



$$OA = 1$$

$$\sin(\alpha + \beta) = AN = AT + TN$$

$$\parallel$$

$$\sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$BM = OB \sin \alpha$$

$$NM = TB = AB \sin \alpha$$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \cos \frac{\pi}{8} = \frac{1}{2} \sqrt{2 + 2 \cos \frac{\pi}{4}} = \frac{1}{2} \sqrt{2 + \sqrt{2}}$
 $\cos \frac{\pi}{16} = \frac{1}{2} \sqrt{2 + 2 \cos \frac{\pi}{8}} = \frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2}}}$
 $\cos \frac{\pi}{32} = \frac{1}{2} \sqrt{2 + 2 \cos \frac{\pi}{16}} = \frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$
 \dots
 $\cos \left(\frac{\pi}{2^{2^k+1}} \right) = \frac{1}{2} \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{2}}}}$

(Left side notes: $\frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2}}}$ \rightarrow ≈ 0.98)
 $\frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2}}}$ \rightarrow ≈ 0.6

$\sin(6.175) \approx 0.1745$

$$\sin x = 2 \cos \frac{x}{2} \sin \frac{x}{2}$$

$$= 2^2 \cos \frac{x}{4} \sin \frac{x}{4} \cos \frac{x}{4}$$

$$= 2^3 \cos \frac{x}{8} \cos \frac{x}{8} \cos \frac{x}{8} \sin \frac{x}{8}$$

$$\dots$$

$$= 2^n \cos \frac{x}{2^n} \cos \frac{x}{2^n} \dots \cos \frac{x}{2^n} \sin \frac{x}{2^n}$$

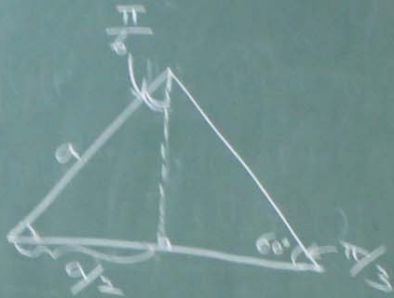
$$\alpha = \frac{\pi}{2^n} \Rightarrow 1 = 2^n \sin \frac{\pi}{2^n}$$

$$n \gg 1 \Rightarrow \frac{2}{\pi} \approx \cos \frac{\pi}{2^n}$$

Vieta's formula

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Set $\alpha = \beta \Rightarrow \frac{\sin(2\alpha)}{2 \cos^2 \alpha} = \dots$



$$(1 + \epsilon)^x = 1 + x\epsilon + \frac{x(x-1)}{2}\epsilon^2 + \dots$$

$$\sin 30^\circ = \sin \frac{\pi}{6} = \frac{1}{2} = \cos 60^\circ = \cos \frac{\pi}{3}$$

$$\text{OR } 30^\circ = \cos \frac{\pi}{2} = \frac{\sqrt{2}}{2} = \sin 60^\circ = \sin \frac{\pi}{3}$$

$$\sin \theta = \cos \left(\frac{\pi}{2} - \theta \right)$$

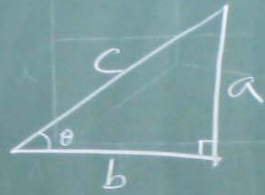
$$\cos \theta = \sin \left(\frac{\pi}{2} - \theta \right)$$

$$(1 + \epsilon)^{\frac{1}{2}} (1 + \epsilon)^{\frac{1}{2}} = 1 + 2\epsilon + 2\epsilon^2 + \dots = 1 + \epsilon$$

$$\sin 45^\circ = \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \tan \frac{\pi}{4} = 1$$

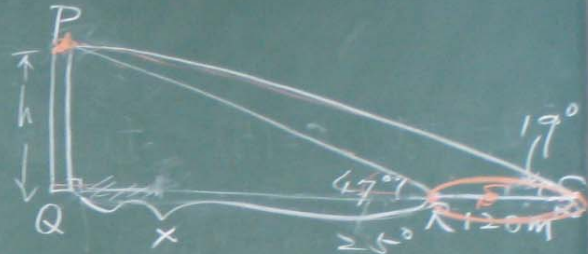
$$\begin{cases} \sin x \approx \tan x \approx x \\ x \text{ (in radians)} \ll 1. \\ \cos x = \sqrt{1 - \sin^2 x} \approx \sqrt{1 - x^2} \\ \cos x \approx 1 - \frac{x^2}{2} \approx 1 - \frac{x^2}{2} + \dots \end{cases}$$



$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

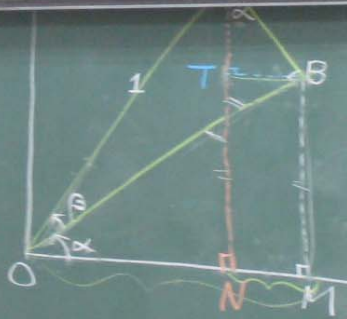
$$\tan \theta = \frac{a}{b}$$



$$\frac{h}{x+120} = \tan 19^\circ = 0.3443$$

$$\Rightarrow h = 0.3443(x+120)$$

$$\frac{h}{x} = \tan 47^\circ = 1.0724$$



$$\sin(\alpha + \beta) = \overline{AN} = \overline{AT} + \overline{TN}$$

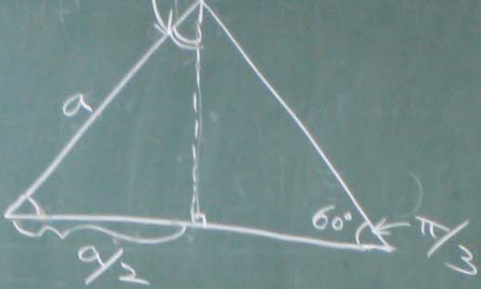
$$\parallel = \overline{AB} \cos \alpha + \overline{AB} \sin \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \frac{\overline{ON}}{\overline{OA}} = \overline{ON} = \overline{OM} - \overline{NM} = \overline{OB} \cos \alpha - \overline{OB} \sin \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\overline{NM} = \overline{TB} = \overline{AB} \sin \alpha$$





$$(1 + \epsilon)^x$$

$$= 1 + x\epsilon + \frac{x(x-1)}{2}\epsilon^2 + \dots$$

$$\sin 30^\circ = \sin \frac{\pi}{6} = \frac{1}{2} = \cos 60^\circ = \cos \frac{\pi}{3}$$

$$\cos 30^\circ = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} = \sin 60^\circ = \sin \frac{\pi}{3}$$

$$(1 + \epsilon)^{\frac{1}{2}} = 1 + c_1 \epsilon + c_2 \epsilon^2 + \dots$$

$$c_1 + 2c_2 = 0$$

$$\sin \theta = \cos \left(\frac{\pi}{2} - \theta \right)$$

$$\cos \theta = \sin \left(\frac{\pi}{2} - \theta \right)$$

$$(1 + \epsilon)^{\frac{1}{2}} (1 + \epsilon)^{\frac{1}{2}} = 1 + 2c_1 \epsilon + \dots = 1 + \epsilon$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

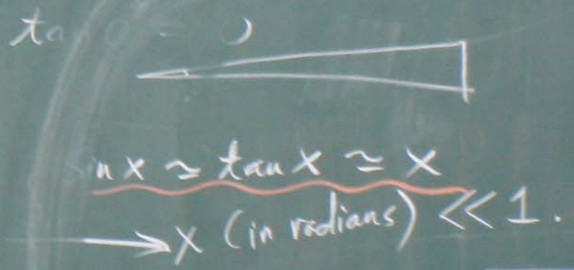
$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(-\beta) = -\sin \beta$$

$$\cos(-\beta) = \cos \beta$$

$$0 = \sin 0$$

$$1 = \cos 0$$



$$\sin x \approx \tan x \approx x$$

$$\cos x = \sqrt{1 - \sin^2 x} \approx \sqrt{1 - x^2}$$

$$\cos x \approx 1 - \frac{x^2}{2} \approx 1 - \frac{x^2}{2} + \dots$$



$$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \cos \frac{\pi}{8} = \frac{1}{2} \sqrt{2 + 2 \cos \frac{\pi}{4}} = \frac{1}{2} \sqrt{2 + \sqrt{2}}$$

$$\cos \frac{\pi}{16} = \frac{1}{2} \sqrt{2 + 2 \cos \frac{\pi}{8}} = \frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2}}}$$

$$\cos \frac{\pi}{32} = \frac{1}{2} \sqrt{2 + 2 \cos \frac{\pi}{16}} = \frac{1}{2} \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$$

$$\cos \left(\frac{\pi}{2^{n+1}} \right) = \frac{1}{2} \sqrt{2 + \sqrt{2 + \dots \sqrt{2 + \sqrt{2}}}}$$

$\sin(\alpha + \beta) = AN = AT + TN$
 \parallel
 $\sin \alpha \cos \beta + \cos \alpha \sin \beta$
 $\cos(\alpha + \beta) = \frac{ON}{OA} = \frac{ON}{OA} = \frac{OM}{OA} - \frac{NT}{OA}$
 $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$NM = TB$
 $= AB \sin \alpha$
 $\sin \beta$

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\sin(-\beta) = -\sin\beta$$

$$\cos(\beta) = \cos\beta$$

$$\sin 0 = 0$$

$$\sin\alpha = 2 \cos \frac{\alpha}{2} \sin \frac{\alpha}{2}$$

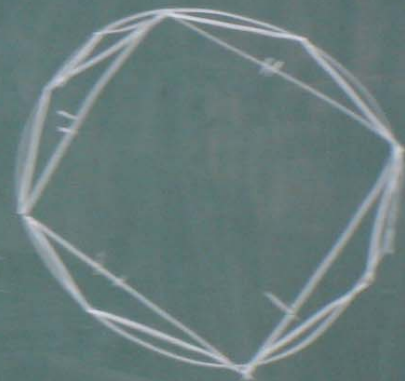
$$= 2^2 \cos \frac{\alpha}{2} \cos \frac{\alpha}{4} \sin \frac{\alpha}{4}$$

$$= 2^3 \cos \frac{\alpha}{2} \cos \frac{\alpha}{4} \cos \frac{\alpha}{8} \sin \frac{\alpha}{8}$$

$$\dots$$

$$= 2^n \cos \frac{\alpha}{2} \cos \frac{\alpha}{2^2} \cos \frac{\alpha}{2^3} \dots \cos \frac{\alpha}{2^n} \sin \frac{\alpha}{2^n}$$

$$\alpha = \frac{\pi}{2^n} \Rightarrow 1 = 2^n \cos \frac{\pi}{2^n} \cos \frac{\pi}{2^{2n}} \dots \cos \frac{\pi}{2^n}$$



A person in a white shirt is standing in front of the chalkboard, writing. The person's back is to the camera. They are pointing towards the right side of the board with their right hand.