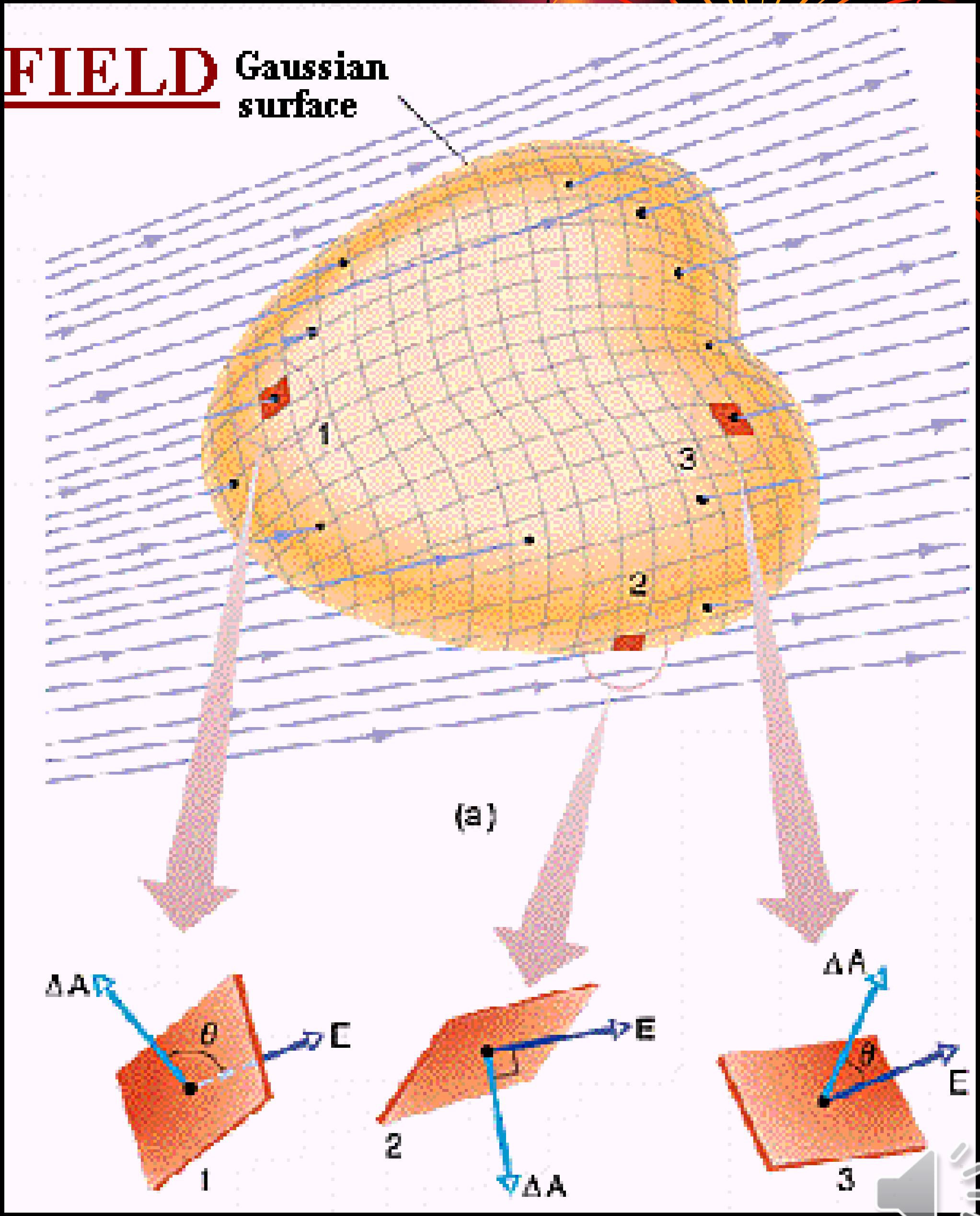
- For a flat surface

$$\Phi = (E\cos\theta)A = EA\cos\theta = \vec{E}\cdot\vec{A}$$

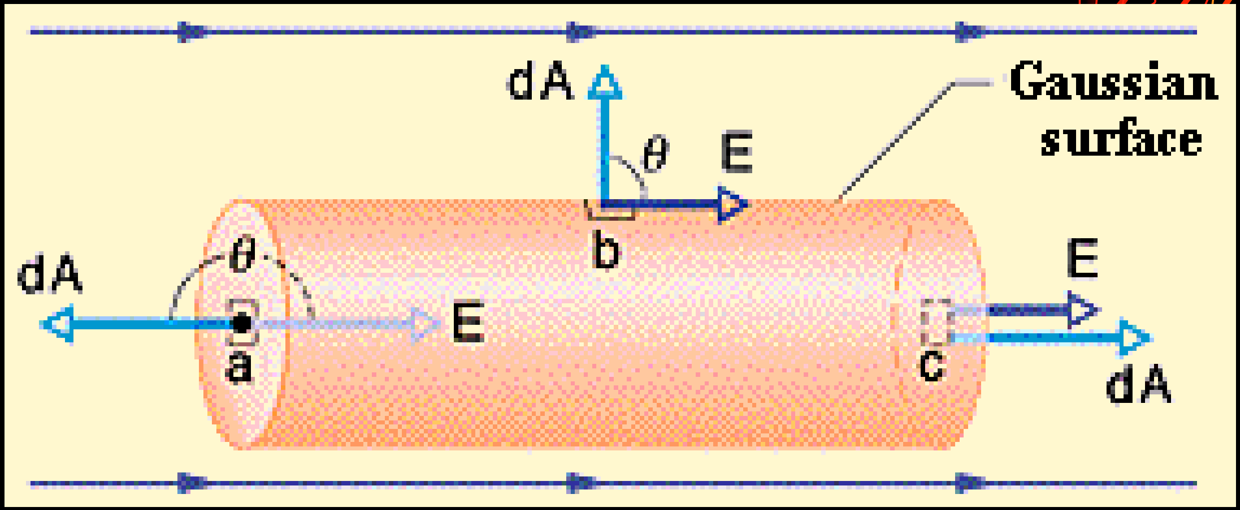
- For a arbitrary (asymmetric) surface

$$\Phi = \sum \vec{E}\cdot\Delta\vec{A} \rightarrow \oint \vec{E}\cdot d\vec{A}$$

- *An arbitrary Gaussian surface*



Ex.3-1 A cylindrical Gaussian surface



$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}$$

$$= -EA + 0 + EA = 0$$

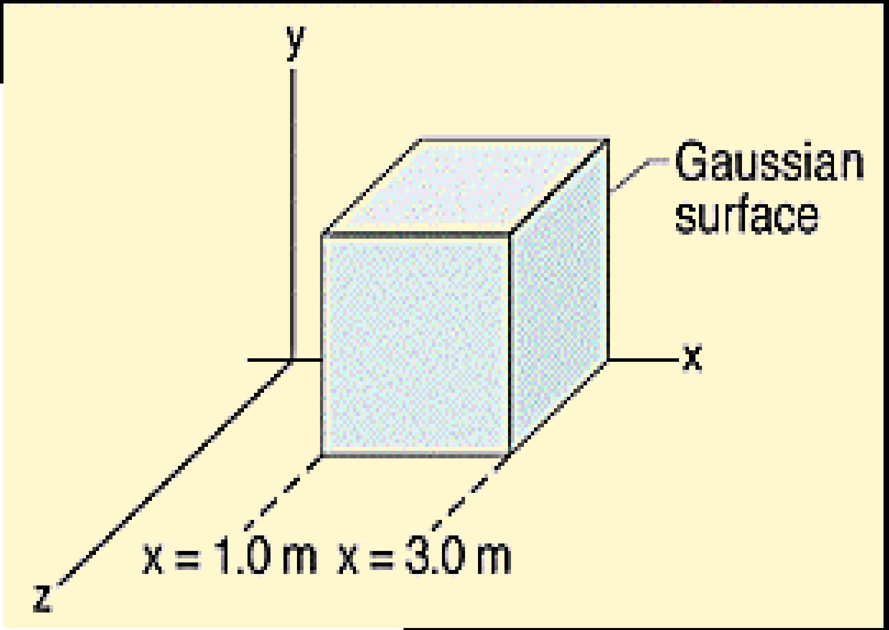
Ex.3-2 A nonuniform electric field and a Gaussian cube

$$\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$$

$$d\vec{A} = dA\hat{i}$$

$$\Phi_r = \int \vec{E} \cdot d\vec{A}$$

$$= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$



Ex.3-2 right face

$$\Phi_r = \int \vec{E} \cdot d\vec{A}$$

$$= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$

$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i})$$

$$= \int (3.0xdA + 0) = 3.0\int xdA$$

$$= 3.0\int 3.0dA = 9.0\int dA = 36N \cdot m^2 / C$$

Ex.3-2 left and top faces

left face: $d\vec{A} = -dA\hat{i}$

$$\Phi_l = 3.0\int 1.0dA = 3.0\int dA = -12N \cdot m^2 / C$$

$$\Phi_t = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j})$$

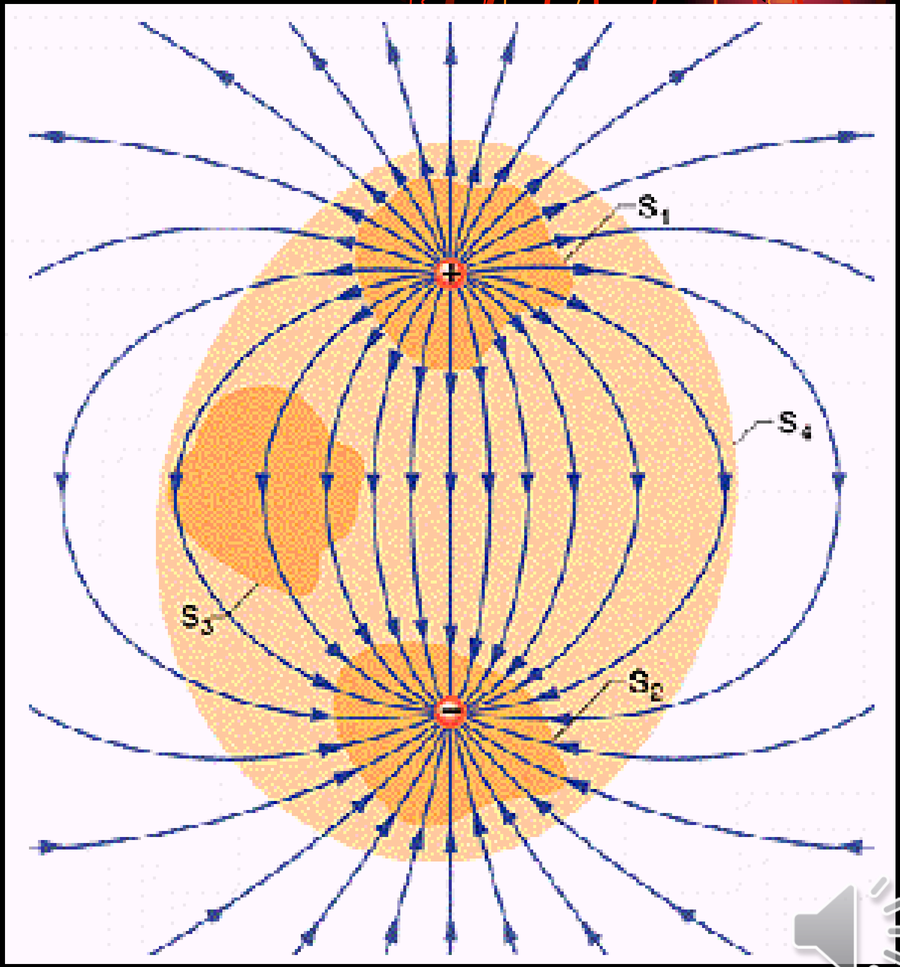
$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{j} + (4.0)(dA)(\hat{j} \cdot \hat{j})]$$

$$= \int (0 + 4.0dA) = 4.0\int dA = 16N \cdot m^2 / C$$

3-4 Gauss' Law

- Flux ↔ enclosed charge

$$\varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$



Ex.3-3 bottom, front and back

$$\Phi_b = -16 N \cdot m^2 / C,\ \Phi_f = \Phi_b = 0$$

$$\Phi_t = 24 N \cdot m^2 / C$$

$$q_{enc} = \varepsilon_0 \Phi_t$$

$$= (8.85 \times 10^{-12} C^2 / N \cdot m^2)(24 N \cdot m^2 / C)$$

$$= 2.1 \times 10^{-10} C$$

3-5 Gauss' Law and Coulomb' Law

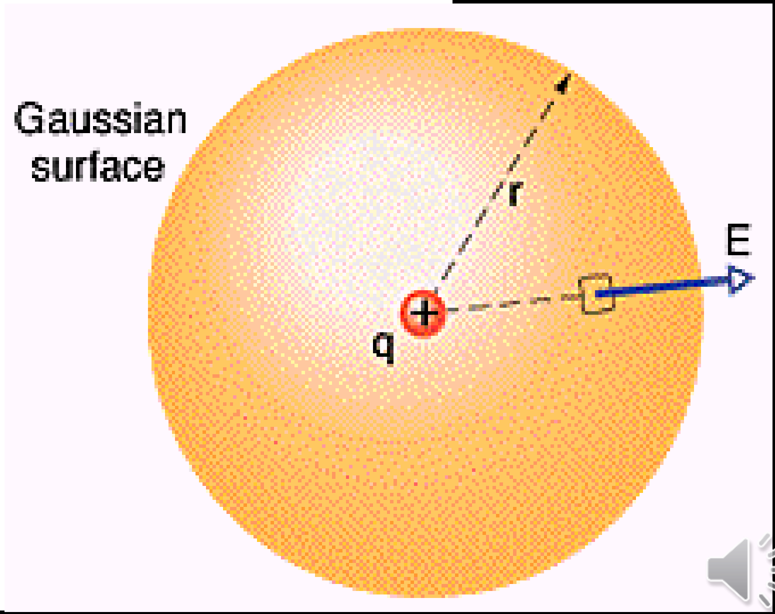
- From G.L. to C.L.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E dA = q$$

$$\varepsilon_0 E \oint dA = q$$

$$\varepsilon_0 E (4\pi r^2) = q$$

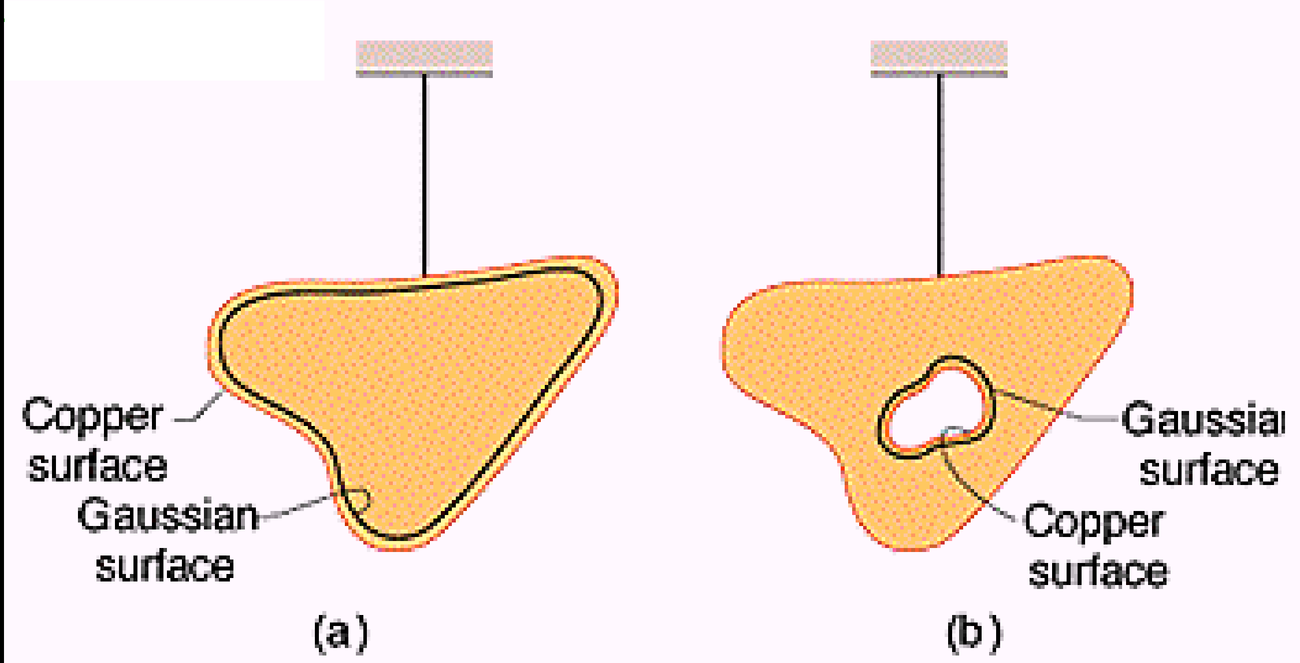
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$



3-6 A Charged Isolated Conductor

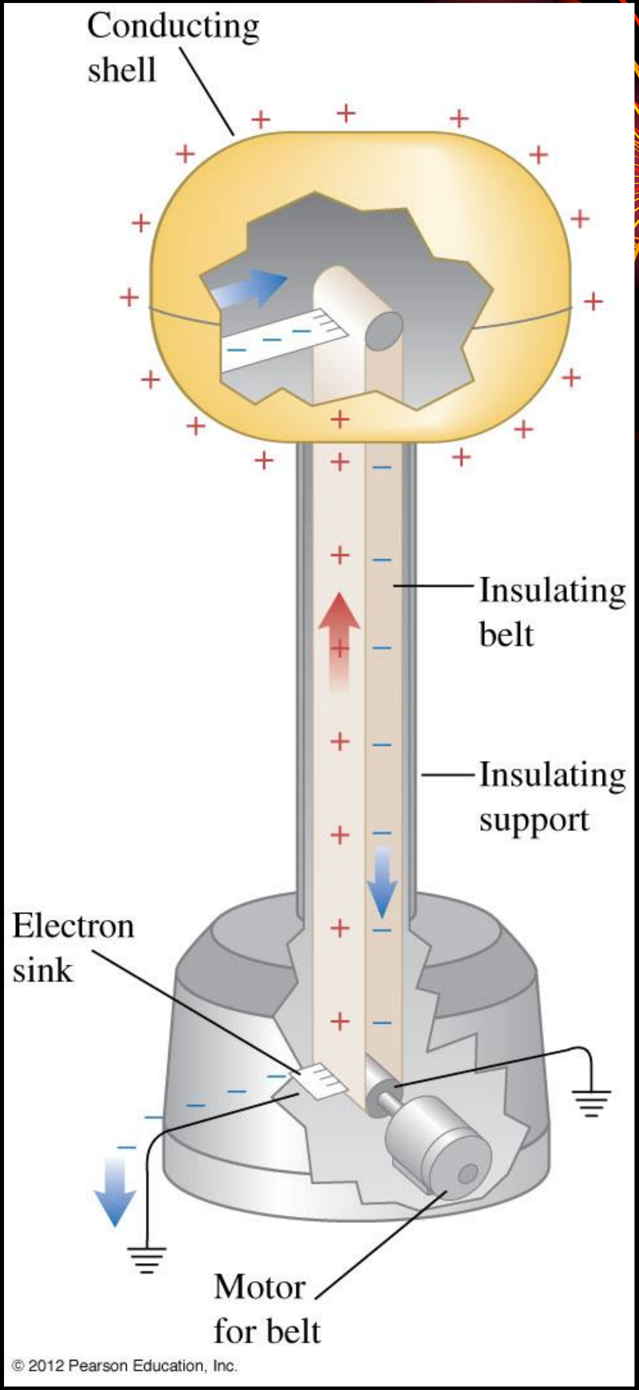
- 同號電荷相斥
- 導體內部無電場

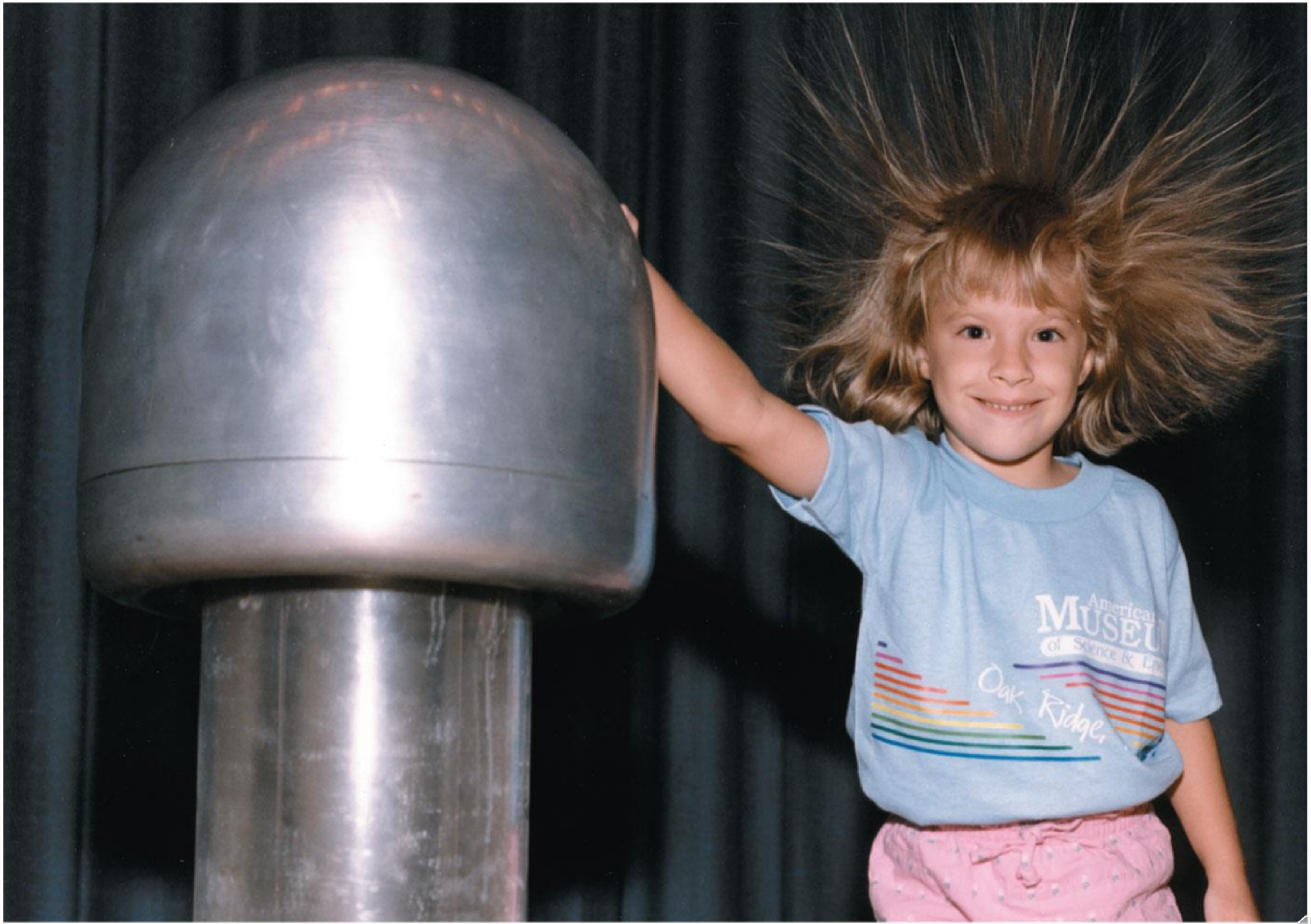
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$



Charge distribution inside a human nerve cell

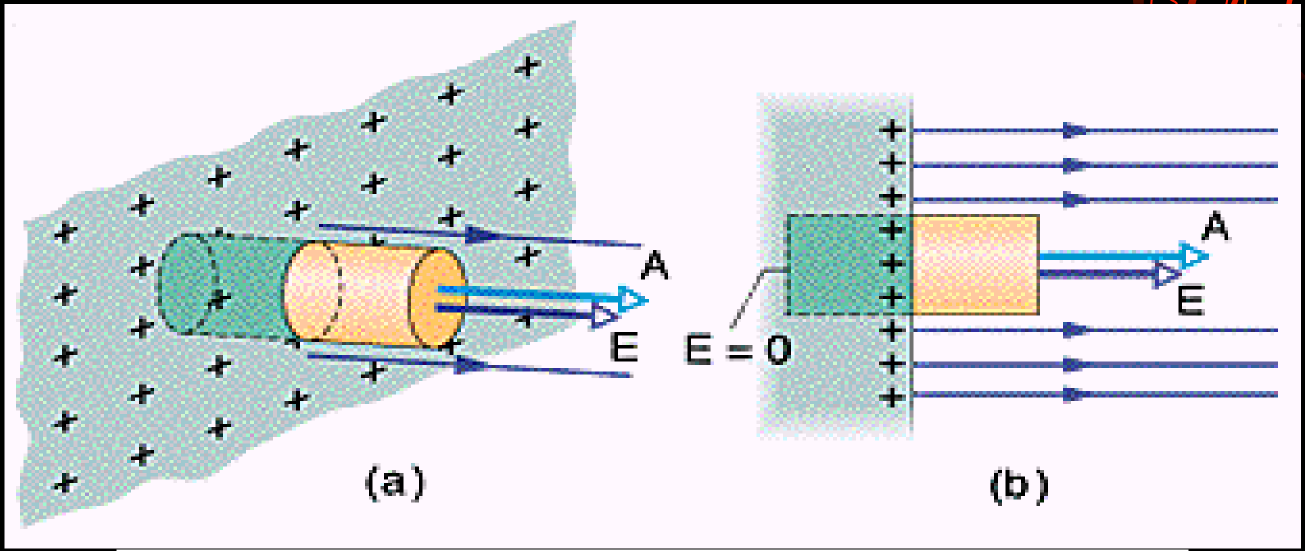
Van de Graaff electrostatic generator




American
MUSEU
of Science & En
Oak Ridge,

ES shielding - Faraday Cage

The external electric field



$$q_{enc} = \sigma A, \Phi = EA$$

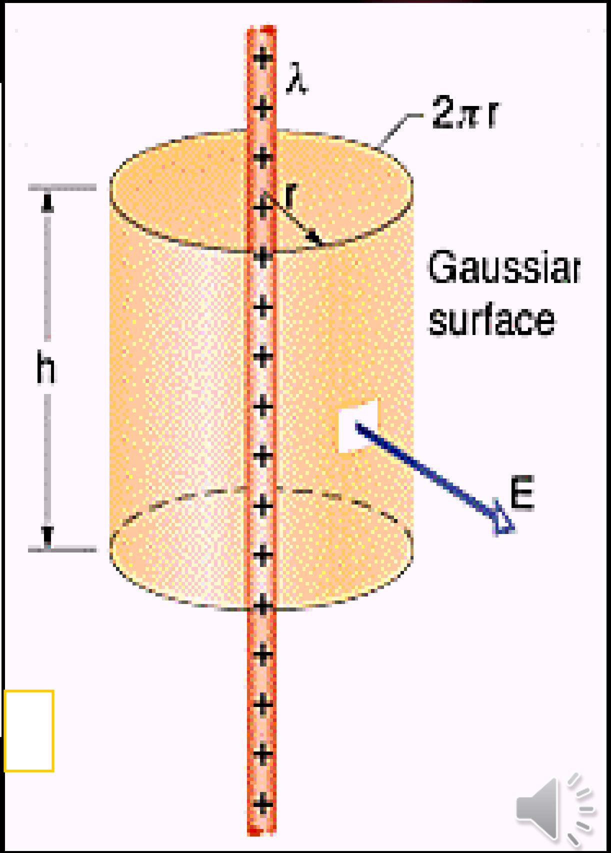
$$\varepsilon_0 EA = \sigma A \rightarrow E = \frac{\sigma}{\varepsilon_0}$$

3-7 Applying Gauss' Law: Cylindrical Symmetry

$$q_{enc} = \lambda h,\ A = 2\pi r h$$

$$\varepsilon_0 E 2\pi r h = \lambda h$$

$$\rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



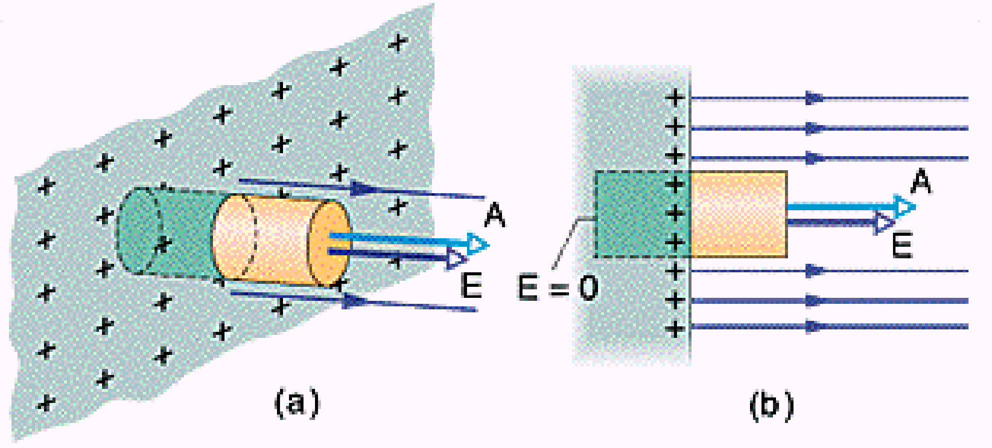
Ex.3-4 A lightning strike

$$r = \frac{\lambda}{2\pi\varepsilon_0 E}$$

$$= \frac{1\times10^{-3}}{2\pi\varepsilon_0(3\times10^{6})}$$

$$= 6m$$

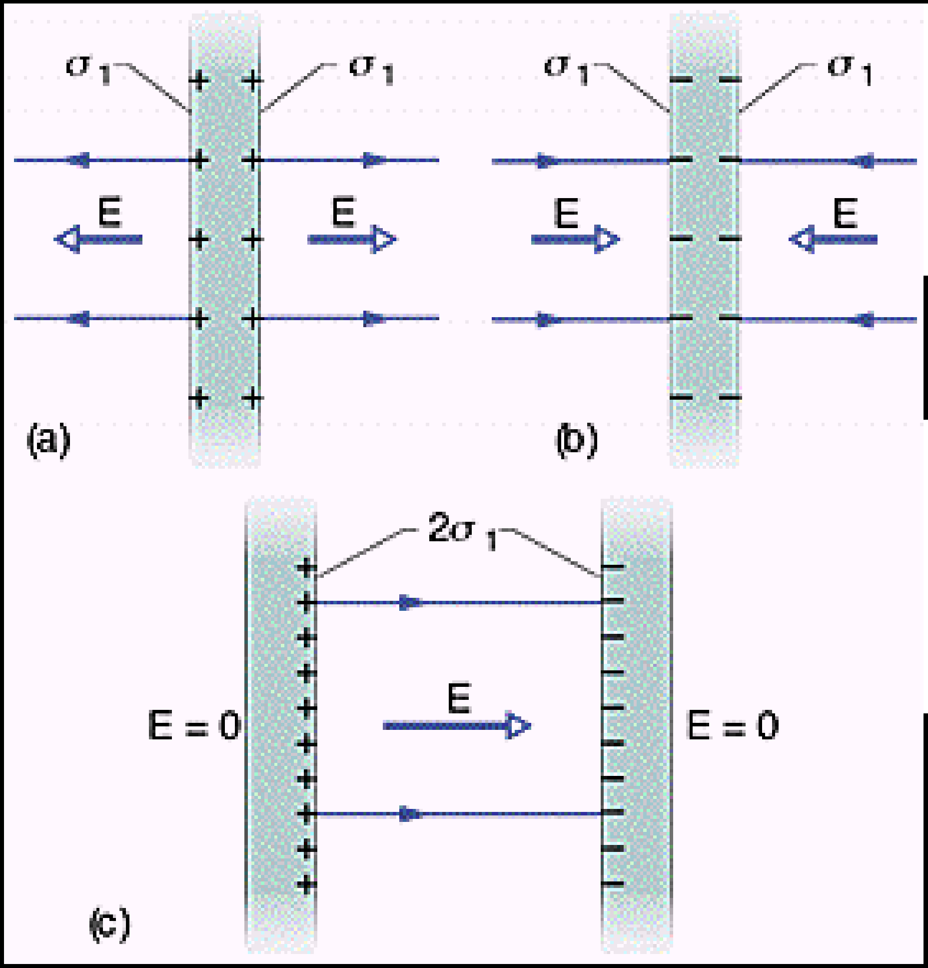
3-8 Applying Gauss' Law: Planar Symmetry



$$\vec{E} \cdot d\vec{A} = EdA$$

$$\varepsilon_0 (EA + EA) = \sigma A \rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

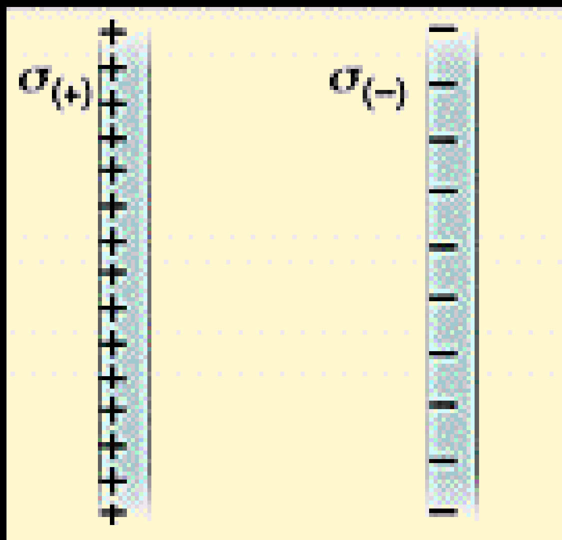
- *Two Conducting Plates*

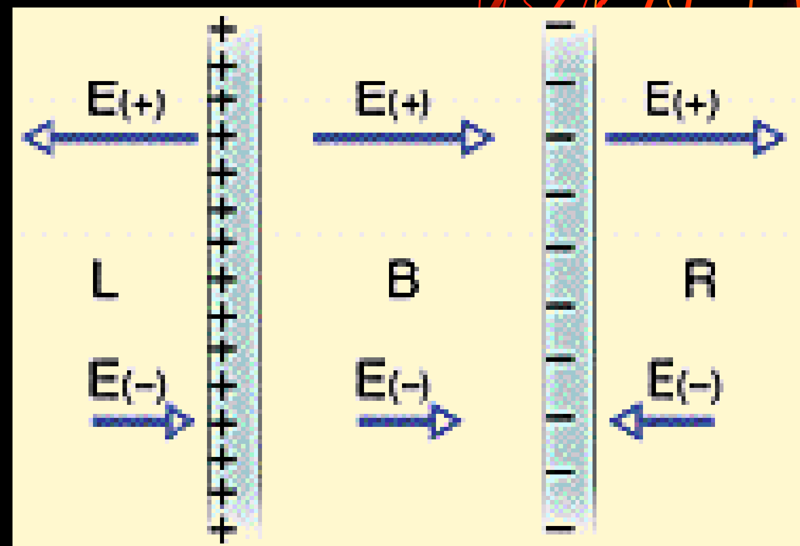


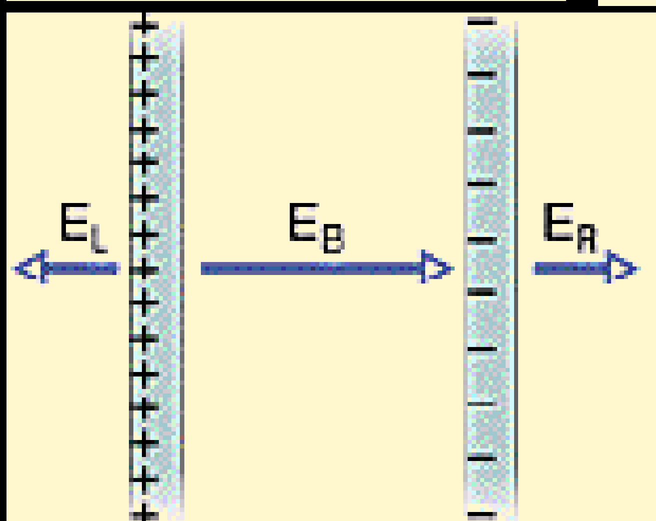
$$E = \frac{\sigma_1}{\varepsilon_0}$$

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

ex.3-5 Two || nonconducting sheets







$$E_+ = \frac{\sigma_+}{2\varepsilon_0}$$

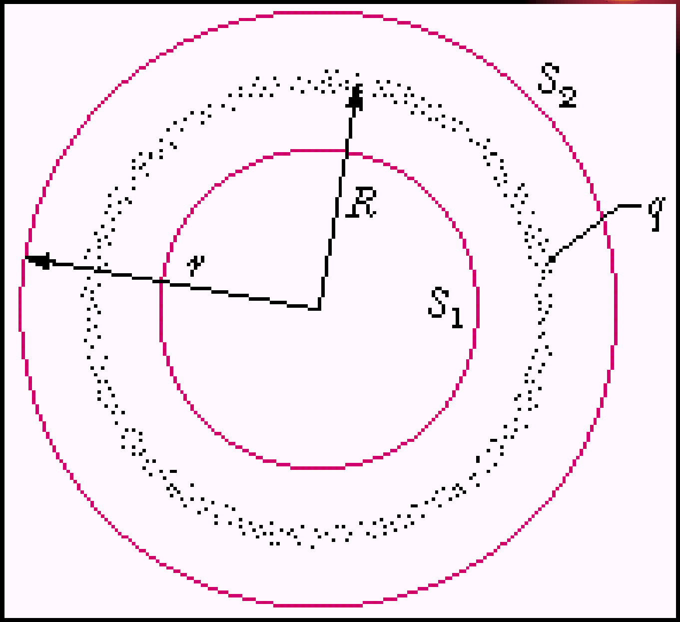
$$E_- = \frac{\sigma_-}{2\varepsilon_0}$$

3-9 Applying Gauss' Law: Spherical Symmetry

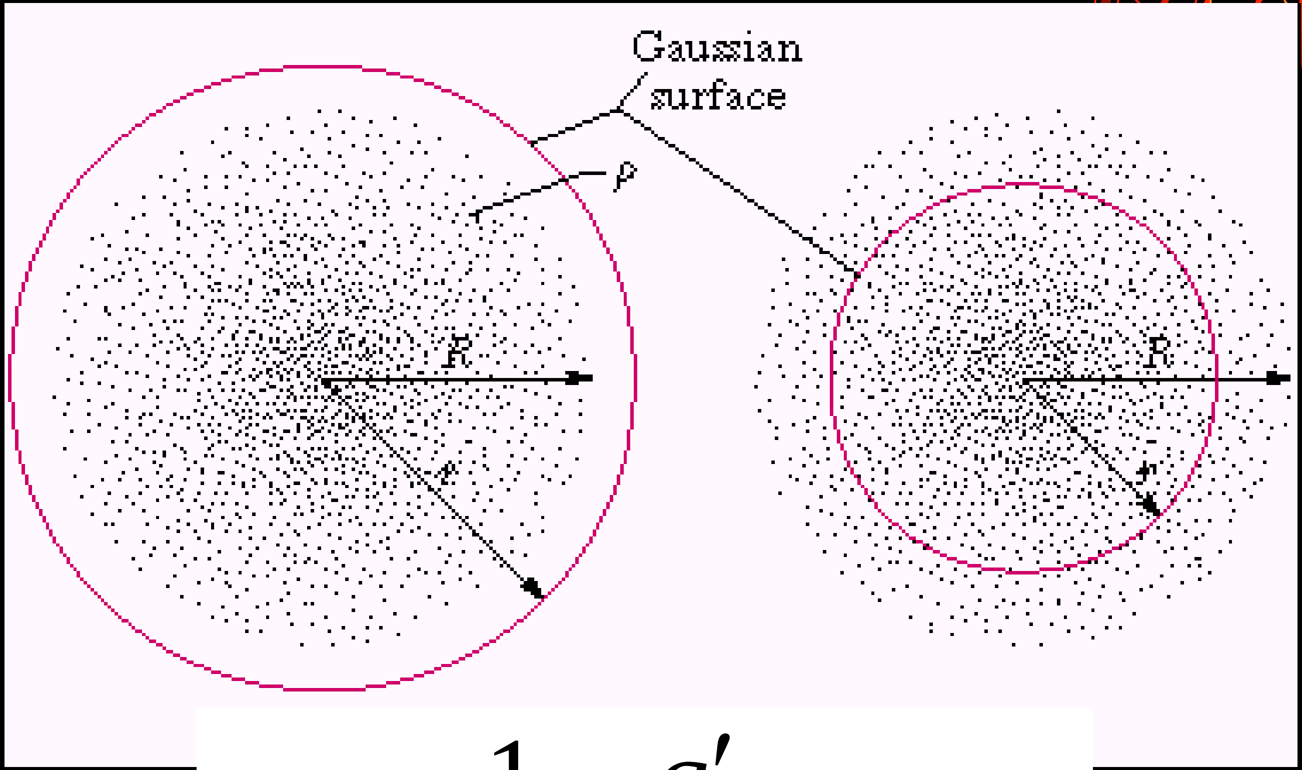
- The Shell Theorems

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (r \geq R)$$

$$E = 0 \quad (r < R)$$



A spherically symmetric charge distribution – ρ (r)



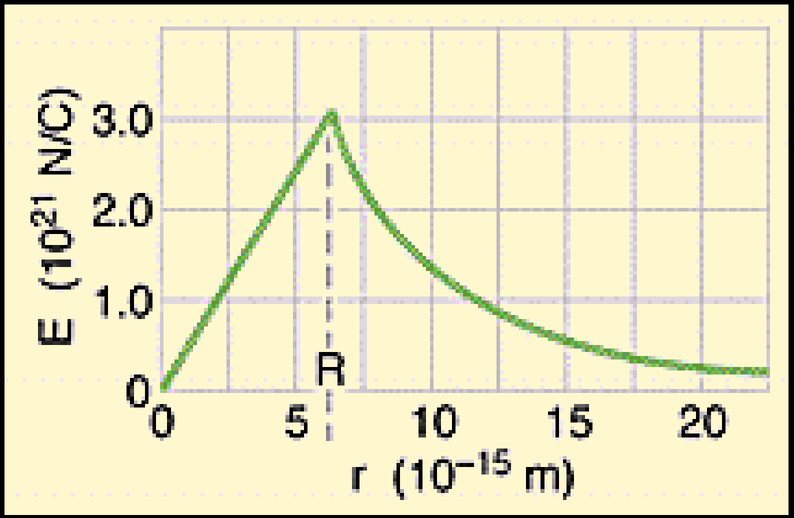
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2}\,(r \le R)$$

Uniform distribution

$$\frac{q'}{q} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} \rightarrow q' = q\frac{r^3}{R^3}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r$$

Ex.3-6 The electric field vs. r



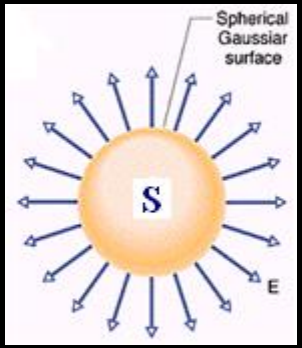
$$q = Ze = (79)e = 1.264 \times 10^{-17} C$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = 3.0 \times 10^{21} N / C$$

3-10 Gauss' Law in Differential Form

$$\varepsilon_0 \Phi = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$\rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$



Divergence Theorem

$$\oint_S \vec{E} \cdot d\vec{A} = \int_V \nabla \cdot \vec{E} dV$$

Total charge

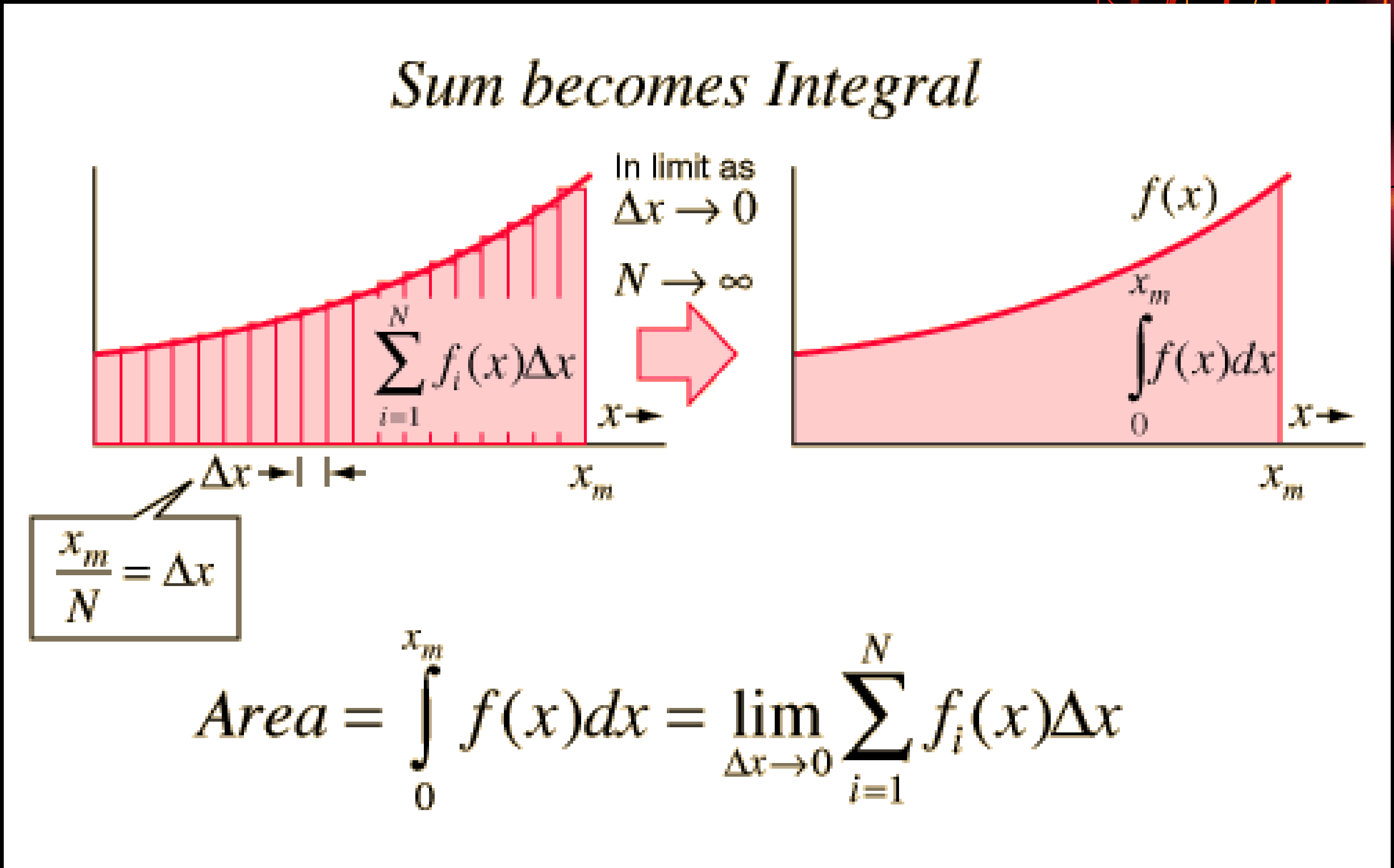
$$\rightarrow \int \nabla \cdot \vec{E} dV = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho dV$$

Charge density

$$\rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

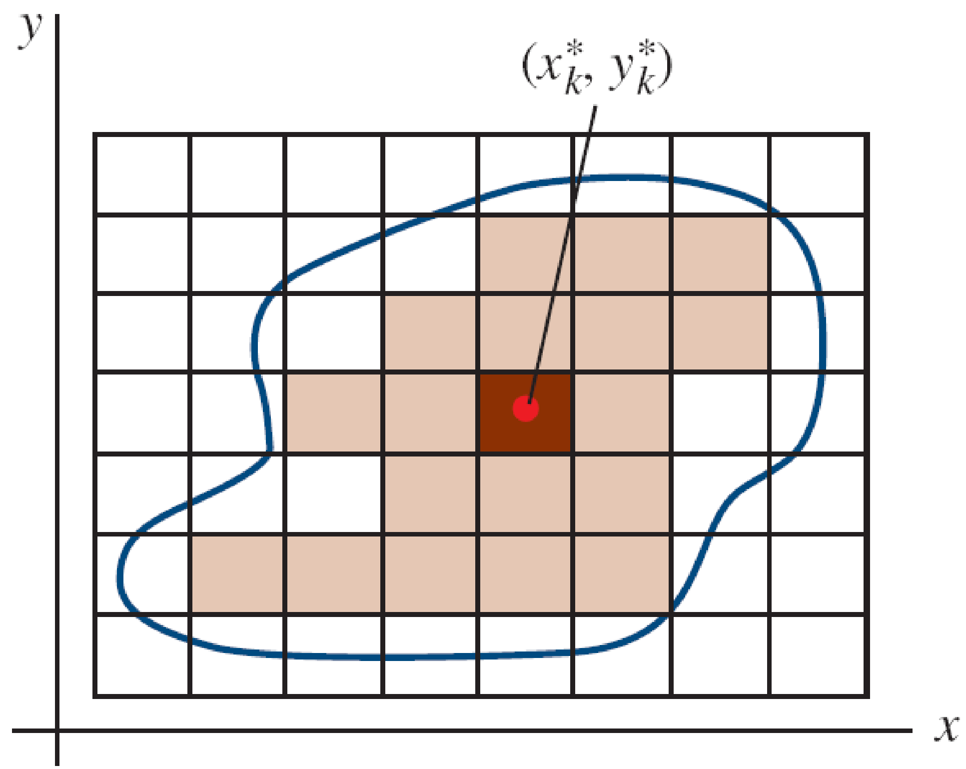
3-11 Vector Calculus

Integral



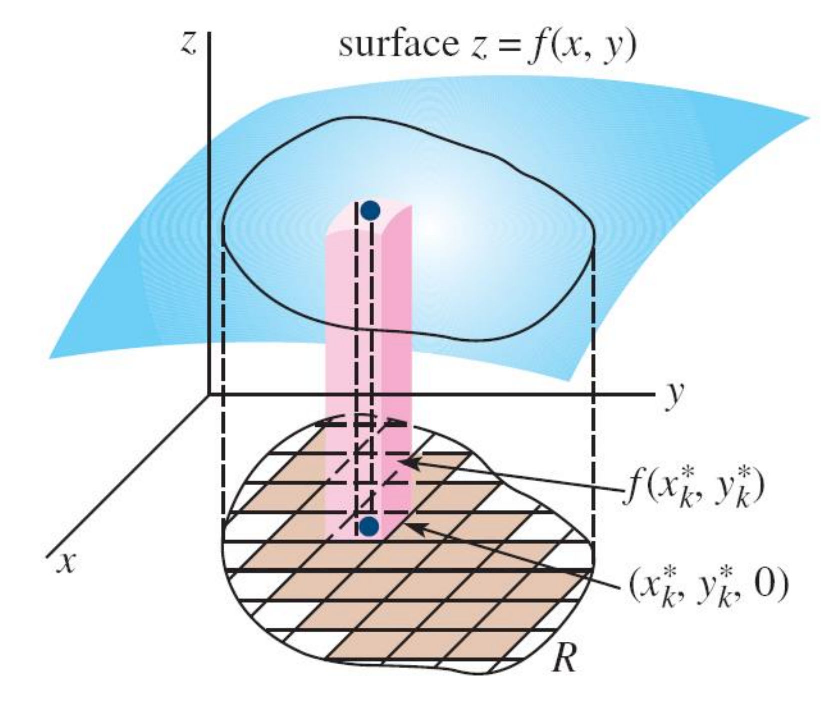
Sum becomes Integral
In limit as
$\Delta x \to 0$
$N \to \infty$
$\sum_{i=1}^{N} f_i(x)\Delta x$
$x \to$
Δx
x_m
$\frac{x_m}{N} = \Delta x$
$f(x)$
$\int_0^{x_m} f(x)dx$
$Area = \int_0^{x_m} f(x)dx = \lim_{\Delta x \to 0} \sum_{i=1}^{N} f_i(x)\Delta x$

Double Integral



$$\iint_R f(x,y)\,dA = \lim_{\|P\|\to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\Delta A_k$$

Volume under a surface



$$V = \iint_R f(x, y)\,dA$$

質心

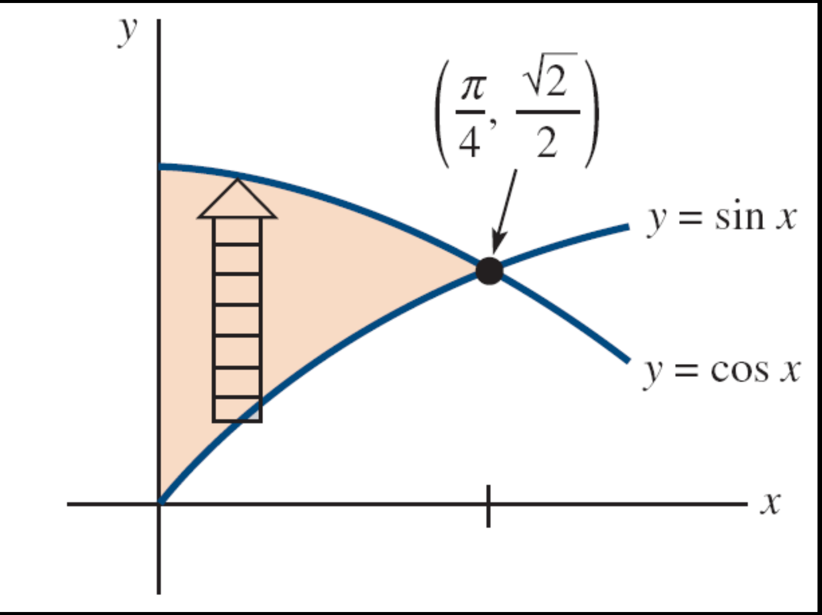
- **Laminas with Variable Density—Center of Mass**

$$m = \lim_{\|P\|\to 0} \sum_{k=1}^{n} \rho(x_k^*, y_k^*)\Delta A_k = \iint_R \rho(x, y)\,dA$$

$$\bar{x} = \frac{M_y}{m}, \quad \bar{y} = \frac{M_x}{m}$$

$$M_y = \iint_R x\rho(x, y)\,dA,$$

$$M_x = \iint_R y\rho(x, y)\,dA$$

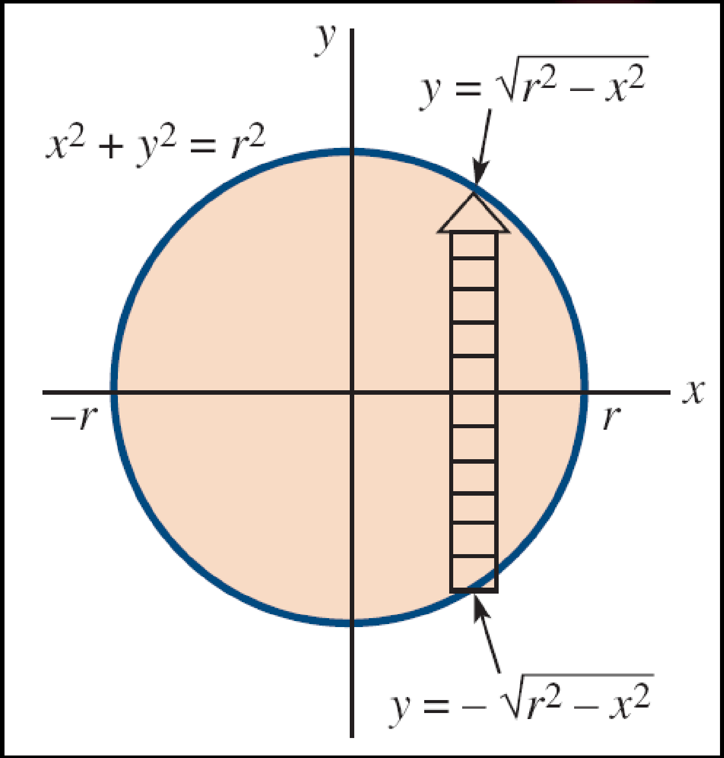


慣性矩(轉動慣量)

- **Moments of Inertia**

$$I_x = \iint_R y^2 \rho(x, y)\, dA$$

$$I_y = \iint_R x^2 \rho(x, y)\, dA$$

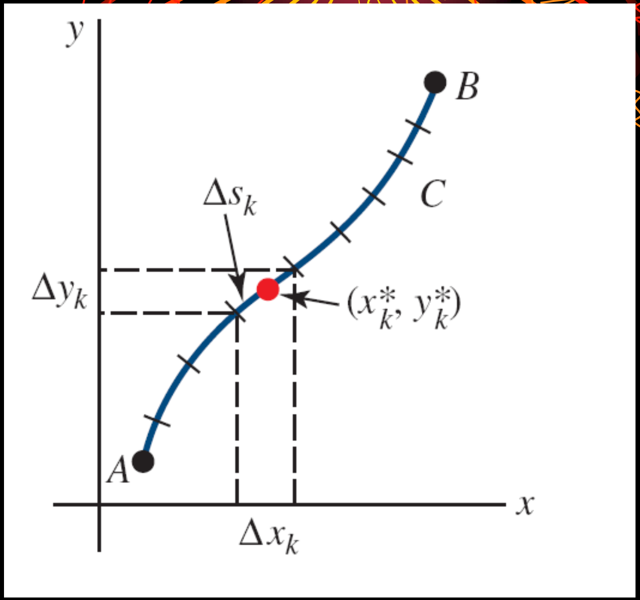


Line Integral

$$\int_C G(x, y)\, dx = \lim_{\|P\| \to 0} \sum_{k=1}^{n} G(x_k^*, y_k^*) \Delta x_k$$

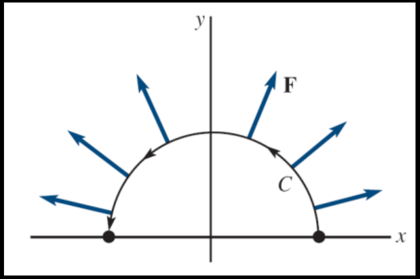
$$\int_C G(x, y)\, dy = \lim_{\|P\| \to 0} \sum_{k=1}^{n} G(x_k^*, y_k^*) \Delta y_k$$

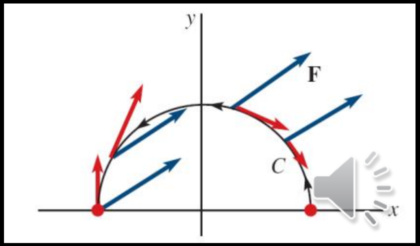
$$\int_C G(x, y)\, ds = \lim_{\|P\| \to 0} \sum_{k=1}^{n} G(x_k^*, y_k^*) \Delta s_k$$



- **Work**
- $\mathbf{F}(x, y) = P(x, y)\,\mathbf{i} + Q(x, y)\,\mathbf{j}$
  $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$
  $d\mathbf{r} = dx\,\mathbf{i} + dy\,\mathbf{j}$

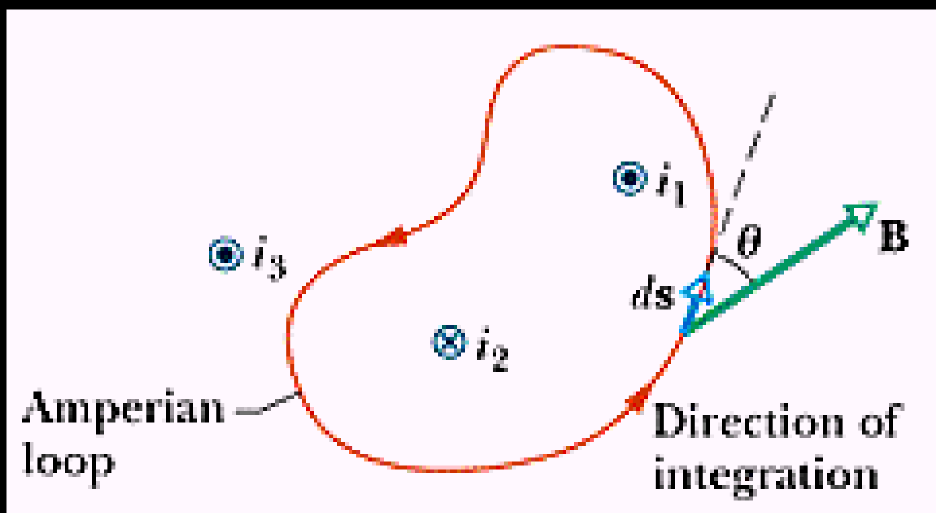
$$\int_C P(x, y, z)\, dx + Q(x, y, z)\, dy = \int_C \mathbf{F} \cdot d\mathbf{r}$$





Circulation

- Circulation of B around *C*



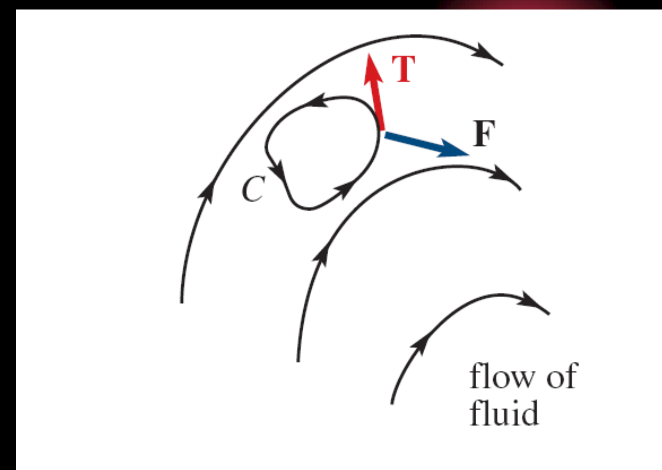
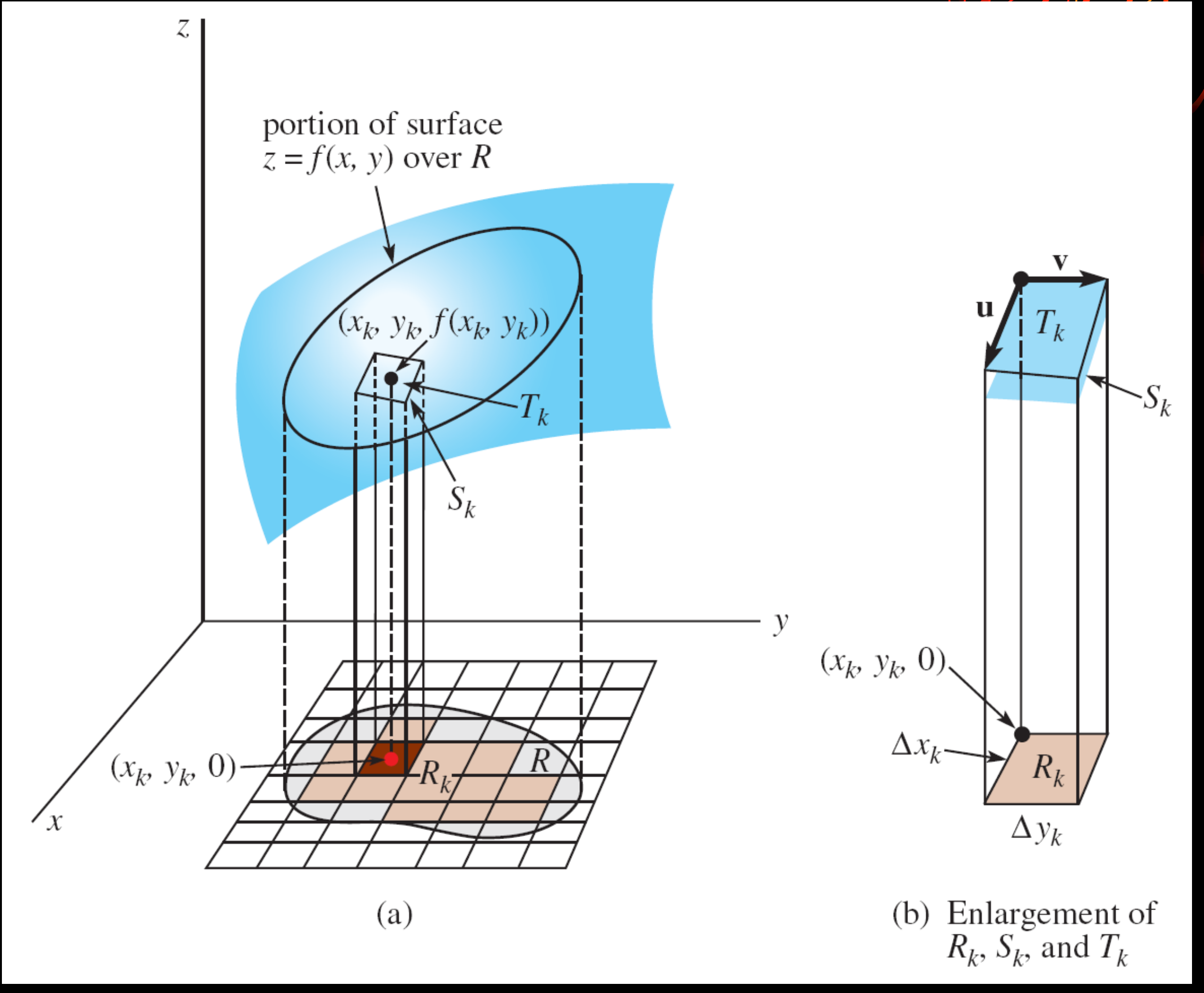


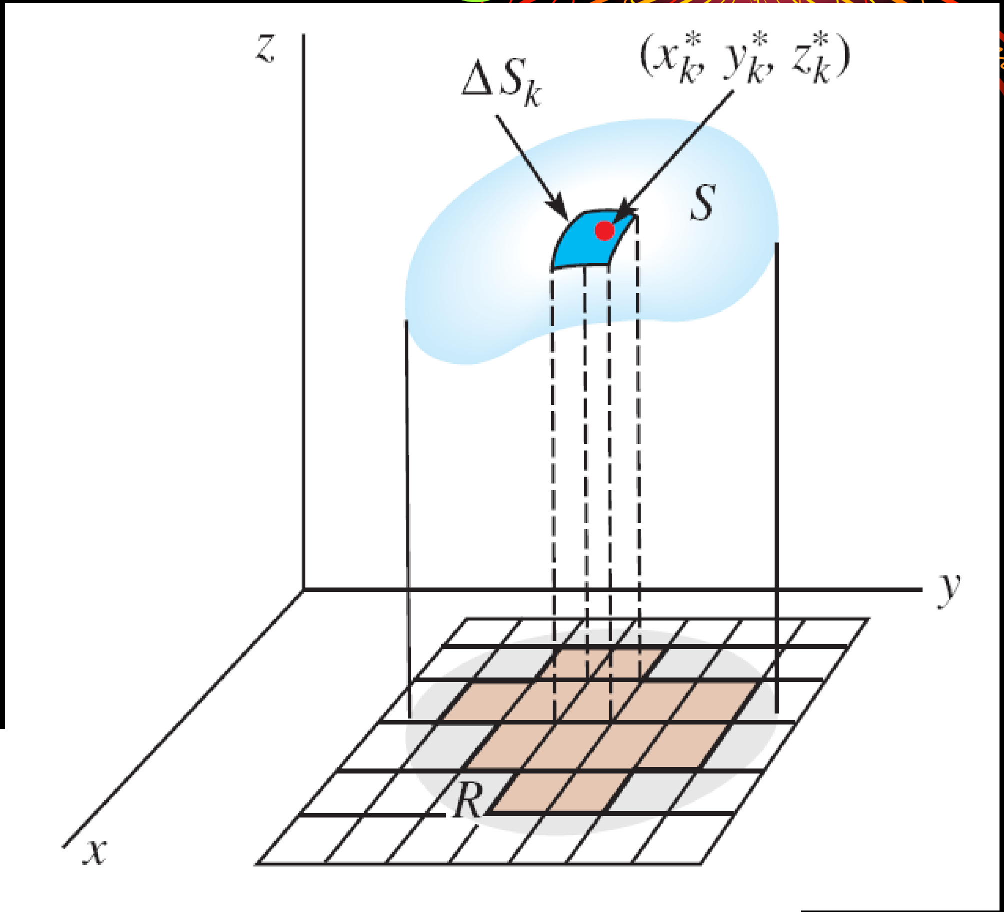
FIGURE 9.8.11 Does the velocity field turn the curve *C*?

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$$

Surface Integrals



Evaluate Surface Integrals



$$\iint_S G(x, y, z)\, dS$$

$$= \iint_R G(x, y, f(x, y))\sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2}\, dA$$