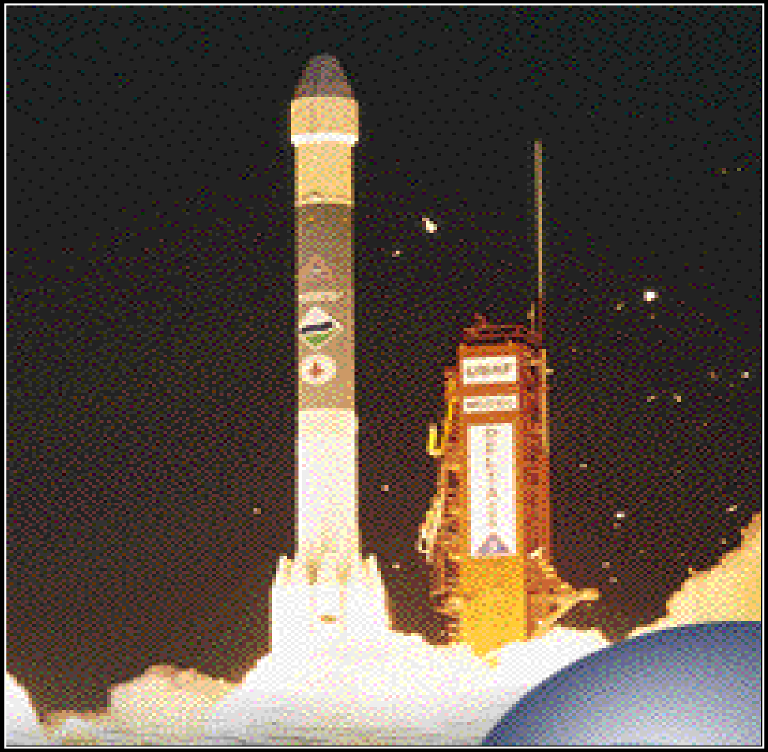
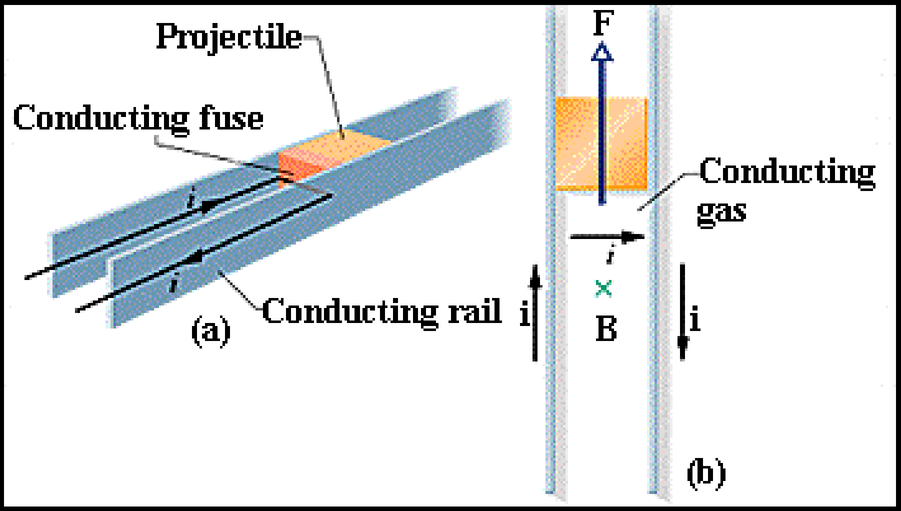
9 電流與磁場、安培定律

Magnetic Fields due to Currents



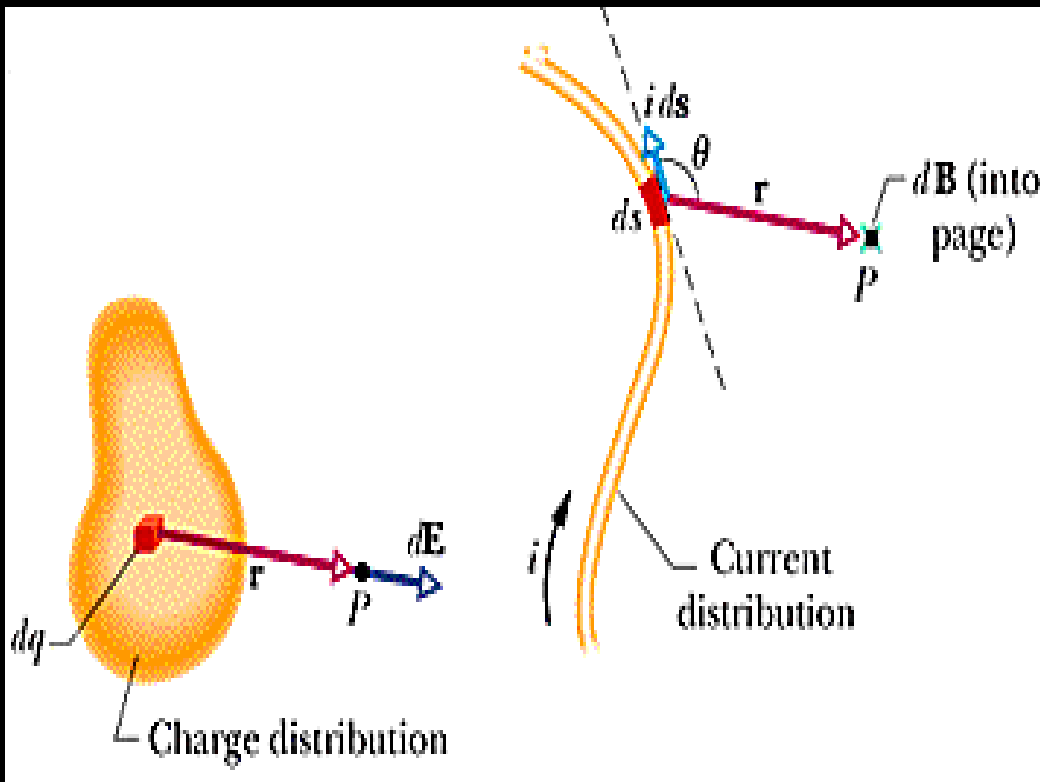


Conventional rocket

The space shuttle

EM Rail Gun

9-1 Calculating the Magnetic Field due to a current



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

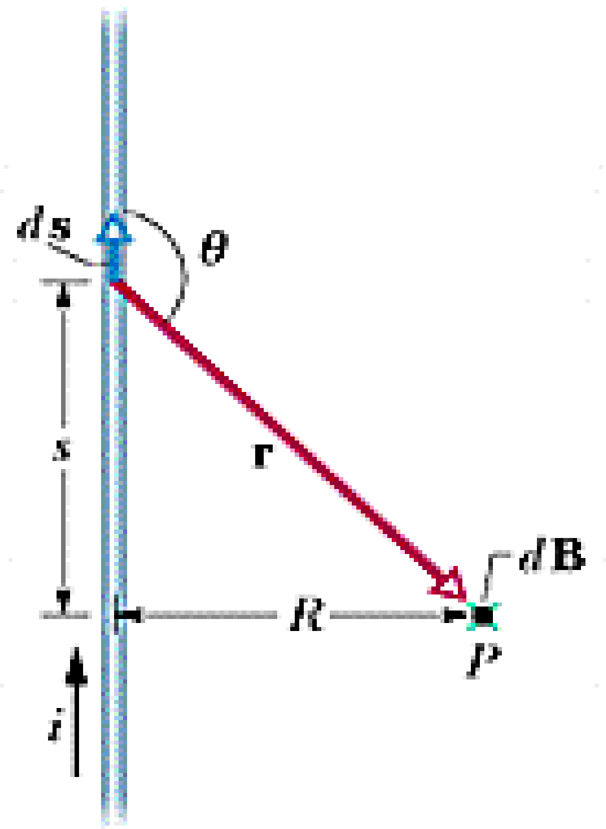
$$dB = \frac{\mu_0}{4\pi}\frac{ids\ \sin\theta}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3}$$

The law of Biot and Savart (必歐-沙伐定律)

Magnetic Field Due to a Current in a Long Straight Wire

- $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ permeability



$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin\theta}{r^2}$$

$$B = \int_{-\infty}^{\infty} dB = 2\int_0^{\infty} dB$$

$$= \frac{\mu_0 i}{2\pi} \int_0^{\infty} \frac{\sin\theta\, ds}{r^2}$$

Integration

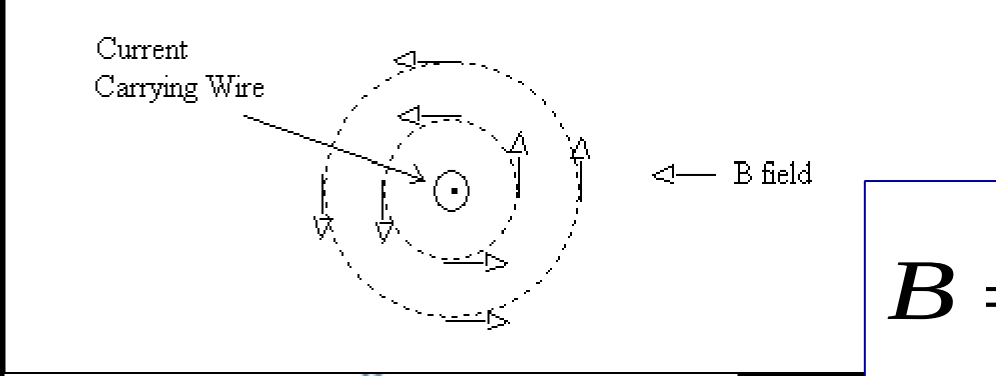
$$r = \sqrt{s^2 + R^2}, \sin\theta = \frac{R}{\sqrt{s^2 + R^2}}$$

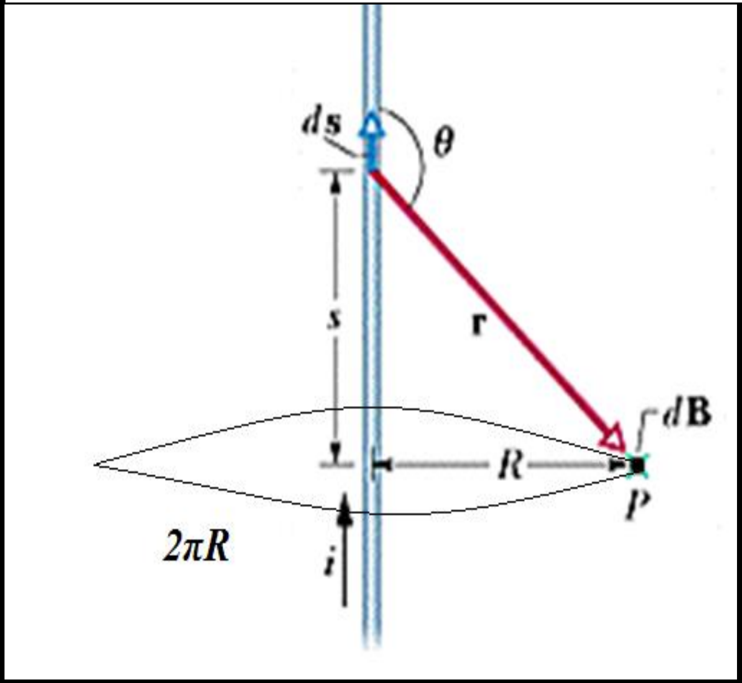
$$B = \frac{\mu_0 i}{2\pi} \int_0^\infty \frac{R ds}{(s^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi R} [\frac{s}{(s^2 + R^2)^{1/2}}]_0^\infty = \frac{\mu_0 i}{2\pi R}$$

A hint of Ampere's Law

Figure 1: B field near a current-carrying wire.





$$B = \frac{\mu_0 i}{2\pi R}$$

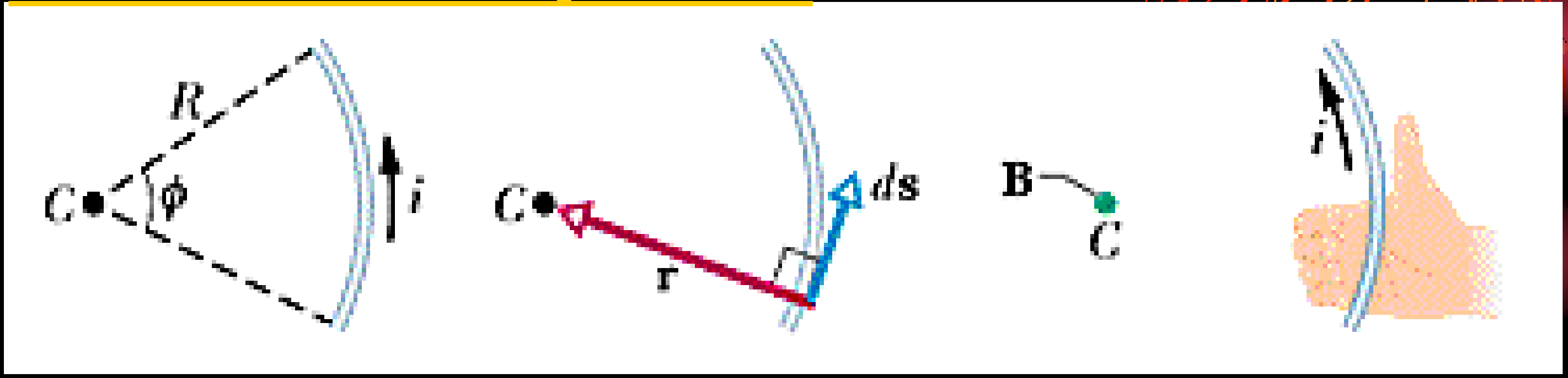
$$B \times 2\pi R = \mu_0 i$$

$$B \times \oint dl = \mu_0 i$$

$$\oint B dl = \mu_0 i$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

Magnetic Field Due to a Current in a Circular Arc of Wire

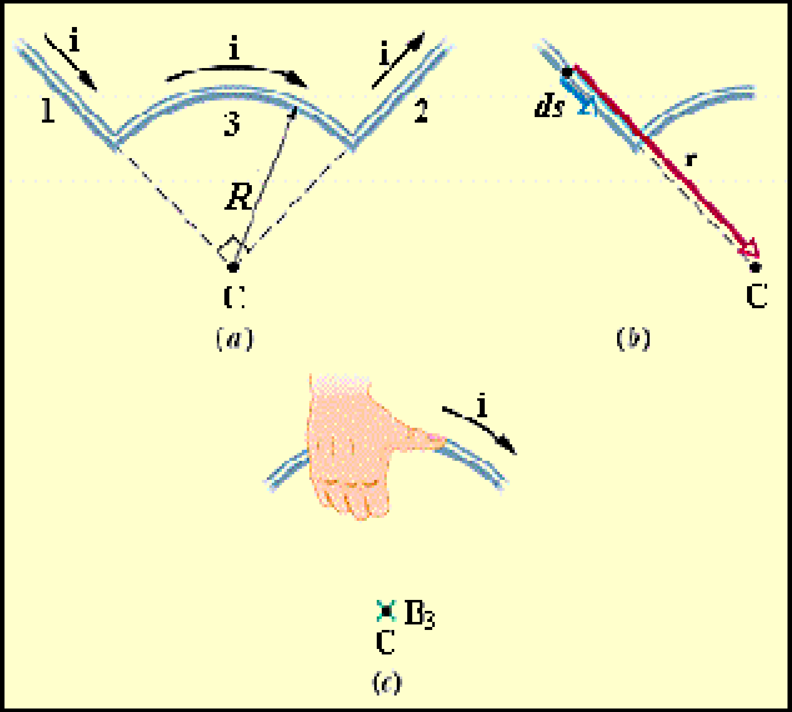


$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{ids}{R^2}$$

$$B = \int dB = \int_0^{\phi}\frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2}$$

$$= \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi = \frac{\mu_0 i\phi}{4\pi R} \xrightarrow{\bigcirc} \frac{\mu_0 i}{2R}$$

例 1 What B does the current produce?

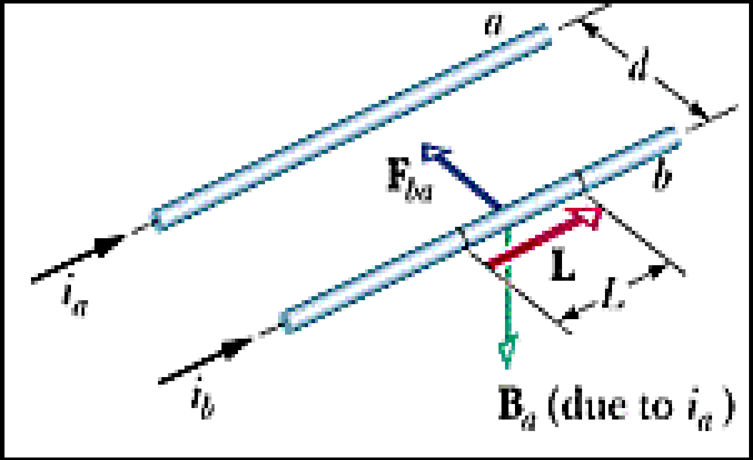


$$d\vec{s} \times \vec{r} = 0$$

$$B_1 = B_2 = 0$$

$$B_3 = \frac{\mu_0 i(\pi / 2)}{4\pi R} = \frac{\mu_0 i}{8R}$$

9-2 Two Parallel Currents



$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

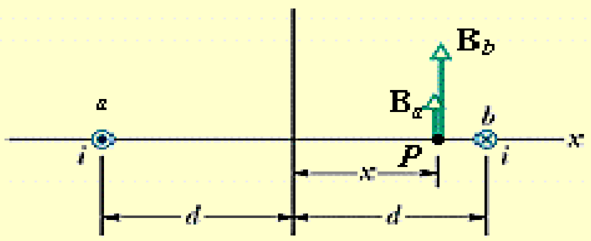
$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a$$

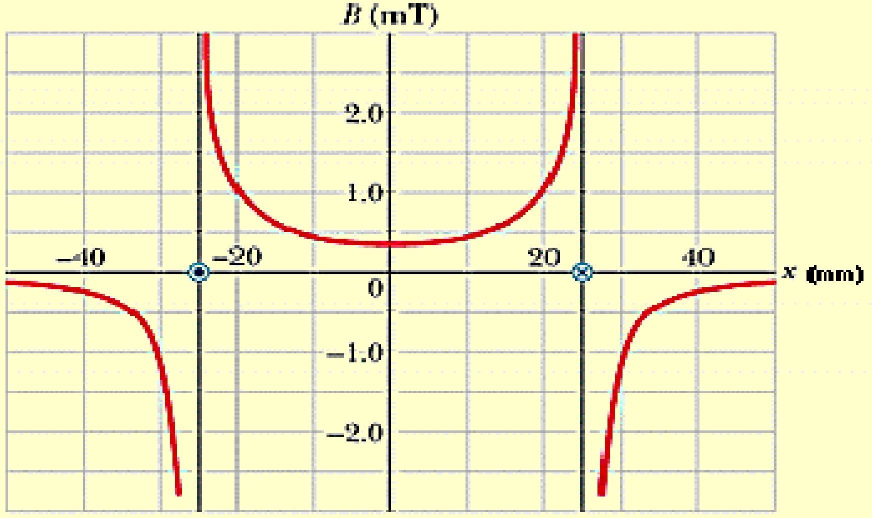
$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}$$

例 2 The Field Between Two wires

$$B(x) = B_a(x) + B_b(x)$$

$$= \frac{\mu_0 i}{2\pi(d+x)} + \frac{\mu_0 i}{2\pi(d-x)} = \frac{\mu_0 i d}{\pi(d^2 - x^2)}$$





9-3 Ampere' s Law

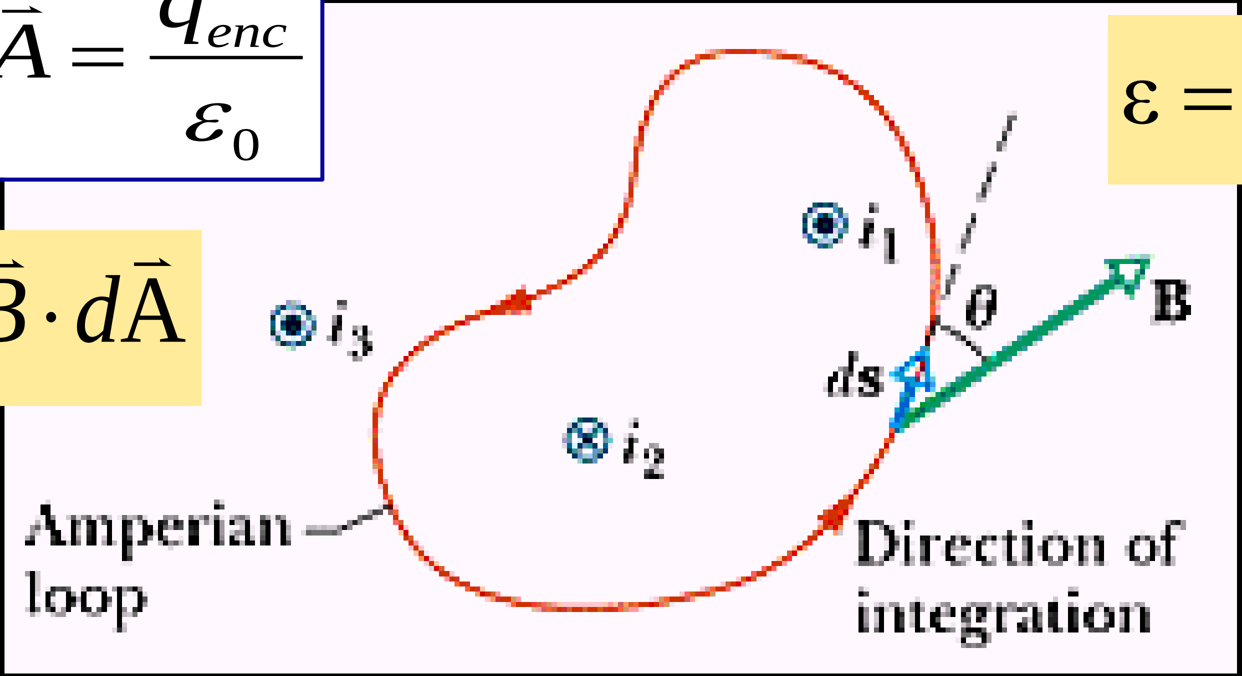
- Comparing Gauss' law and Ampere's law
- Ampere' s law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

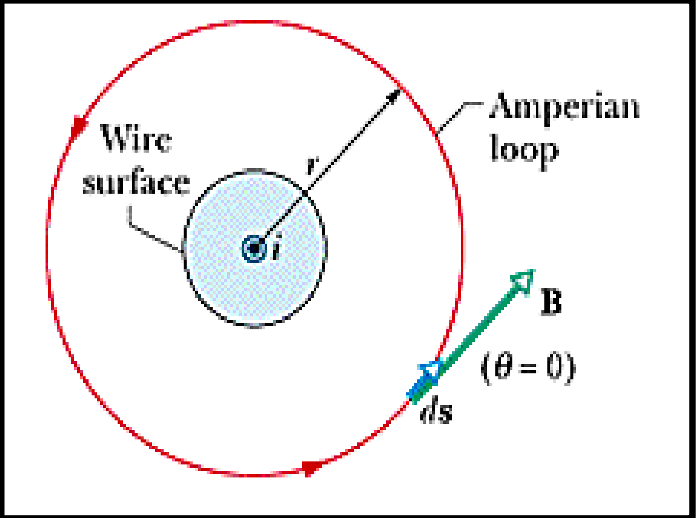
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\varepsilon = \oint \vec{E} \cdot d\vec{s}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$



The Magnetic Field Outside a Long Straight Wire with Current

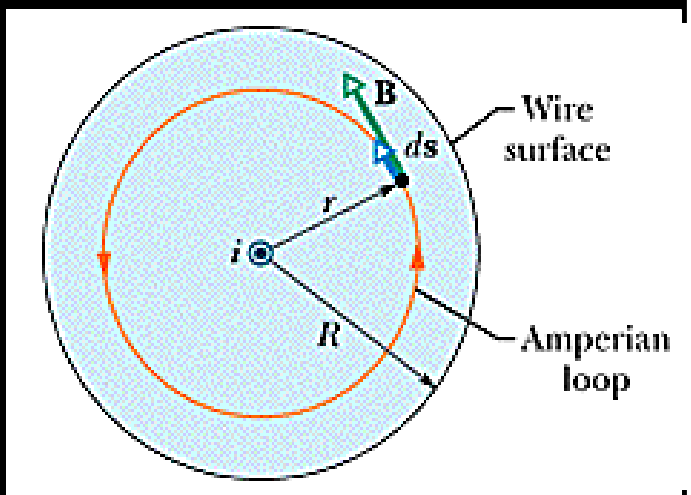


$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta ds$$

$$= B\oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i$$

The Magnetic Field Inside a Long Straight Wire with Current

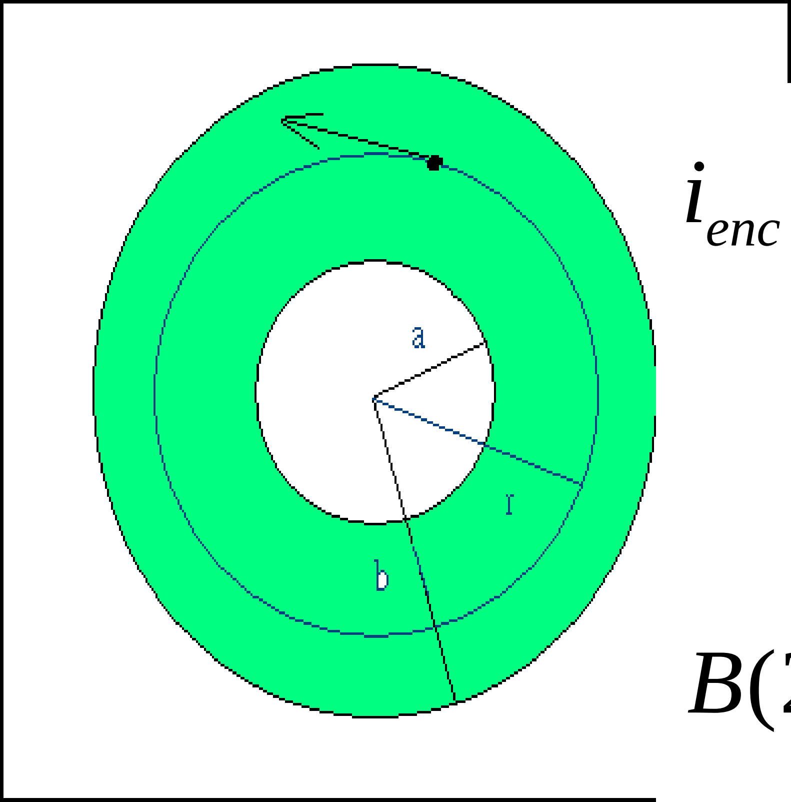


$$\oint \vec{B} \cdot d\vec{s} = B(2\pi r)$$

$$i_{enc} = i \frac{\pi r^2}{\pi R^2}$$

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

例 3 A hollow conducting cylinder



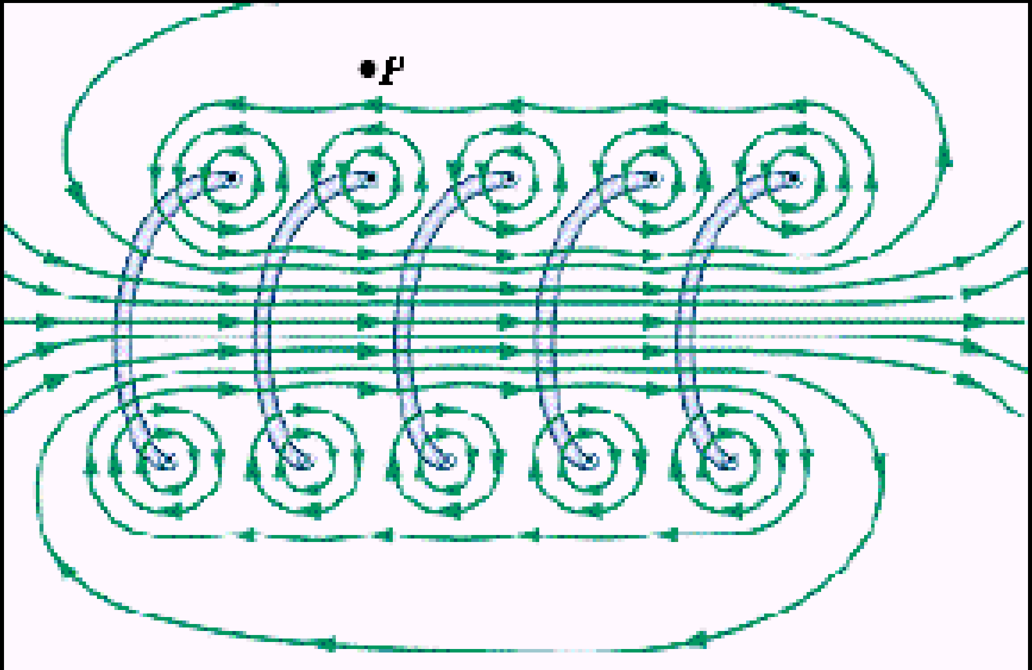
$$i_{enc} = \int J dA = \int_a^r cr^2 (2\pi r dr)$$

$$= \frac{\pi c (r^4 - a^4)}{2}$$

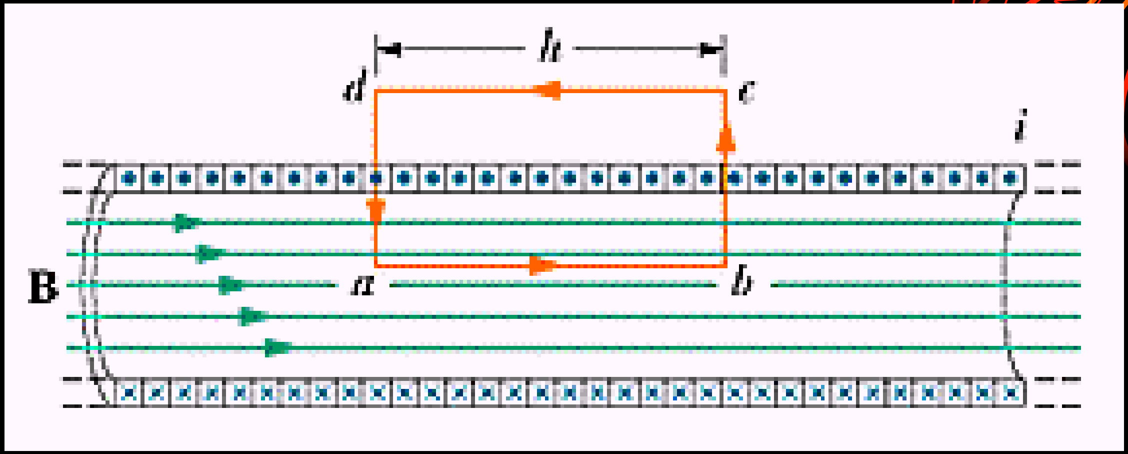
$$B(2\pi r) = -\frac{\mu_0 \pi c (r^4 - a^4)}{2}$$

9-4 Solenoids and Toroids

- Magnetic Field of a Solenoid (螺線管)
- Magnetic Field of a Toroid (螺線環)



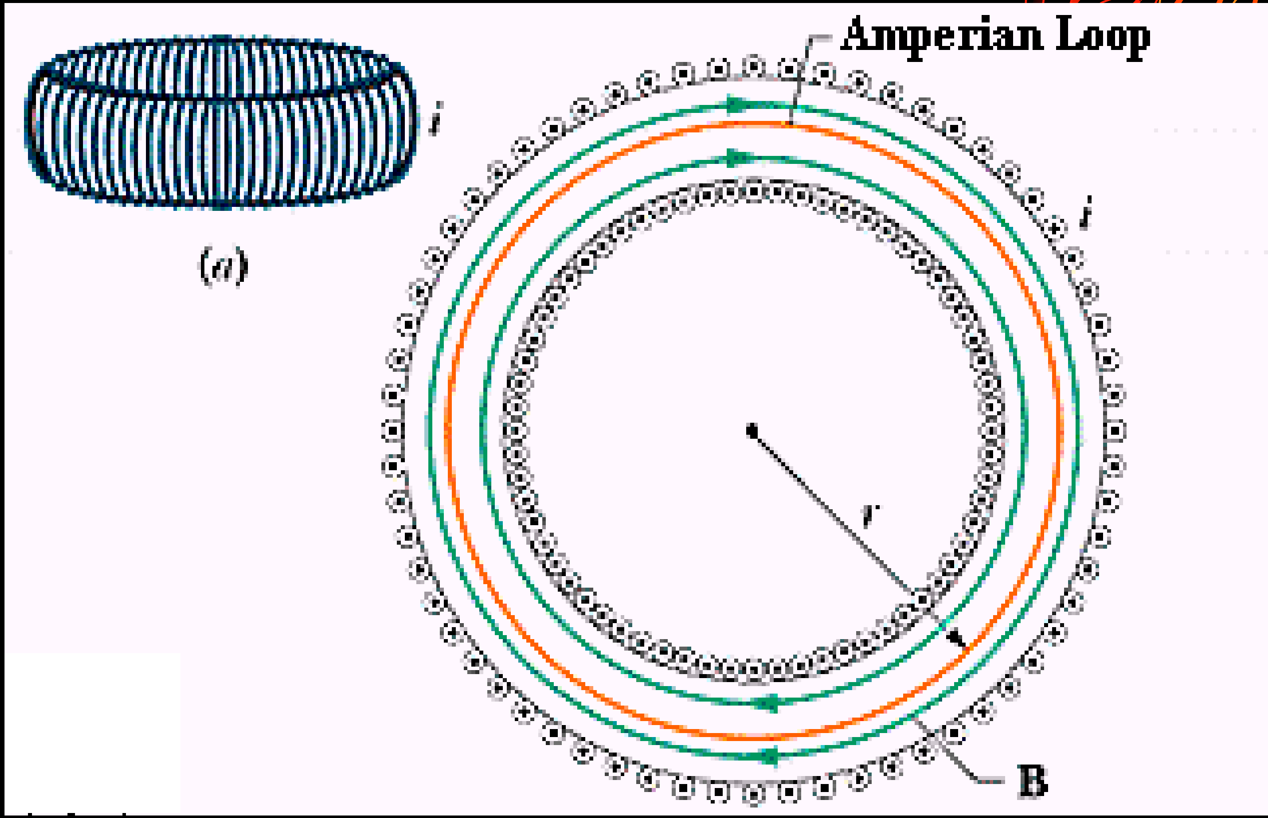
Magnetic Field of a Solenoid



$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}$$

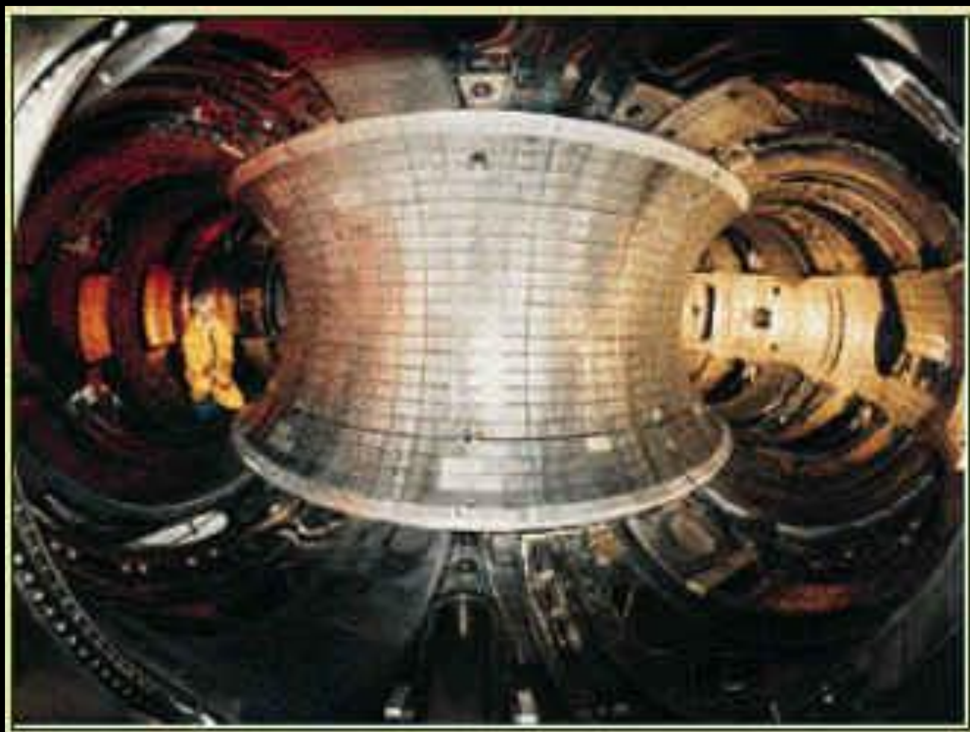
$$Bh = \mu_0 inh \qquad B = \mu_0 in$$

Magnetic Field of a Toroid

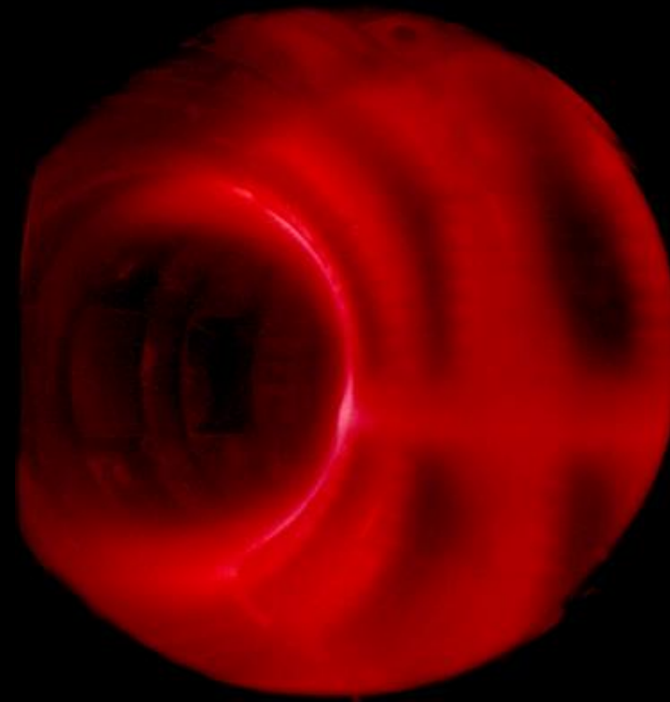


$$B(2\pi r) = \mu_0 iN \quad B = \frac{\mu_0 iN}{2\pi r}$$

磁圍阻核融合反應器



Tokamak Fusion Test Reactor 1982 - 1997

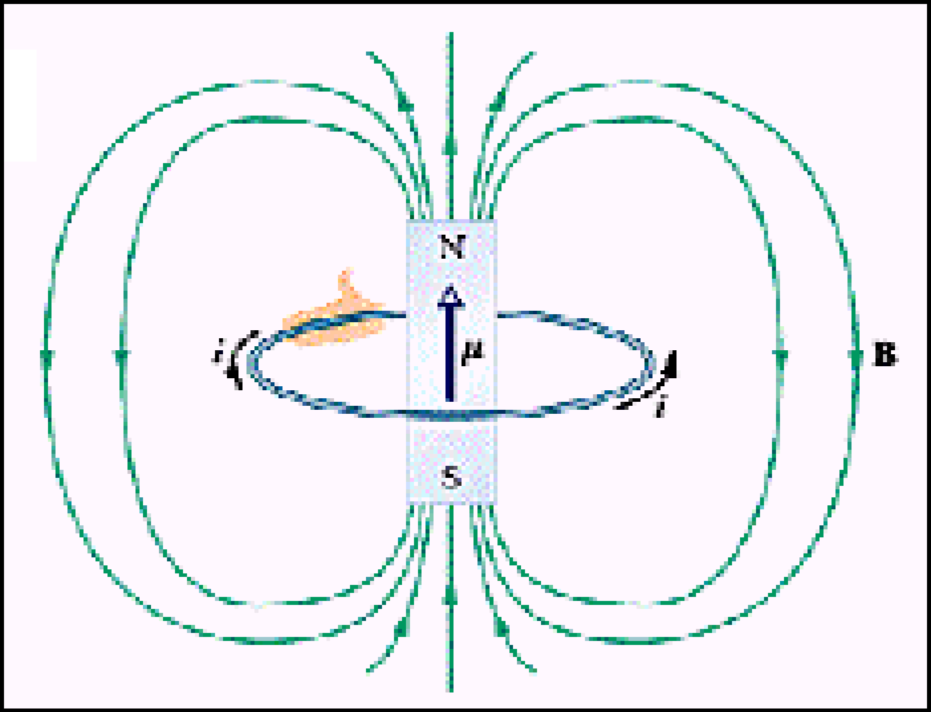


JET **ITER**

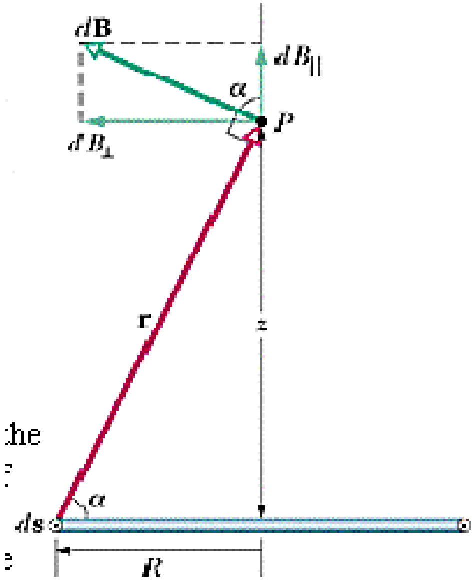
- **Joint European Torus**
- **International Thermonuclear Experimental Reactor**

9-5 A Current Carrying Coil as a Magnetic Dipole

- A current loop and a bar magnet



Magnetic Field of a Coil



$$B = \int dB \cos\alpha$$

$$= \int \frac{\mu_0 i ds}{4\pi r^2} \frac{R}{r}$$

$$= \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} 2\pi R$$

$$= \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$

$$\rightarrow \frac{\mu_0 i}{2R} \text{(at center of coil)}$$