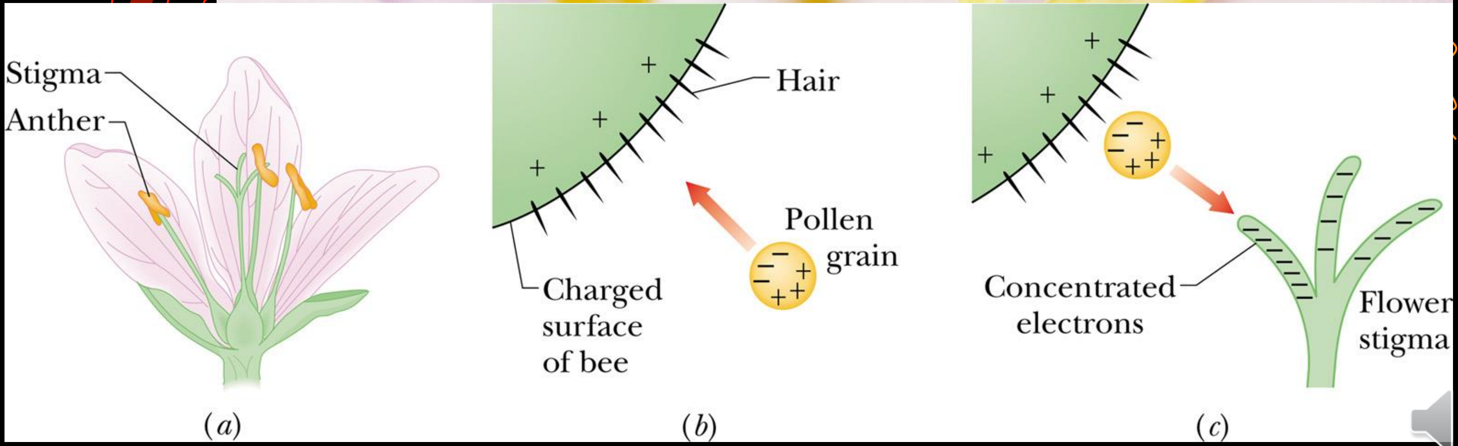
2 電場



Stigma
Anther
(a)
Hair
Charged surface of bee
Pollen grain
(b)
Concentrated electrons
Flower stigma
(c)

Contents

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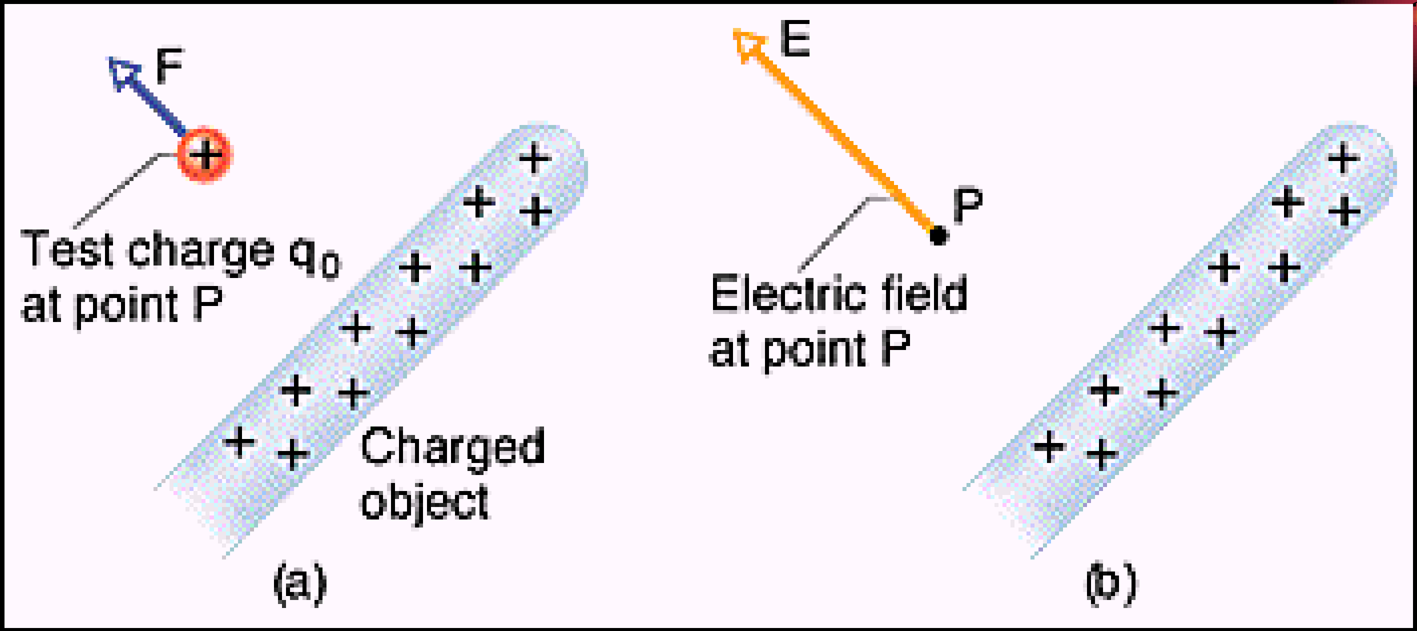
- 2-2電力線 Lines of force
- 2-3 Electric-field vector of a point charge
- 2-4 電場的計算

2-1電場的定義

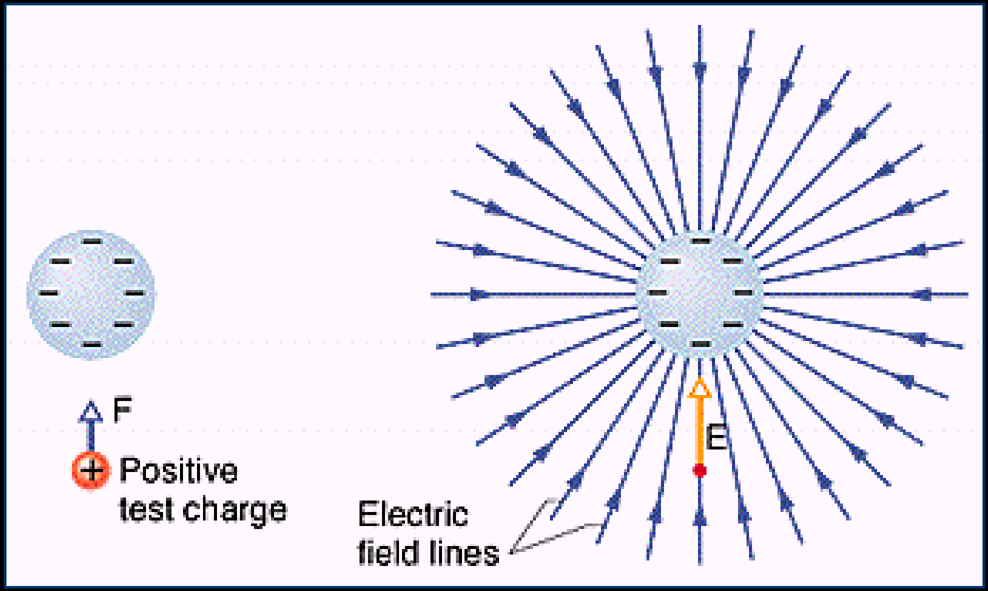
- 電力：q與q_0之間的電力 or
  q在q_0所建立的電場中受到的電力

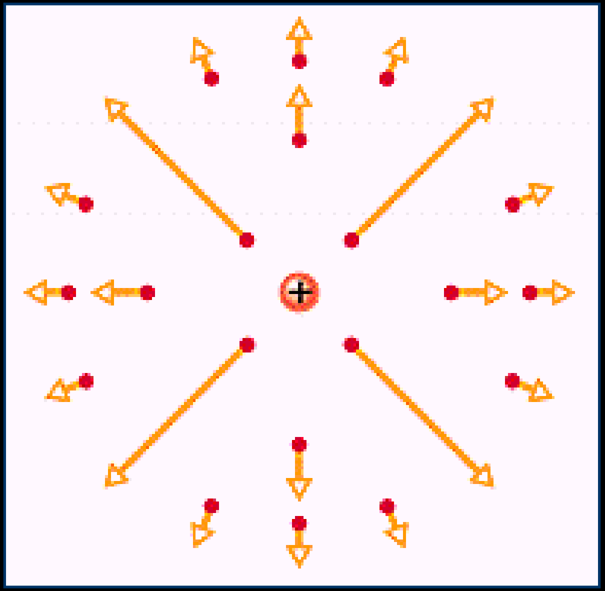
$$\vec{E} = \frac{\vec{F}}{q_0} \,(\mathrm{N/C})\,(\mathrm{V \cdot m})$$

2-2電力線 Lines of force

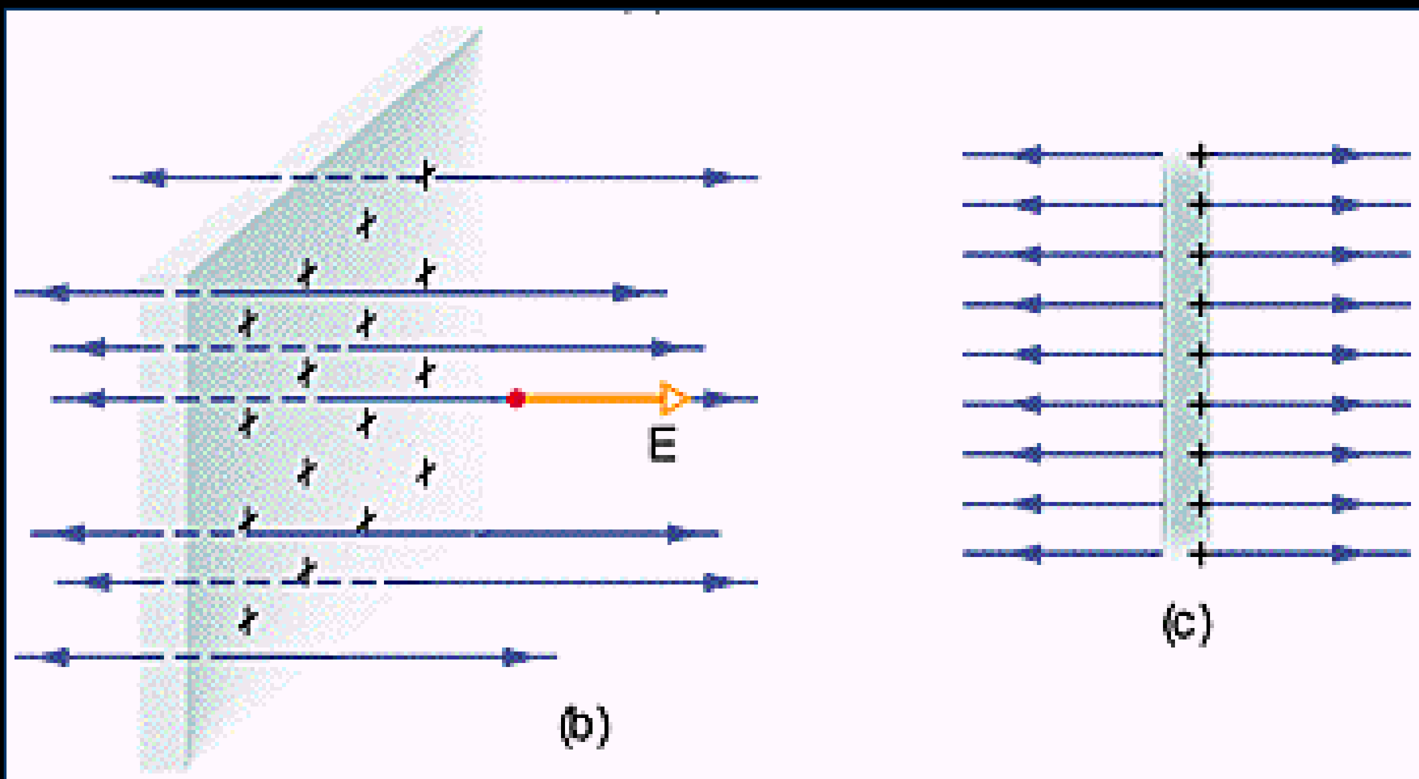


點電荷



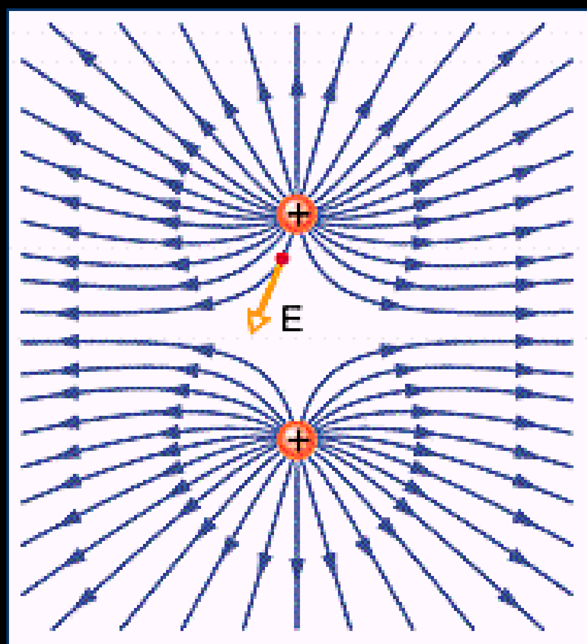


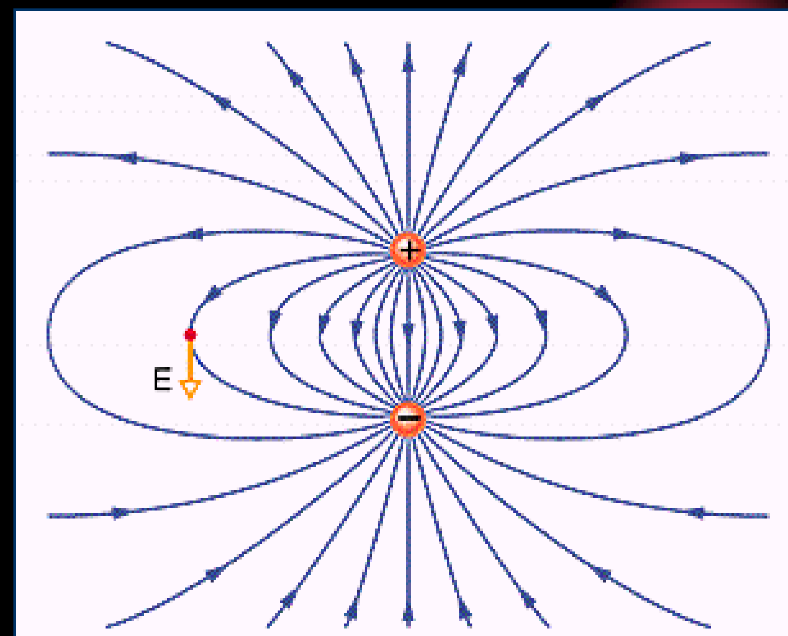
帶電平板



二正電荷與電雙極

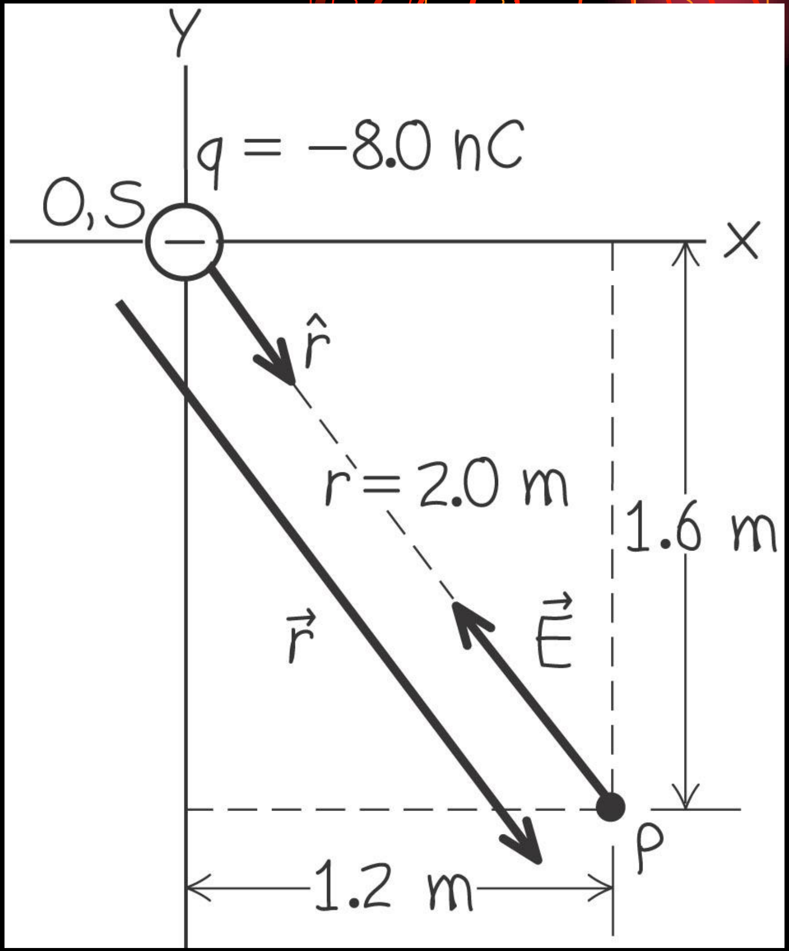
q_0





2-3 Electric-field vector of a point charge

- Follow Example 21.6 to see the vector nature of the electric field. Use Figure 21.19 at the right.



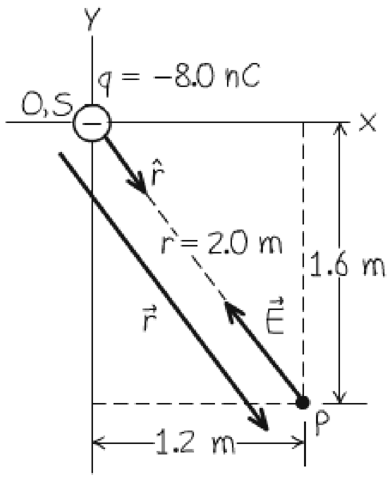
Example 21.6 Electric-field vector of a point charge

A point charge $q = -8.0$ nC is located at the origin. Find the electric-field vector at the field point $x = 1.2$ m, $y = -1.6$ m.

SOLUTION

IDENTIFY and SET UP: We must find the electric-field *vector* $\vec{E}$ due to a point charge. Figure 21.19 shows the situation. We use Eq. (21.7); to do this, we must find the distance r from the source point S (the position of the charge q, which in this example is at the origin O) to the field point P, and we must obtain an expression for the unit vector $\hat{r} = \vec{r}/r$ that points from S to P.

21.19 Our sketch for this problem.



EXECUTE: The distance from S to P is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\text{ m})^2 + (-1.6\text{ m})^2} = 2.0\text{ m}$$

The unit vector $\hat{r}$ is then

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{r}$$

$$= \frac{(1.2\text{ m})\hat{\imath} + (-1.6\text{ m})\hat{\jmath}}{2.0\text{ m}} = 0.60\hat{\imath} - 0.80\hat{\jmath}$$

Then, from Eq. (21.7),

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$= (9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-8.0 \times 10^{-9}\text{ C})}{(2.0\text{ m})^2}(0.60\hat{\imath} - 0.80\hat{\jmath})$$

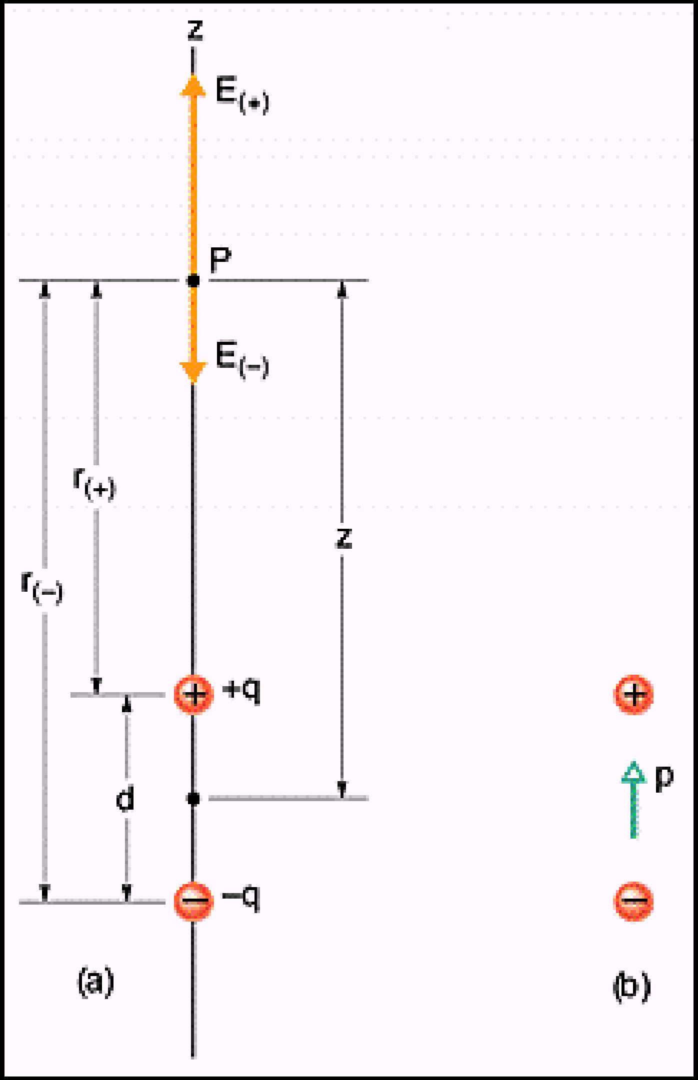
$$= (-11\text{ N/C})\hat{\imath} + (14\text{ N/C})\hat{\jmath}$$

EVALUATE: Since q is negative, $\vec{E}$ points from the field point to the charge (the source point), in the direction opposite to $\hat{r}$ (compare Fig. 21.17c). We leave the calculation of the magnitude and direction of $\vec{E}$ to you (see Exercise 21.36).

2-4 電場的計算

- 電雙極
- 均勻帶電圓環
- 均勻帶電圓盤
- Field of a charged line segment
- Electron in a uniform field
- The electric field on an oil drop

電雙極



兩電荷的電場相加

$$E = E_+ - E_-$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^{\ 2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^{\ 2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-d/2)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(z+d/2)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 z^2}[(1-\frac{d}{2z})^{-2} - (1+\frac{d}{2z})^{-2}]$$

電雙極的電場

$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1+\frac{2d}{2z(1!)}+\ldots\right)-\left(1-\frac{2d}{2z(1!)}+\ldots\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1+\frac{d}{z}+\ldots\right)-\left(1-\frac{d}{z}+\ldots\right)\right]$$

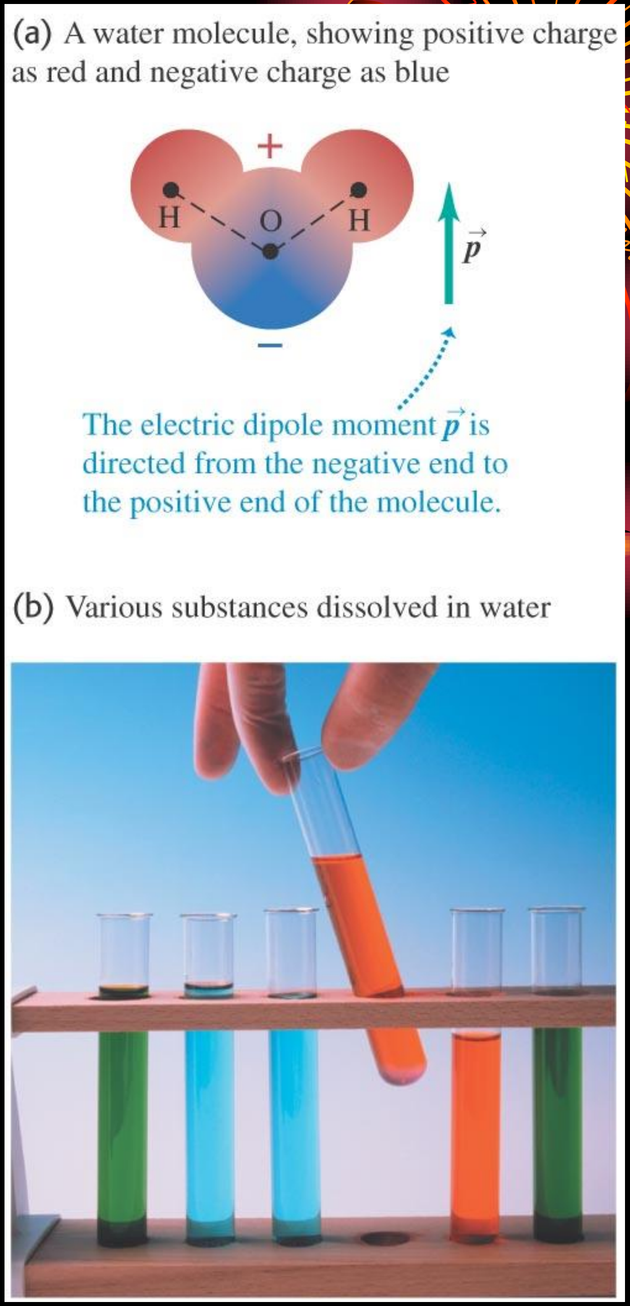
$$= \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d}{z} = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole)}$$

$\vec{p}$ = elcectric dipole moment (電雙極矩)

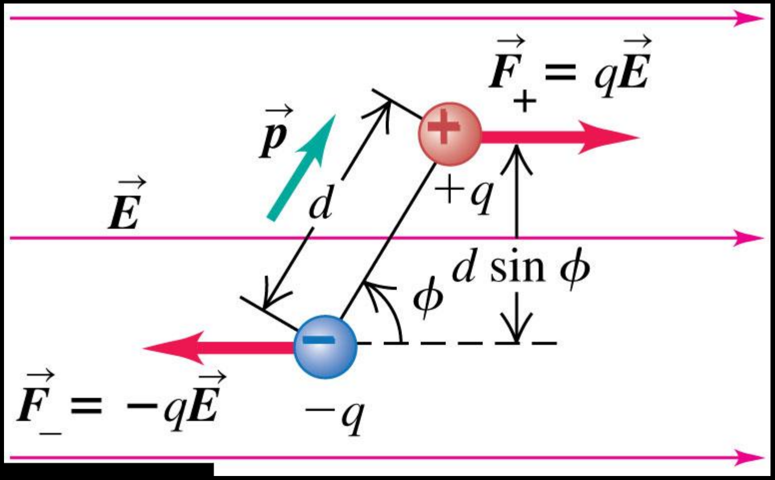
Electric dipoles

- **An *electric dipole* is a pair of point charges having equal but opposite sign and separated by a distance.**
- **Figure 21.30 at the right illustrates the water molecule, which forms an electric dipole.**



Force and torque on a dipole

- Figure 21.31 below left shows the force on a dipole in an electric field.



$$\tau = (qE)(d \sin \phi)$$

$$p = qd$$

electric dipole moment

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on an electric dipole, in vector form)}$$

Potential energy of a dipole

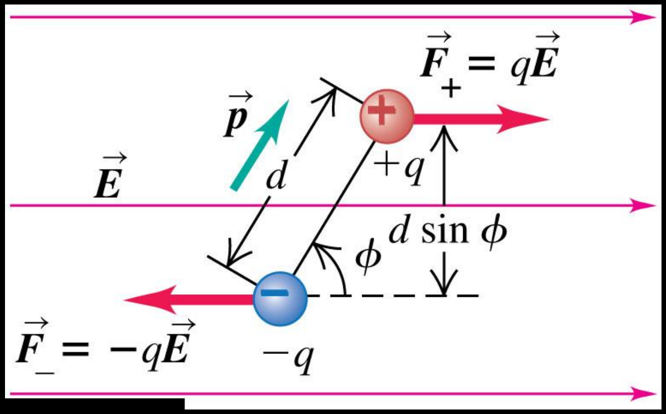
$$dW = \tau\, d\phi = -pE \sin\phi\, d\phi$$

$$W = \int_{\phi_1}^{\phi_2} (-pE\sin\phi)\, d\phi = pE\cos\phi_2 - pE\cos\phi_1$$

$$W = U_1 - U_2.$$

$$U(\phi) = -pE\cos\phi$$

$$U = -\vec{p}\cdot\vec{E} \quad \text{(potential energy for a dipole in an electric field)}$$

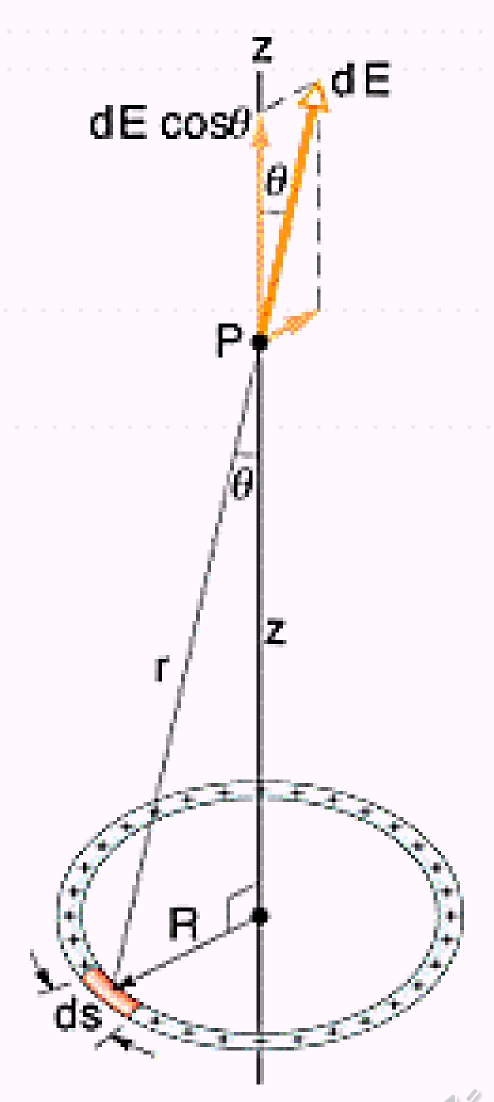


均勻帶電圓環

$$dq = \lambda ds$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{r^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{(z^2 + R^2)}$$



均勻帶電圓環的電場

$$dE \to dE\cos\theta,\ \cos\theta = \frac{z}{r} = \frac{z}{(z^2+R^2)^{1/2}}$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}ds$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} = \frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}$$

極限情形

- If z » R, $z^2 + R^2 \rightarrow z^2$

$$E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}$$

$$\rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \text{ (like a point charge)}$$

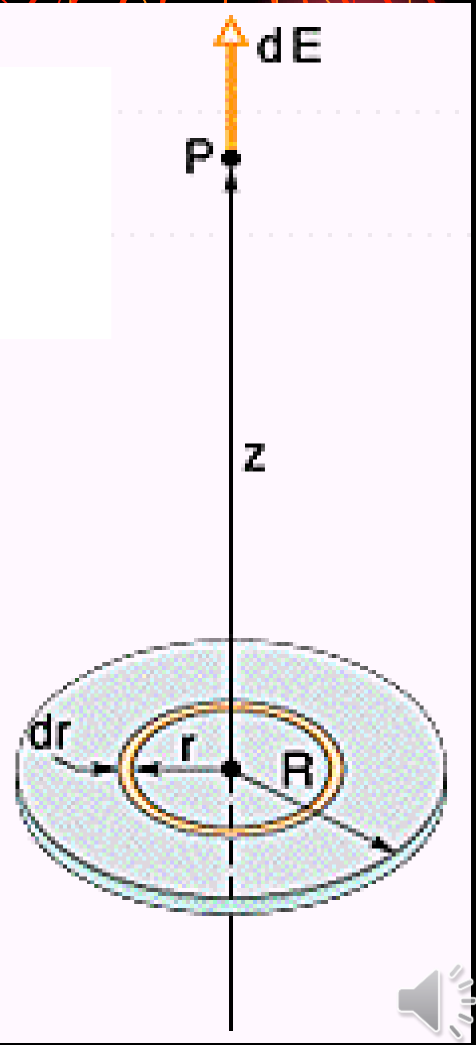
均勻帶電圓盤

$$dq = \sigma dA = \sigma 2\pi r dr$$

$$dE = \frac{z\sigma 2\pi r dr}{4\pi\varepsilon_0 (z^2 + r^2)^{3/2}}$$

$$dE = \frac{\sigma z}{4\varepsilon_0} \frac{2r dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2} (2r) dr$$



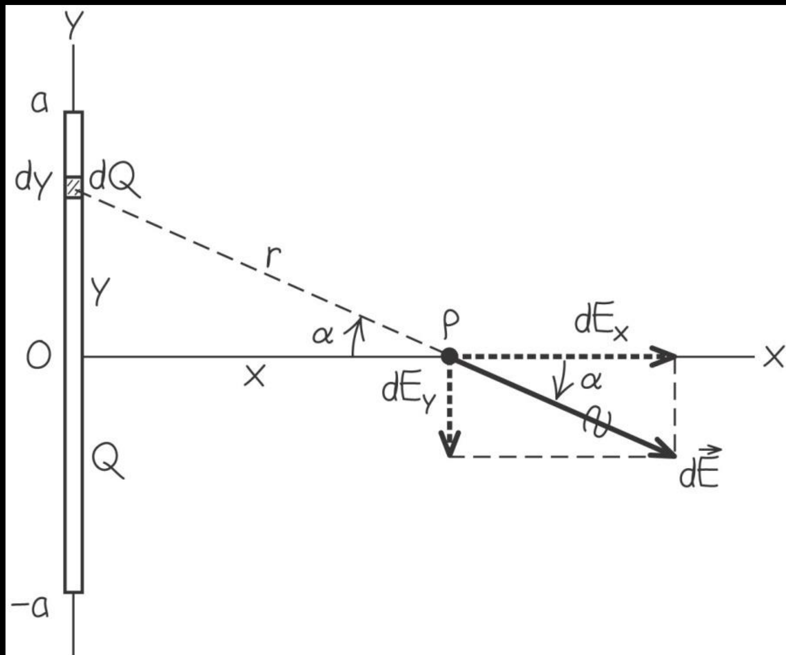
均勻帶電圓盤的電場

$$E=\int dE=\frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2+r^2)^{-3/2}(2r)dr$$

$$=\frac{\sigma z}{4\varepsilon_0}[\frac{(z^2+r^2)^{-1/2}}{-1/2}]_0^R$$

$$=\frac{\sigma}{2\varepsilon_0}(1-\frac{z}{\sqrt{z^2+R^2}})\rightarrow\frac{\sigma}{2\varepsilon_0}$$

Field of a charged line segment



$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{(x^2+y^2)}$$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{x\,dy}{(x^2+y^2)^{3/2}}$$

$$dE_y = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{y\,dy}{(x^2+y^2)^{3/2}}$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{x\,dy}{(x^2+y^2)^{3/2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{x\sqrt{x^2+a^2}}$$

$$E_y = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{+a}\frac{y\,dy}{(x^2+y^2)^{3/2}} = 0$$

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{\boldsymbol{\imath}}$$

Limiting cases

- **For x >> a**

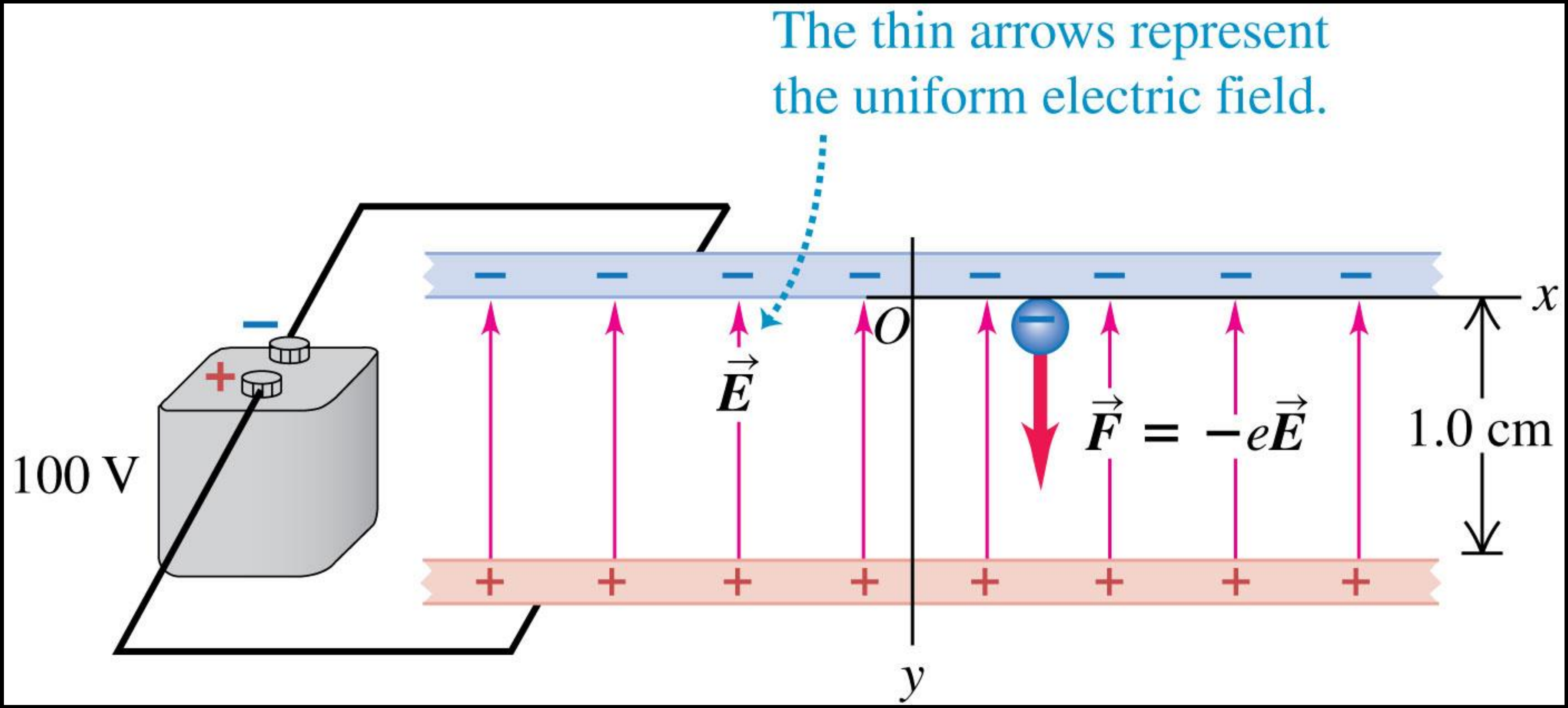
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2 + a^2}}\hat{\imath} \longrightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\hat{\imath}$$

- **For very long segment (a >> x)**

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x^2/a^2) + 1}}\hat{\imath} \longrightarrow \vec{E} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{\imath}$$

Electron in a uniform field

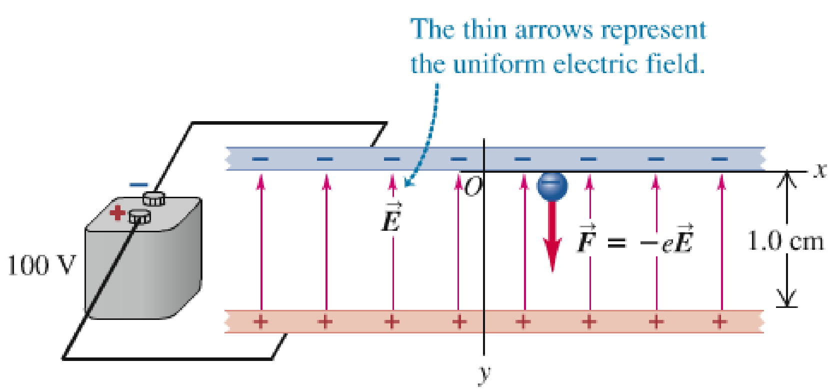
- Example 21.7 requires us to find the force on a charge that is in a known electric field. Follow this example using Figure 21.20 below.



Example 21.7 Electron in a uniform field

When the terminals of a battery are connected to two parallel conducting plates with a small gap between them, the resulting charges on the plates produce a nearly uniform electric field $\vec{E}$ between the plates. (In the next section we'll see why this is.) If the plates are 1.0 cm apart and are connected to a 100-volt battery as shown in Fig. 21.20, the field is vertically upward and has magnitude $E = 1.00 \times 10^4$ N/C. (a) If an electron (charge $-e = -1.60 \times 10^{-9}$ C, mass $m = 9.11 \times 10^{-31}$ kg) is released from rest at the upper plate, what is its acceleration? (b) What speed and kinetic energy does it acquire while traveling 1.0 cm to the lower plate? (c) How long does it take to travel this distance?

21.20 A uniform electric field between two parallel conducting plates connected to a 100-volt battery. (The separation of the plates is exaggerated in this figure relative to the dimensions of the plates.)



SOLUTION

IDENTIFY and SET UP: This example involves the relationship between electric field and electric force. It also involves the relationship between force and acceleration, the definition of kinetic energy, and the kinematic relationships among acceleration, distance, velocity, and time. Figure 21.20 shows our coordinate system. We are given the electric field, so we use Eq. (21.4) to find the force on the electron and Newton's second law to find its acceleration. Because the field is uniform, the force is constant and we can use the constant-acceleration formulas from Chapter 2 to find the electron's velocity and travel time. We find the kinetic energy using $K = \frac{1}{2}mv^2$.

Solution

EXECUTE: (a) Although $\vec{E}$ is upward (in the $+y$-direction), $\vec{F}$ is downward (because the electron's charge is negative) and so F_y is negative. Because F_y is constant, the electron's acceleration is constant:

$$a_y = \frac{F_y}{m} = \frac{-eE}{m} = \frac{(-1.60 \times 10^{-19}\ \text{C})(1.00 \times 10^{4}\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}$$
$$= -1.76 \times 10^{15}\ \text{m/s}^2$$

(b) The electron starts from rest, so its motion is in the y-direction only (the direction of the acceleration). We can find the electron's speed at any position y using the constant-acceleration Eq. (2.13), $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. We have $v_{0y} = 0$ and $y_0 = 0$, so at $y = -1.0$ cm $= -1.0 \times 10^{-2}$ m we have

$$|v_y| = \sqrt{2a_y y} = \sqrt{2(-1.76 \times 10^{15}\ \text{m/s}^2)(-1.0 \times 10^{-2}\ \text{m})}$$
$$= 5.9 \times 10^{6}\ \text{m/s}$$

The velocity is downward, so $v_y = -5.9 \times 10^6$ m/s. The electron's kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(5.9 \times 10^{6}\ \text{m/s})^2$$
$$= 1.6 \times 10^{-17}\ \text{J}$$

(c) From Eq. (2.8) for constant acceleration, $v_y = v_{0y} + a_y t$,

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{(-5.9 \times 10^{6}\ \text{m/s}) - (0\ \text{m/s})}{-1.76 \times 10^{15}\ \text{m/s}^2}$$
$$= 3.4 \times 10^{-9}\ \text{s}$$

EVALUATE: Our results show that in problems concerning subatomic particles such as electrons, many quantities—including acceleration, speed, kinetic energy, and time—will have *very* different values from those typical of everyday objects such as baseballs and automobiles.

The electric field on an oil drop

A drop of R = 2.76μm has 3 excess electrons, ρ = 920kg/m^3

$$F_g = \frac{4}{3}\pi R^3 \rho g = (3e)E = F_e$$

$$E = \frac{4\pi R^3 \rho g}{9e} = 1.65\times 10^6\, N/C$$