10 電磁感應

How did the electric guitar revolutionize rock?

From acoustic to electric guitars

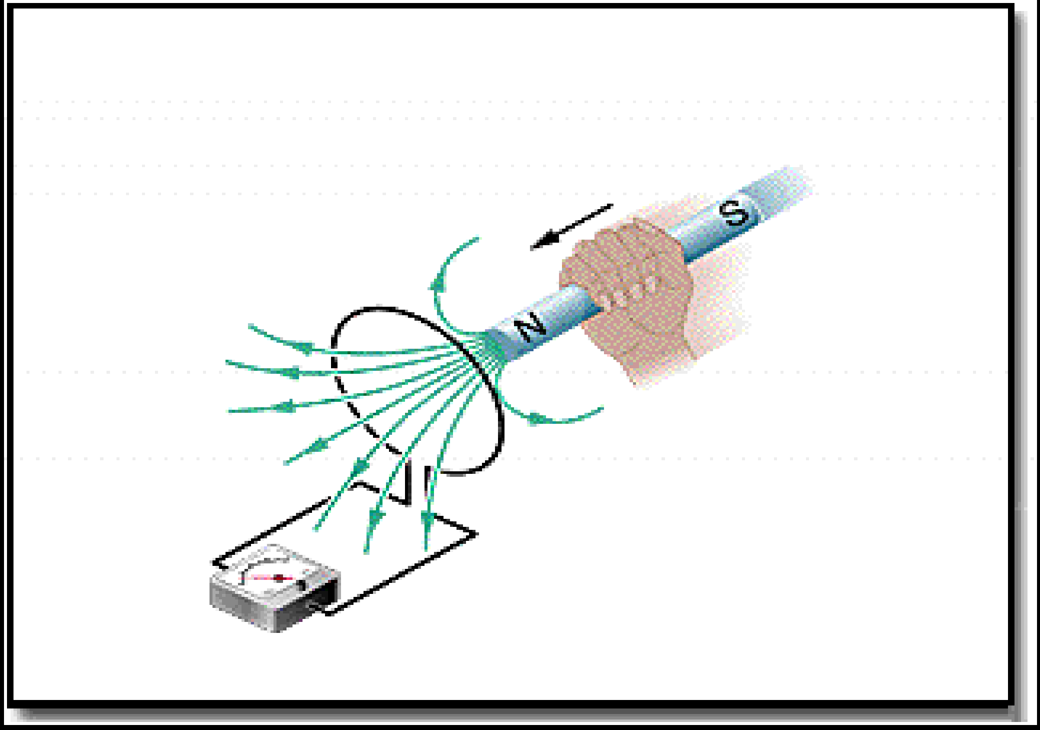
10-1 Two symmetric Situations

- current loop + magnetic field → torque
- torque + magnetic field → current

10-2 Two Experiments

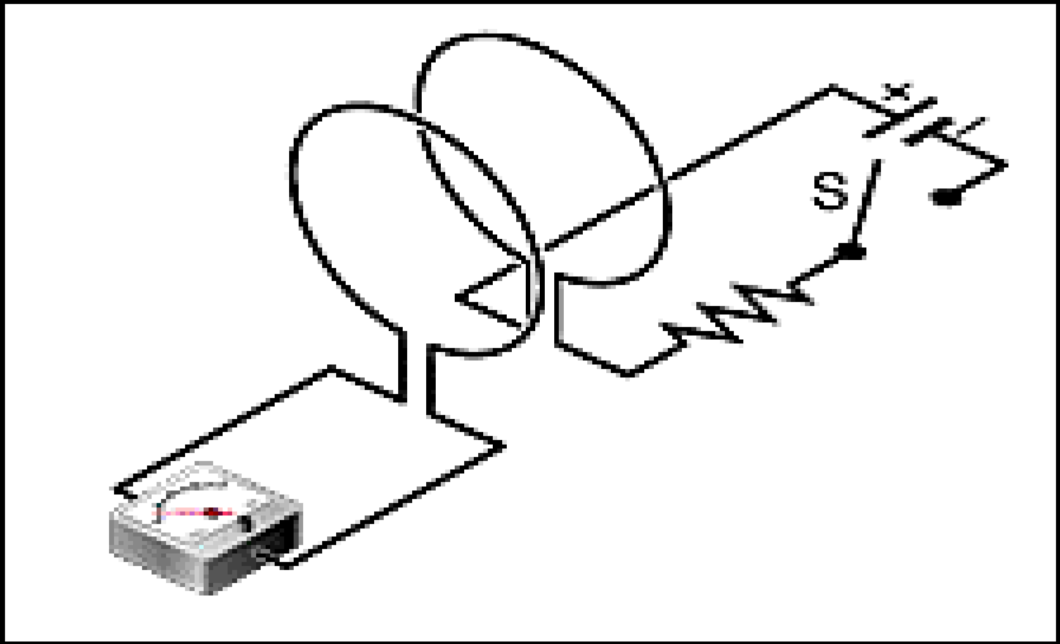
First Experiment

A Magnet is moving with respect to the loop.



Second Experiment

The switch S is closed or opend.



10-3 Faraday's Law of Induction

Electromotive force

- An emf is induced when the number of magnetic field lines that pass through the loop is changing.
  - A Quantitative Treatment
  - The magnetic flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad 1\text{Weber} = 1\text{Wb} = 1\text{T} \cdot \text{m}^2$$

Faraday's law

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad 1\,\text{Weber} = 1\,\text{Wb} = 1\,\text{T} \cdot \text{m}^2$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} \quad \text{(N turns)}$$

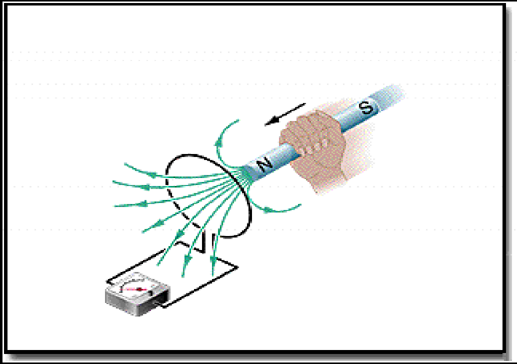
$$\varepsilon = \oint \vec{E} \cdot d\vec{s}$$

emf

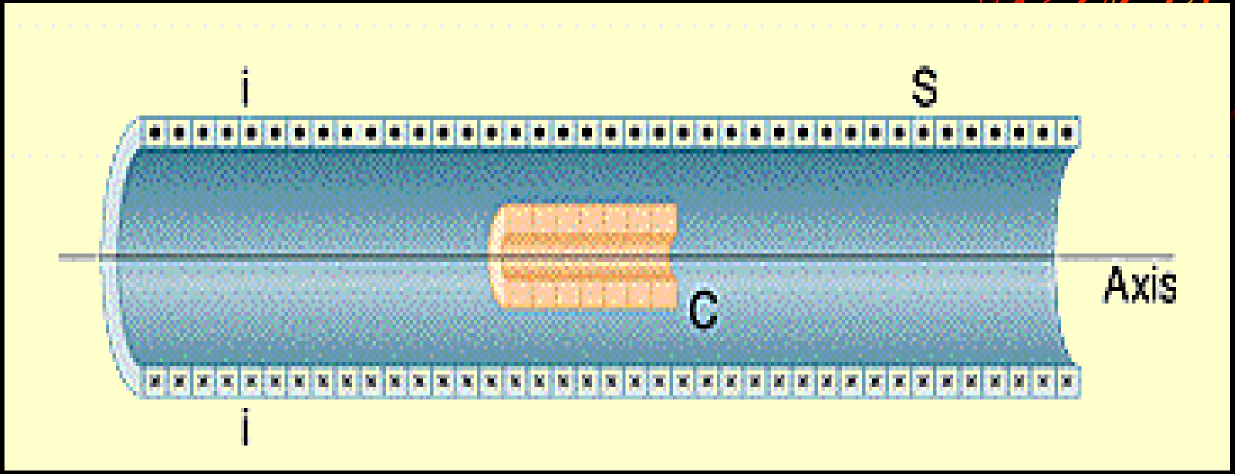
magnetic flux

induced emf

induced electric field



Ex.1 A solenoid contains a coil



- $i = 1.5\ A$ $n_s = 2.2 \times 10^4$ turns / m
- $A = 3.46 \times 10^{-4} m^2$ $N_c = 130$ turns
- 電流在25ms內穩定降至0

計算

- $i = 1.5\,A$　$n = 2.2 \times 10^4$ turns / m
- $A = 3.46 \times 10^{-4} m^2$　$N = 130$ turns
- 電流在25ms內穩定降至0

$$B_i = \mu_0 i n = 4.15 \times 10^{-2}\,\mathrm{T}$$

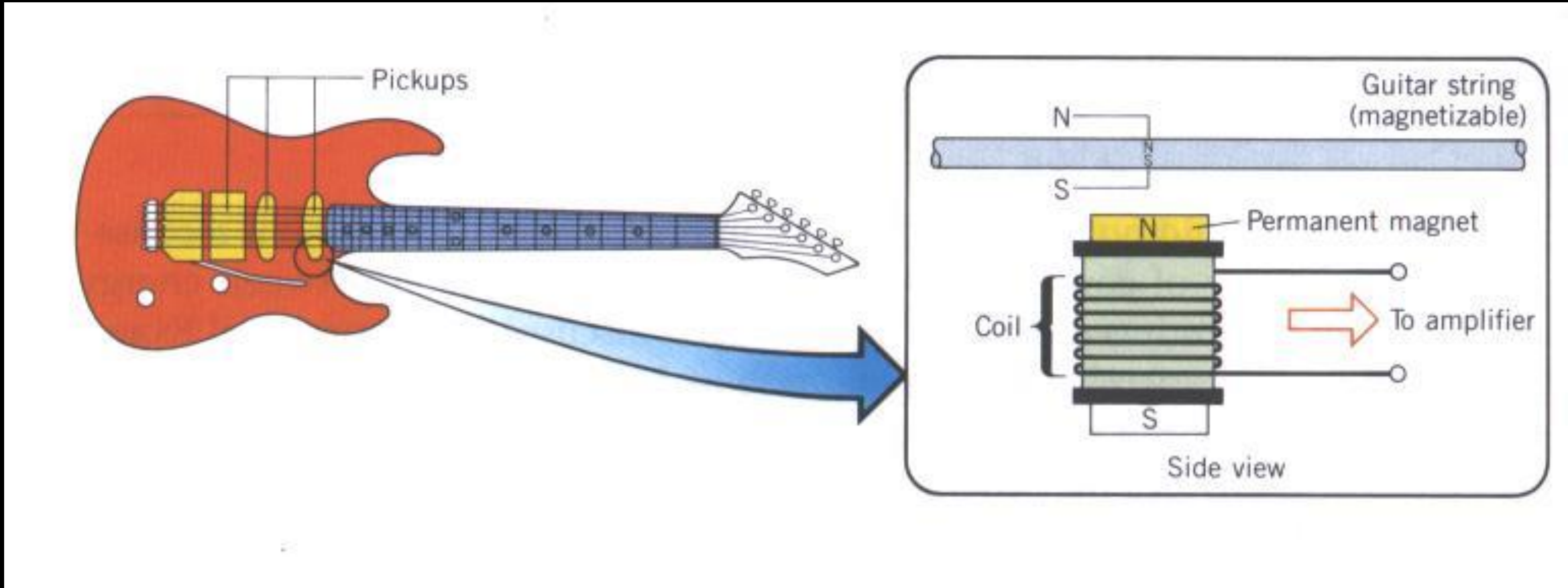
$$\Phi_{B,i} = BA = 14.4\,\mu\mathrm{Wb}$$

$$\varepsilon = N\Delta\Phi_B / \Delta t = 75\,\mathrm{mV}$$

10-4 Lenz's Law

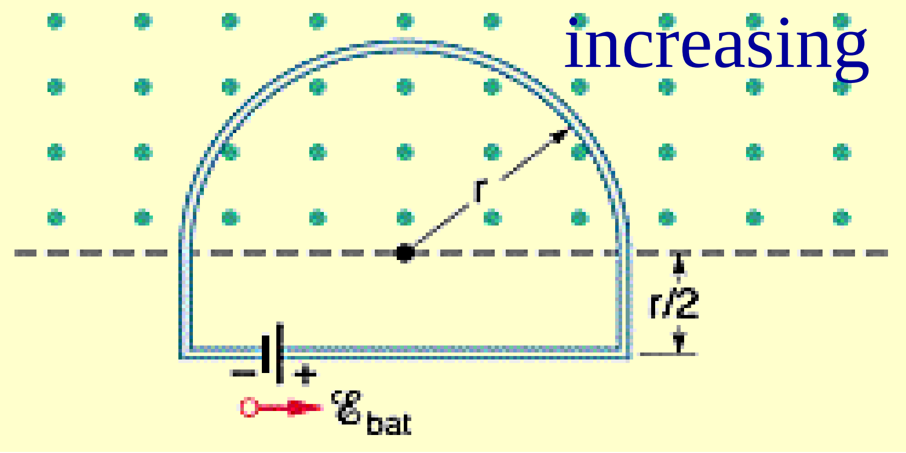
感應電流所生的磁場與感應出該電流的磁場反向

- Opposition to pole Movement
- Opposition to flux change
- Electric Guitars



Ex.2 $B = 4.0\,t^2 + 2.0\,t + 3.0$

$\mathcal{E}_{bat} = 2.0V \quad r = 0.20m \quad R = 2.0\Omega$



$$\varepsilon = \frac{d\Phi_B}{dt} = A\frac{dB}{dt}$$

$$= \frac{\pi r^2}{2}(8.0t^2 + 2.0)$$

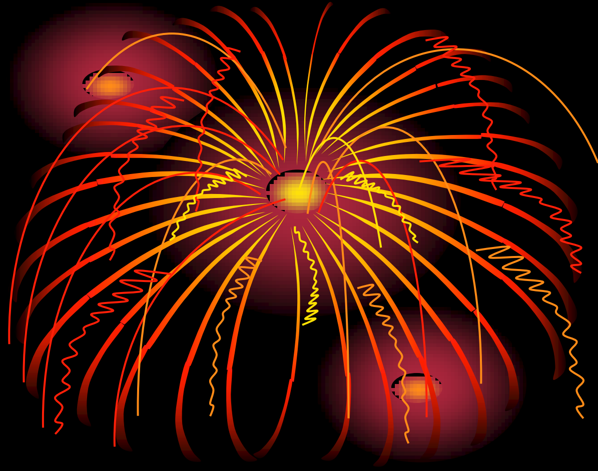
$$= 5.2V \quad (t = 10)$$

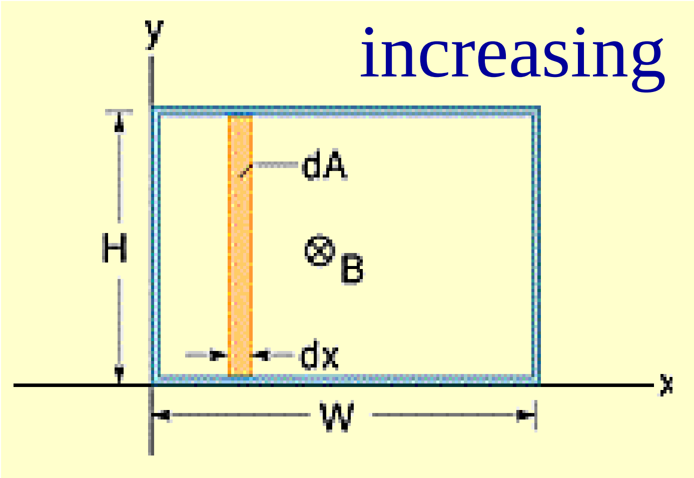
$$i = \frac{\mathcal{E}\text{net}}{R} = \frac{\mathcal{E}\text{ind} - \mathcal{E}\text{bat}}{R} = 1.6\text{A}$$

clockwise

Ex.3 $B = 4t^2x^2$

$W = 3.0\text{m} \quad H = 2.0\text{m} \quad t = 0.1\text{s}$





$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\,dA$$

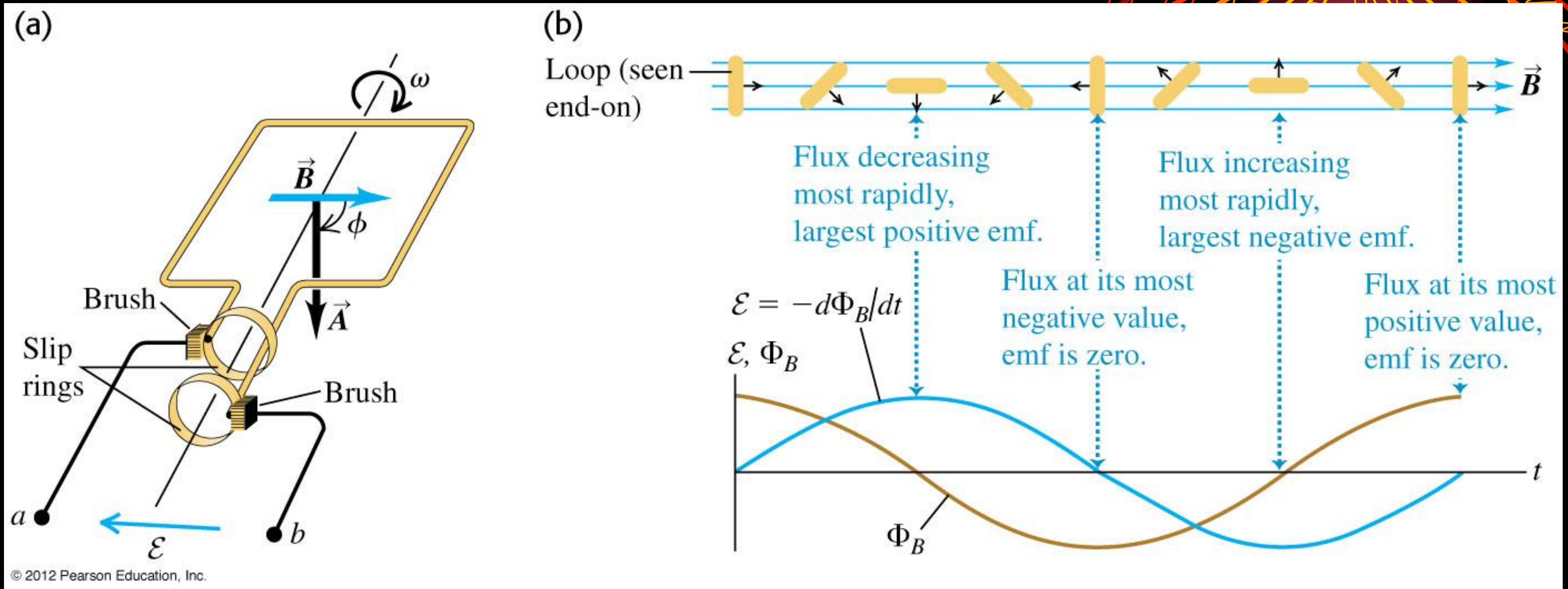
$$= \int BH\,dx = \int 4t^2x^2H\,dx$$

$$= 4t^2H\int_0^{3.0} x^2\,dx = 72t^2$$

$$\varepsilon' = \frac{d\Phi_B}{dt} = 144t = 14\text{V}$$

counterclockwise

Generator I: A simple alternator

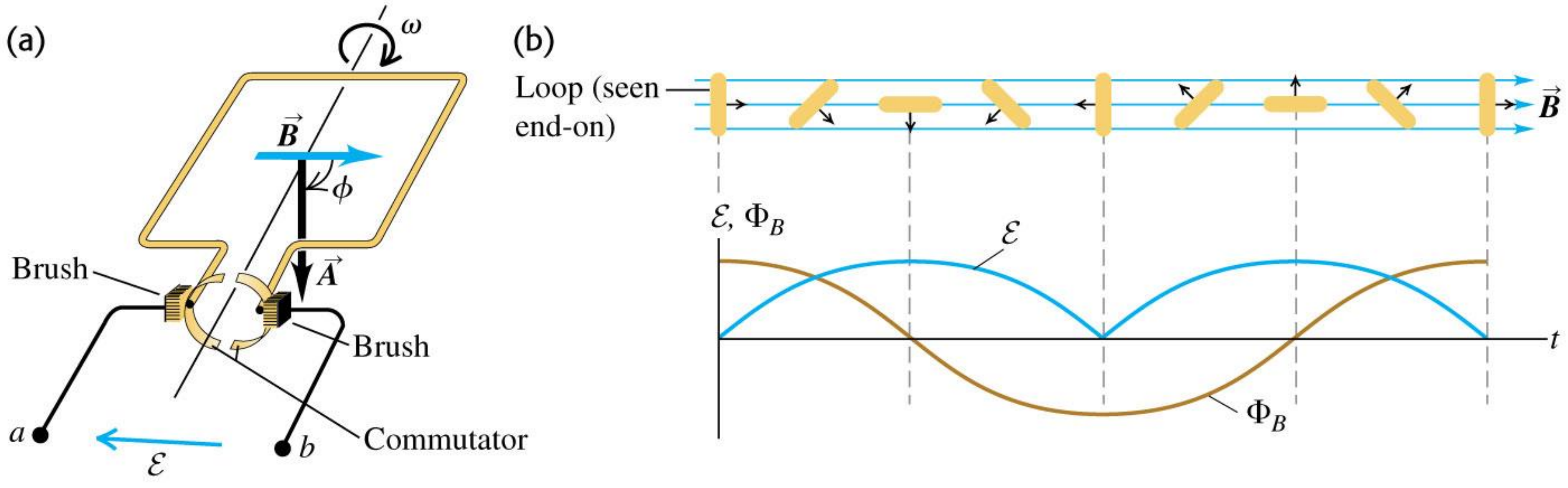


29.8 (a) Schematic diagram of an alternator. A conducting loop rotates in a magnetic field, producing an emf. Connections from each end of the loop to the external circuit are made by means of that end's slip ring. The system is shown at the time when the angle $\phi = \omega t = 90°$. (b) Graph of the flux through the loop and the resulting emf between terminals a and b, along with the corresponding positions of the loop during one complete rotation.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\omega t) = \omega BA\sin\omega t$$

A commercial alternator

Generator II: A DC generator and back emf in a motor



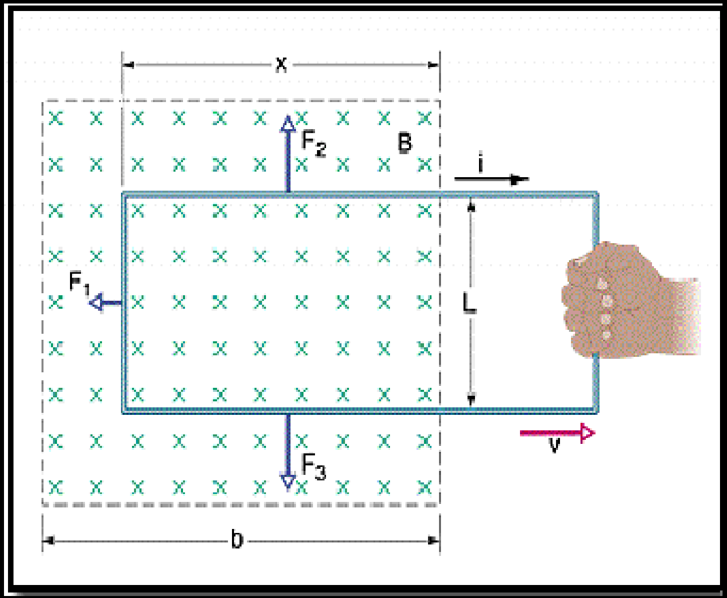
$$|\mathcal{E}| = N\omega BA|\sin\omega t|$$

$$(|\sin\omega t|)_{av} = \frac{\int_0^{\pi/\omega} \sin\omega t\, dt}{\pi/\omega} = \frac{2}{\pi}$$

$$\omega = \frac{\pi\mathcal{E}_{av}}{2NBA} = \frac{\pi(112\text{ V})}{2(500)(0.200\text{ T})(0.100\text{ m})^2} = 176\text{ rad/s}$$

10-5 Induction and Energy Transfers

The work done by the applied force and the thermal energy produced in the wire.



$$P = Fv\,,\ \Phi = BA = Blx$$

$$\varepsilon = d\Phi / dt = d(Blx)/dt$$

$$\varepsilon = Bldx/dt = Blv$$

$$i = Blv/R$$

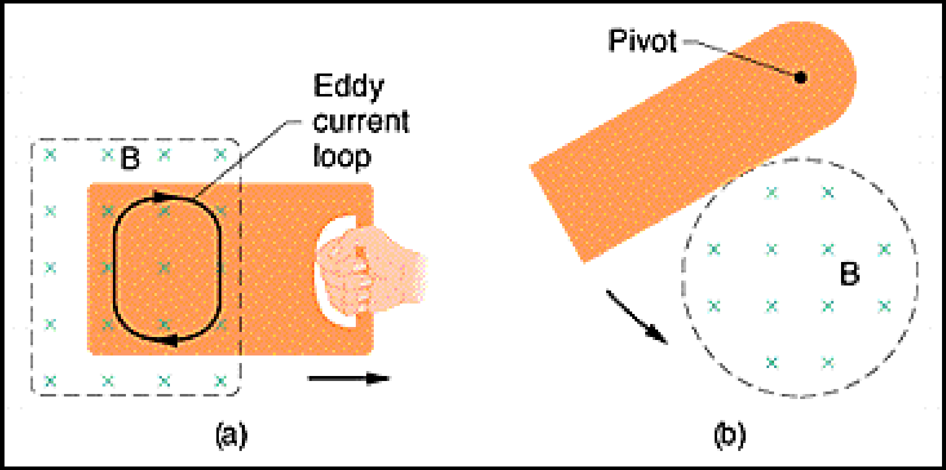
The Power

$$F = iLB = B^2L^2v / R$$

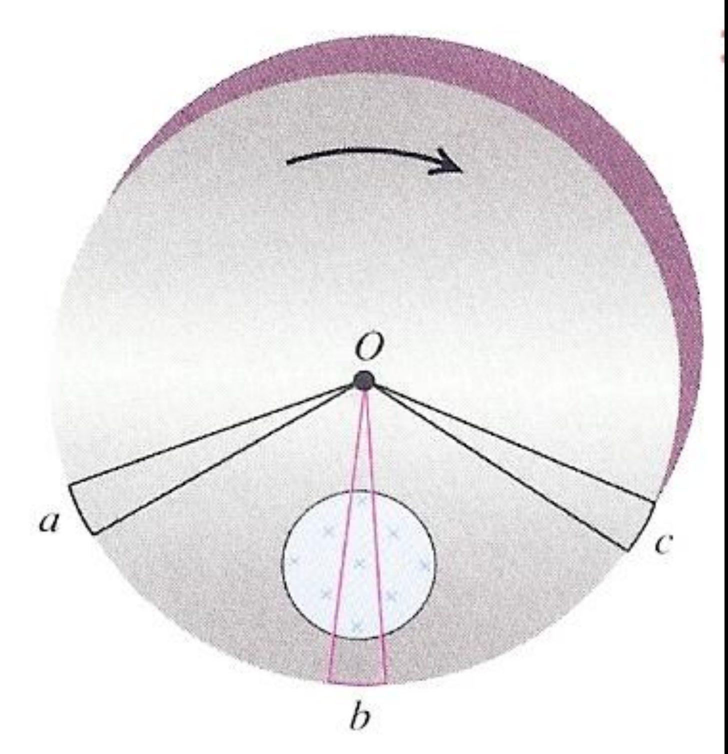
$$P_m = Fv = B^2L^2v^2 / R$$

$$P_{th} = i^2R = (BLv / R)^2 R = B^2L^2v^2 / R$$

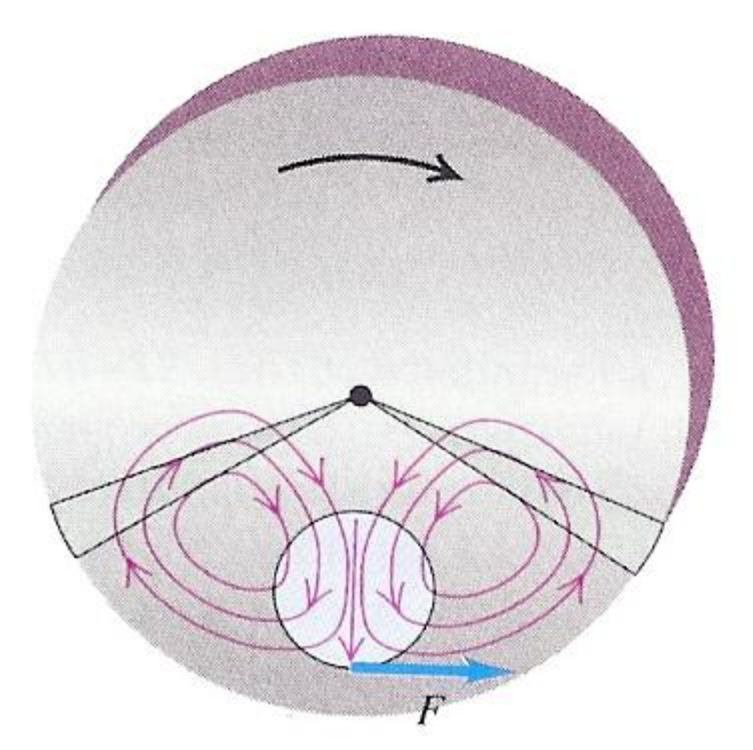
Eddy currents - induction stoves and EM braking



Eddy currents - EM braking

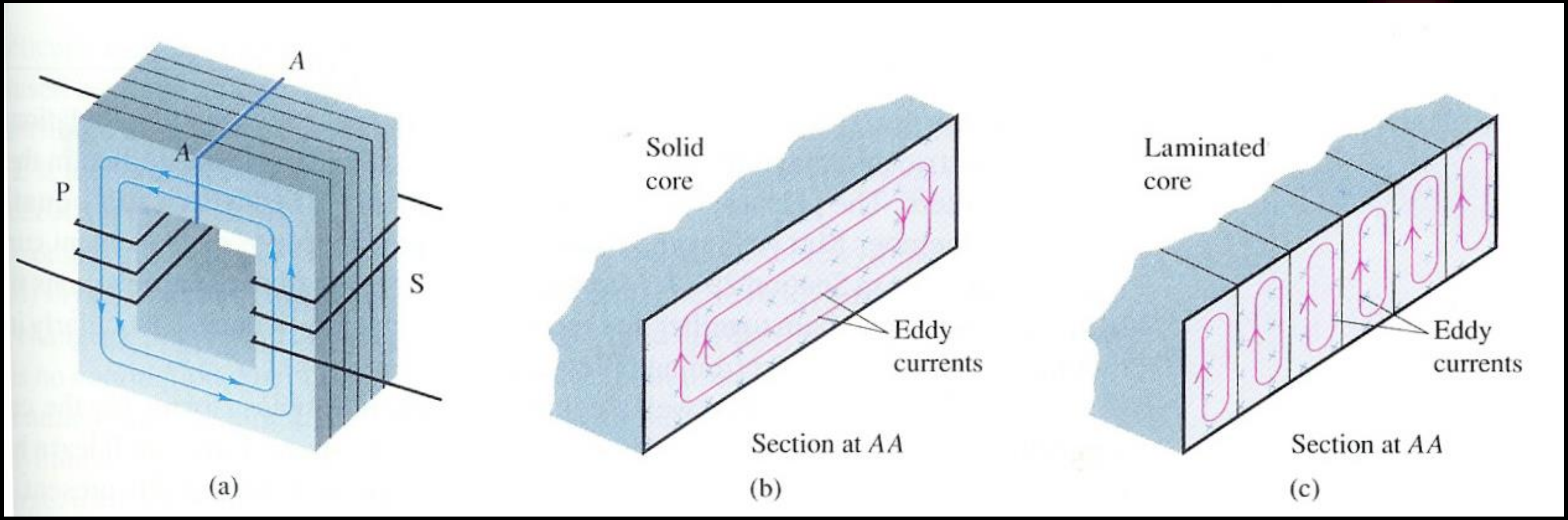


(a)

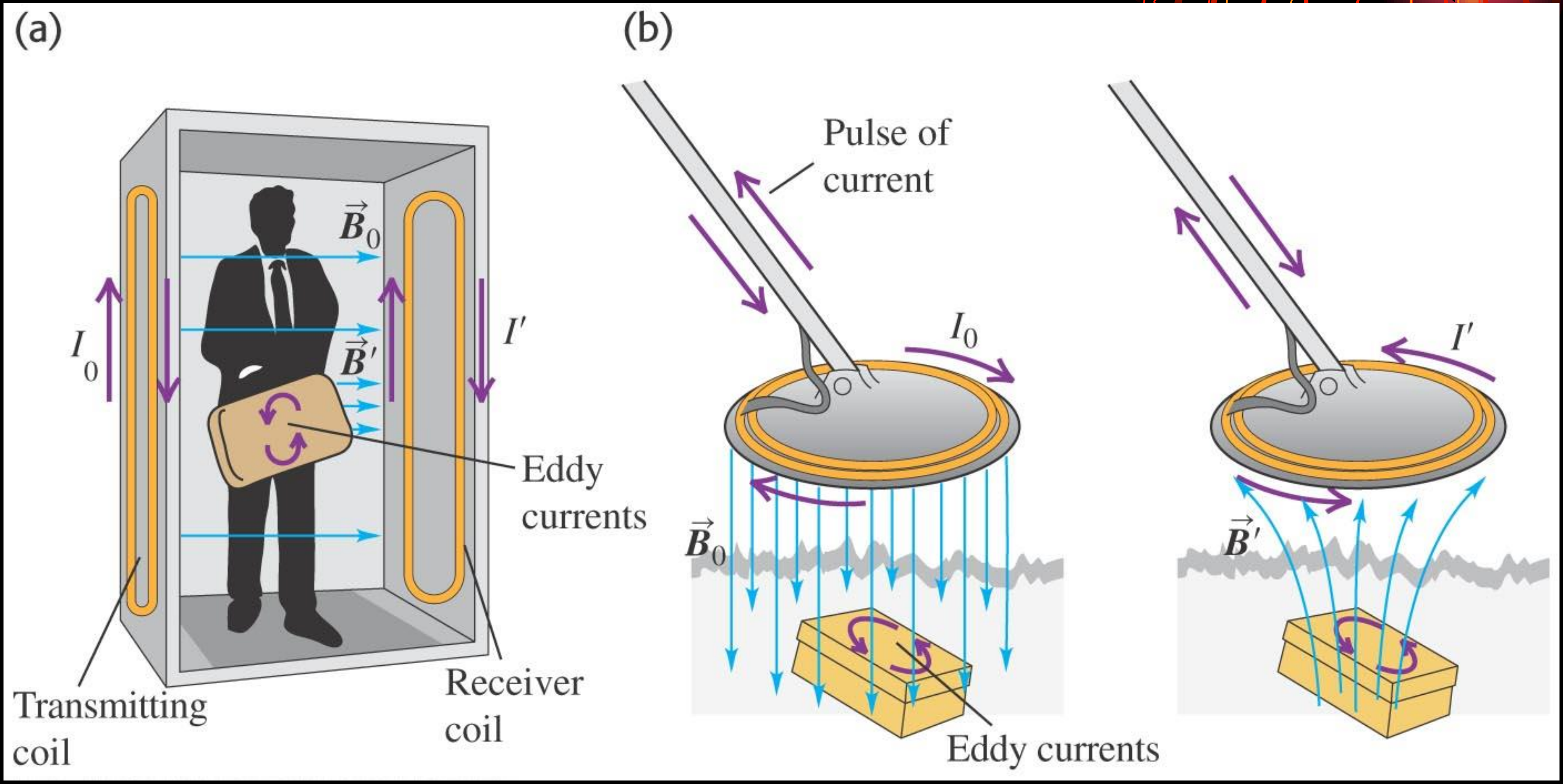


(b)

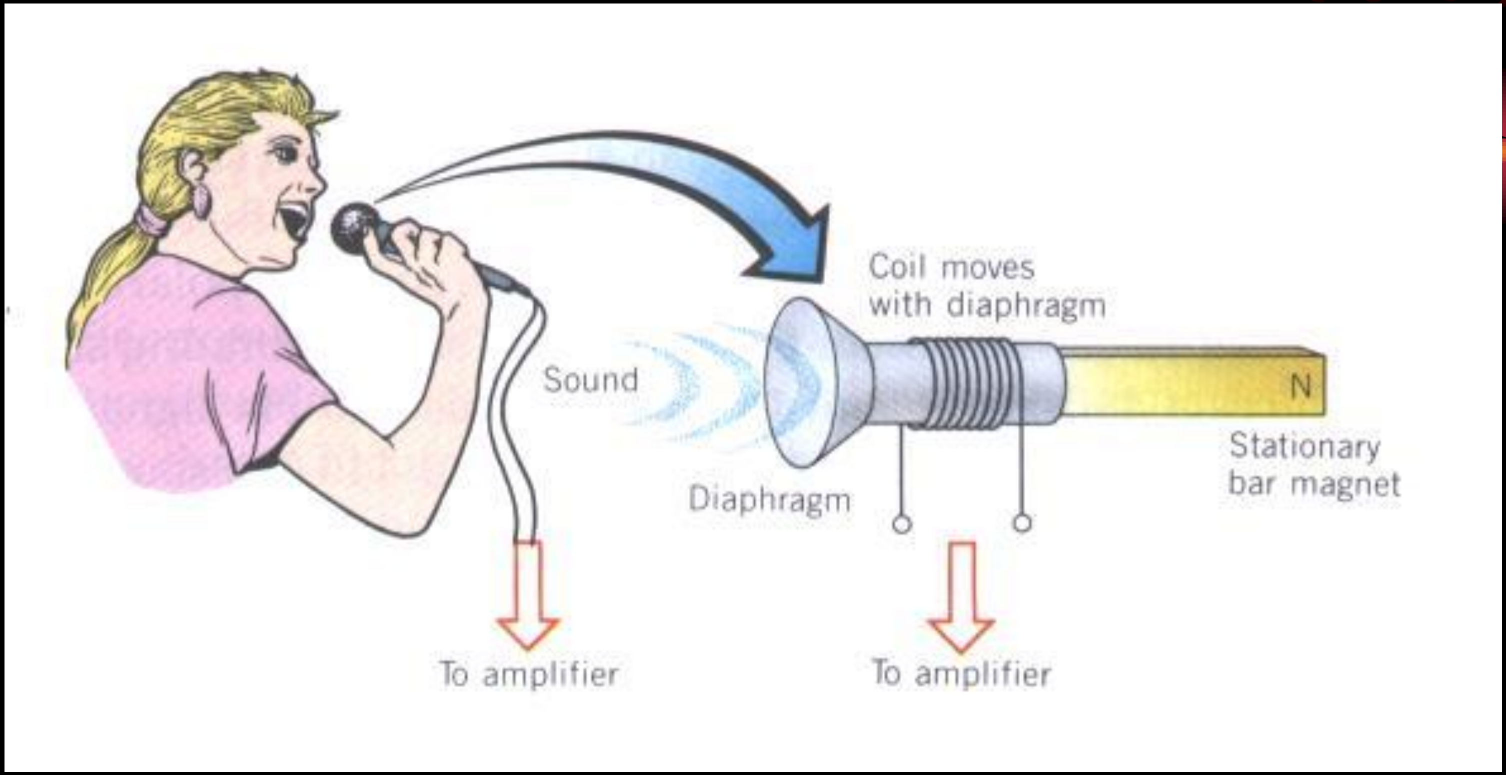
Eddy currents – transformers



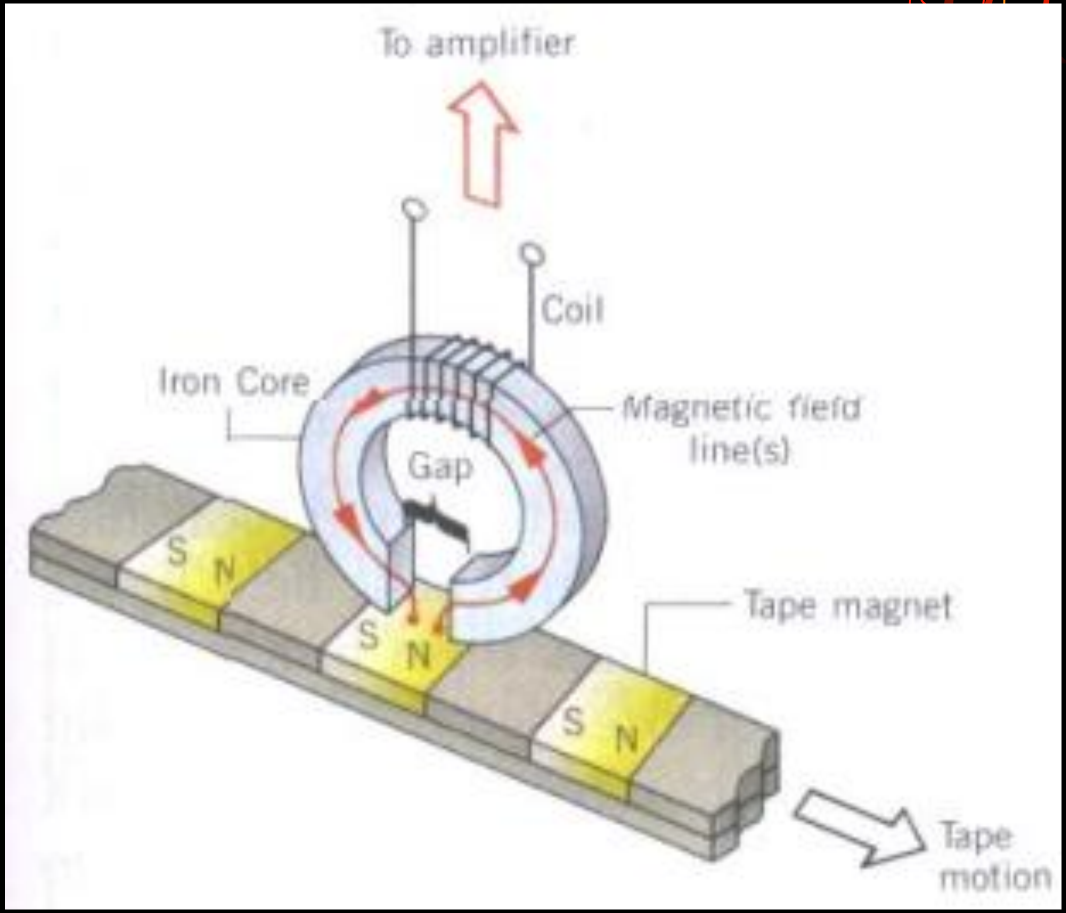
Eddy currents - metal detectors



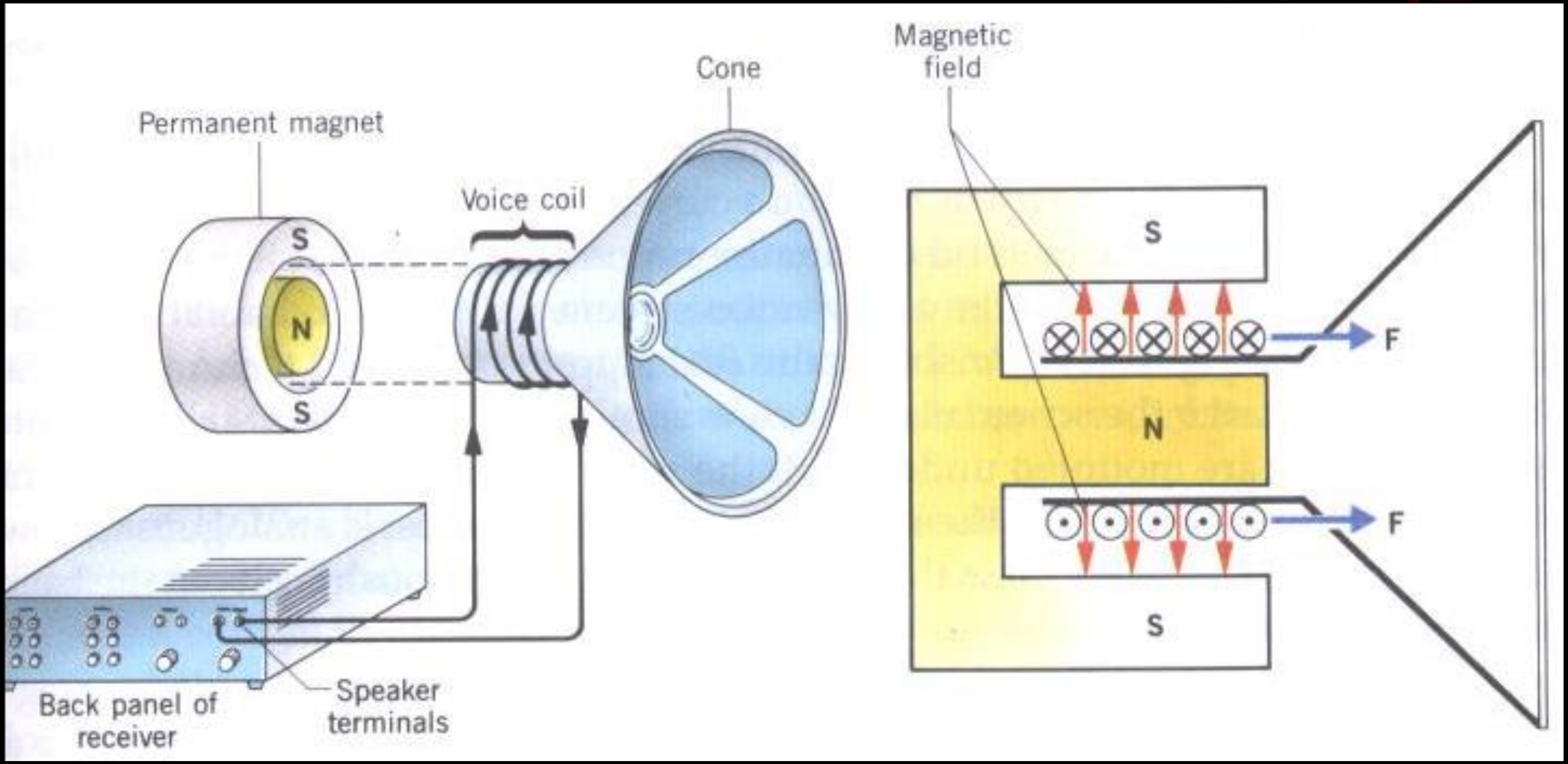
The microphone



The tape recorder



Speaker



Permanent magnet
Voice coil
Cone
Magnetic field
S
N
S
S
N
S
F
F
Back panel of receiver
Speaker terminals

Voice-coil positioner of a HDD

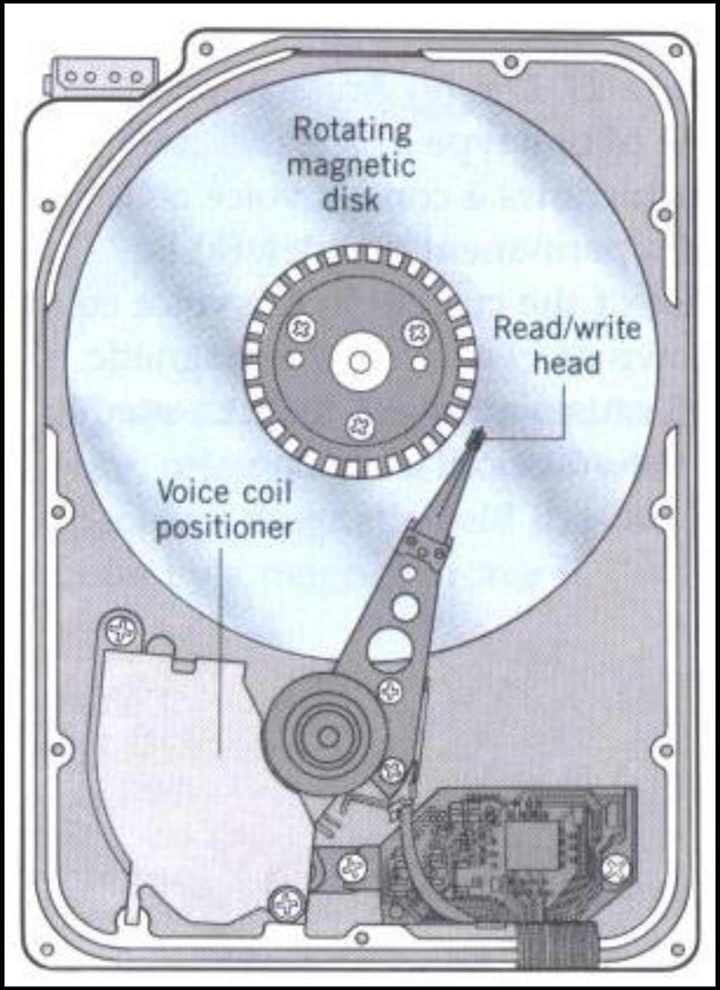
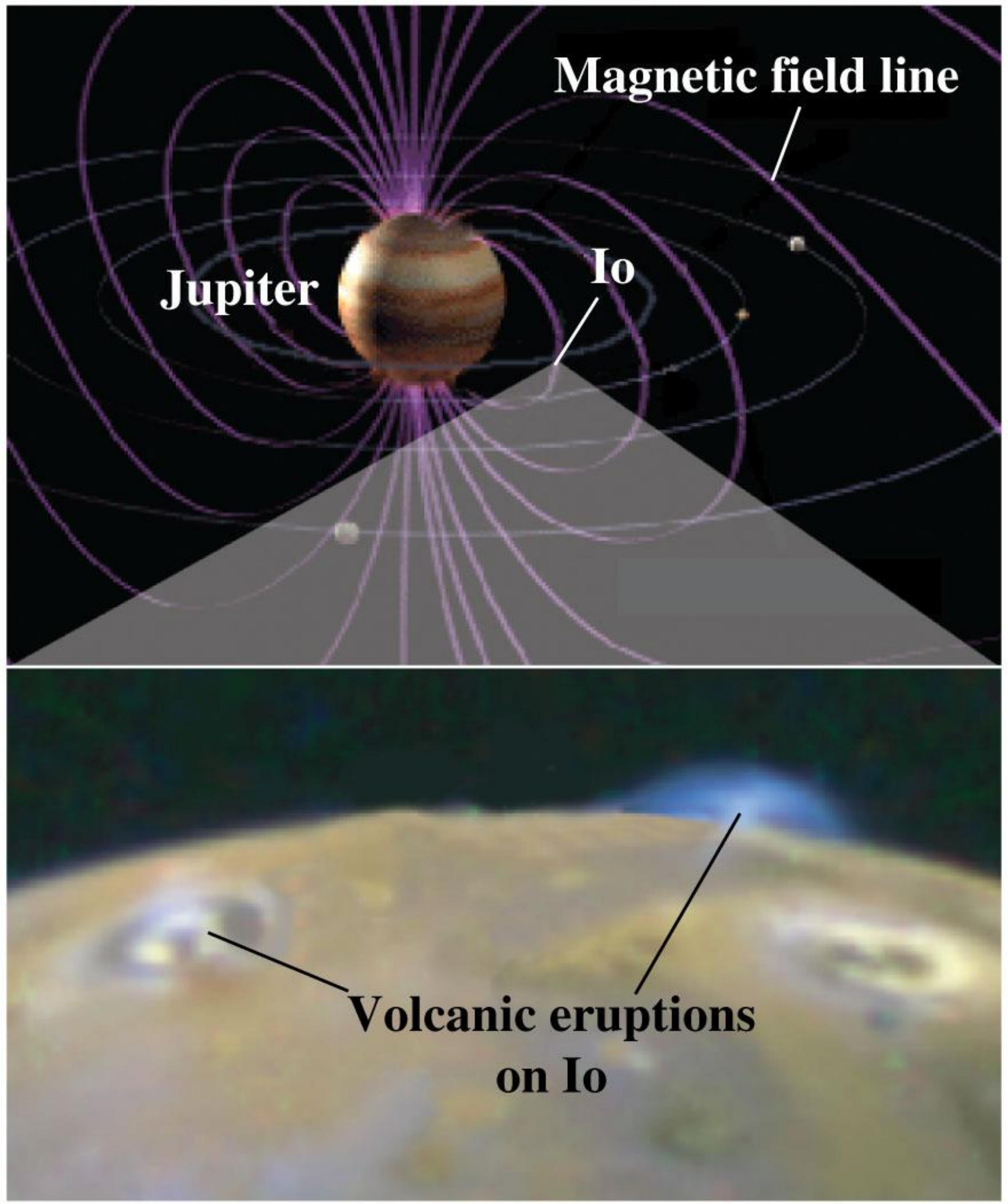


Figure 29.18

Figure 29.18a

Figure 29.18b

Figure 29.18c


Magnetic field line
Jupiter
Io
Volcanic eruptions on Io

10-6 Inductors and Inductance

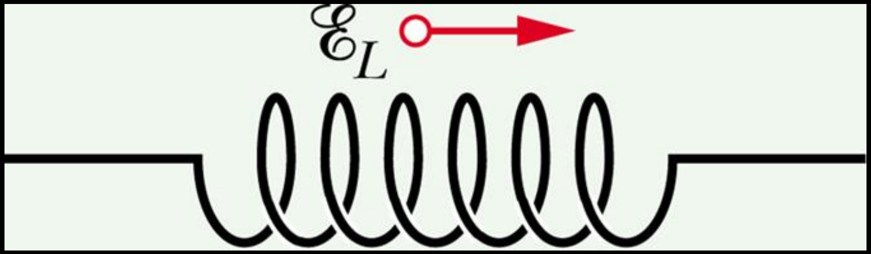
- **Inductance:**

$$L = \frac{N\Phi}{i}$$

$N\Phi$: flux linkage

1 henry=1H= 1T · m^2/A

Ex.4 Inductance of a solenoid:

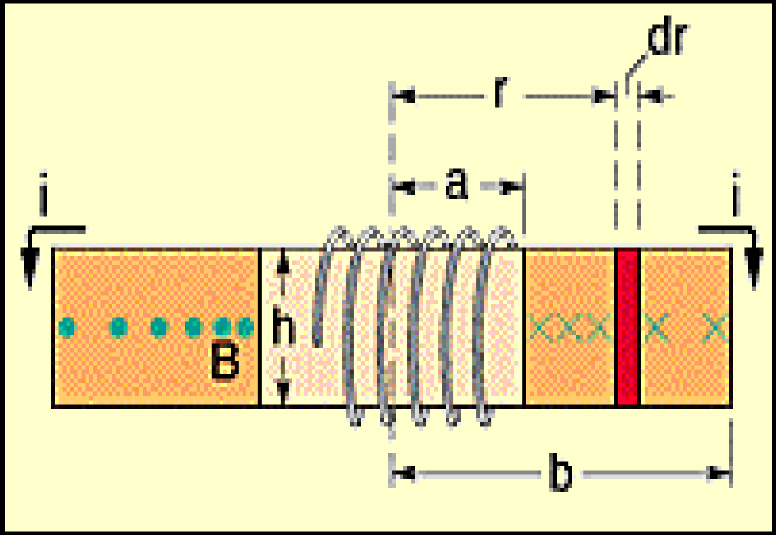


$$N\Phi = (nl)(BA),\ B = \mu_0 in$$

$$L = N\Phi / i = (nl)(BA)/i == \mu_0 n^2 lA$$

$$L/l = \mu_0 n^2 A$$

Ex.5 Inductance of a toroid



$$B = \frac{\mu_0 iN}{2\pi r}, \Phi = \int \vec{B} \cdot d\vec{A}$$

$$\Phi = \int_a^b Bhdr = \int_a^b \frac{\mu_0 iN}{2\pi r} hdr$$

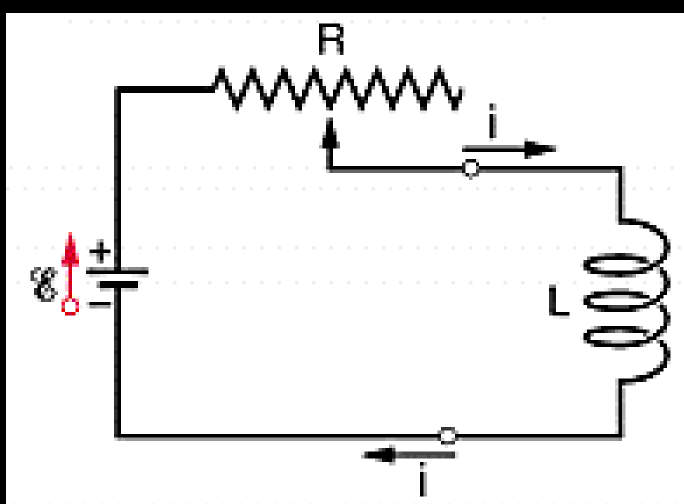
$$= \frac{\mu_0 iNh}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 iNh}{2\pi} \ln \frac{b}{a}$$

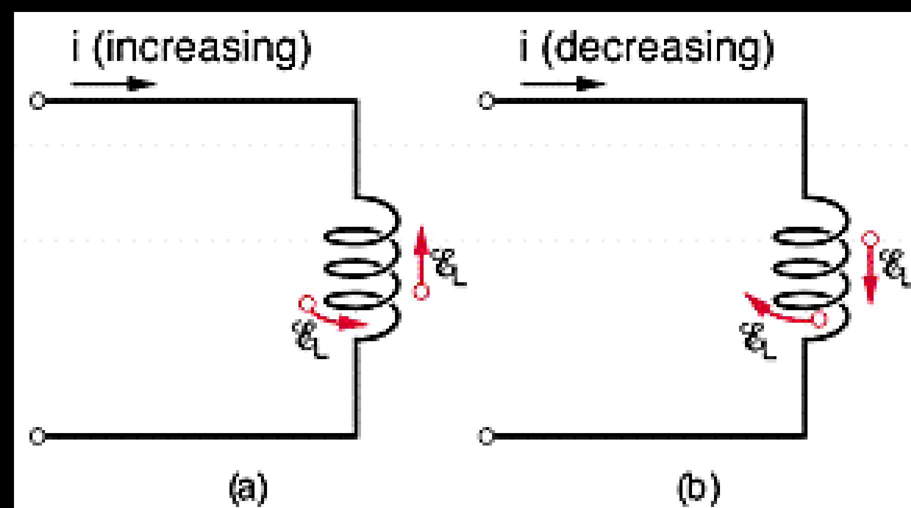
$$L = \frac{N\Phi}{i} = \frac{N}{i} \frac{\mu_0 iNh}{2\pi} \ln \frac{b}{a} = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a} = 2.5mH$$

10-7 Self-Induction

- *An induced emf appears in any coil in which the current is changing.*

$$N\Phi = Li, \quad \mathcal{E} = -\frac{d(N\Phi)}{dt} = -L\frac{di}{dt}$$





10-8 RL Circuits

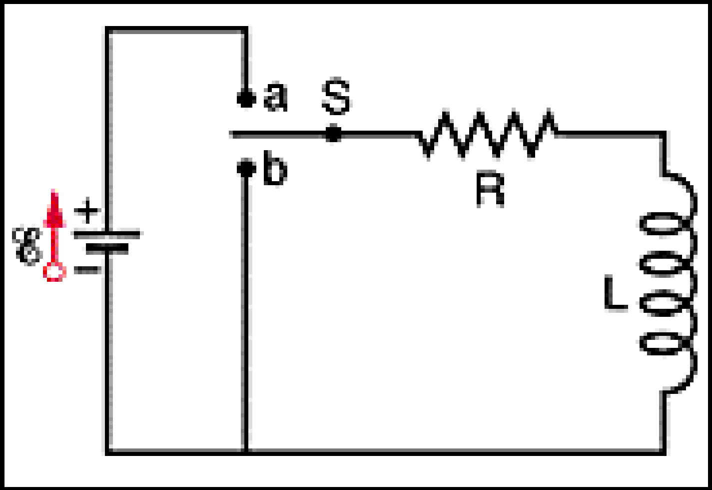
- RC Circuits

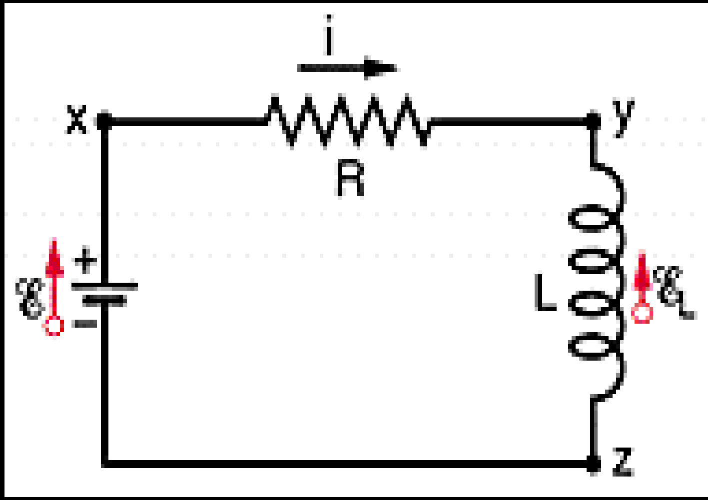
$$q = C\mathcal{E}(1 - e^{-t/\tau_C}),\ \ \tau_{\mathrm{C}} = RC$$

$$q = q_0 e^{-t/\tau_C}$$

- RL Circuits

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau_L}),\ \ \tau_{\mathrm{L}} = \frac{L}{R}$$





Initially, an inductor acts to oppose changes in the current thru it.

$$-iR - L\frac{di}{dt} + \mathcal{E} = 0$$

$$L\frac{di}{dt} + Ri = \mathcal{E}$$

- The inductive time constant

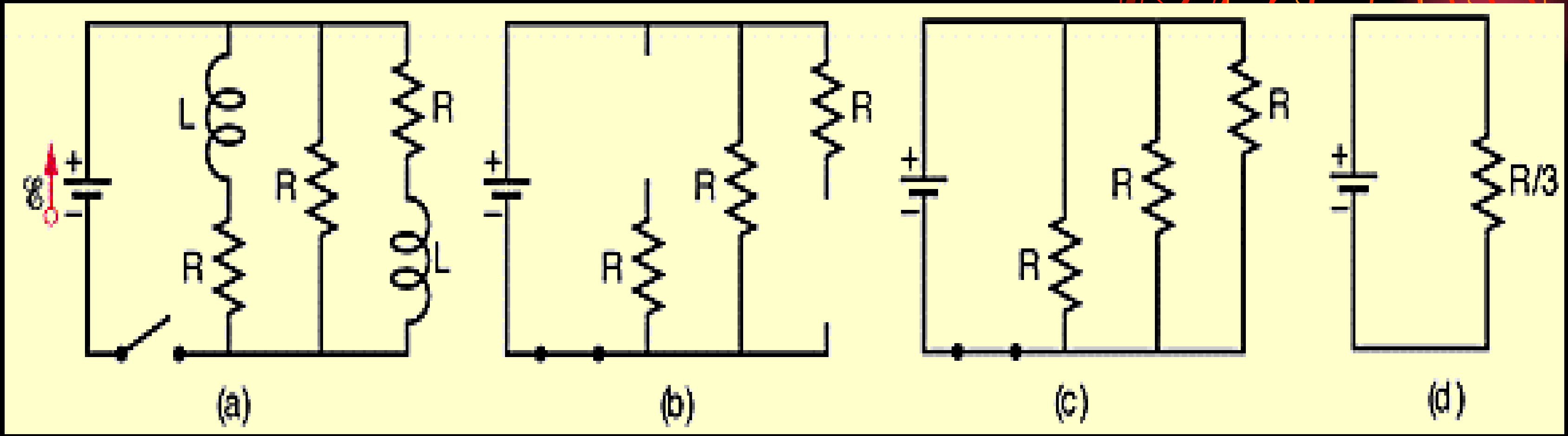
$$\tau_L = \frac{L}{R}, \qquad \text{unit: } 1\frac{\text{H}}{\Omega} = 1\,\text{s}$$

- The decay of current

$$L\frac{di}{dt} + iR = 0$$

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$

Ex.6 A multiloop RL circuit



- What is the current right after and long after the switch is closed?

$$i = \frac{\mathcal{E}}{R} = 2.0A, \quad i = \frac{\mathcal{E}}{R_{eq}} = 6.0A$$

Ex.7 The equilibrium value of current

$$\frac{1}{2}\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}(1 - e^{-t_0/\tau_L})$$

$$t_0 = \tau_L \ln 2 = 0.10\text{s}$$

10-9 Energy Stored in a Magnetic Field

$$\mathcal{E} = L\frac{di}{dt} + iR \rightarrow \mathcal{E} = Li\frac{di}{dt} + i^2R$$

$$\frac{dU_B}{dt} = Li\frac{di}{dt} \rightarrow dU_B = Lidi$$

$$\int_0^{U_B} dU_B = \int_0^i Lidi \rightarrow U_B = \frac{1}{2}Li^2 \quad (U_E = \frac{q^2}{2C})$$

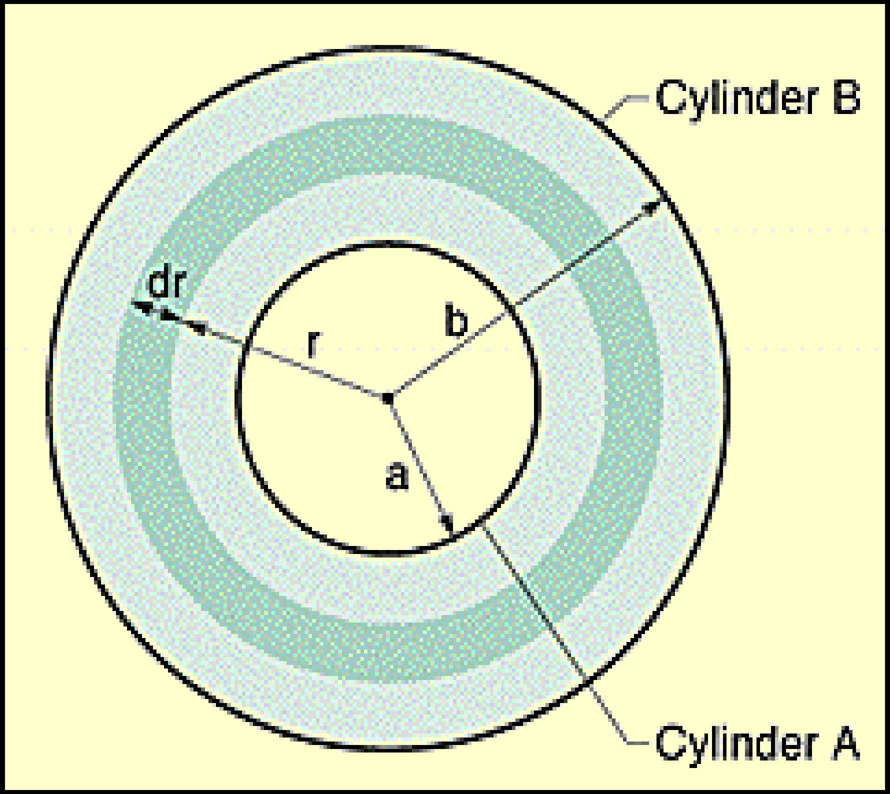
10-10 Energy Density of a Magnetic Field

$$u_B = \frac{U_B}{Al}, \quad U_B = \frac{1}{2}Li^2$$

$$u_B = \frac{Li^2}{2Al} = \frac{L}{l}\frac{i^2}{2A} \qquad (\frac{L}{i} = \mu_0 n^2 A)$$

$$= \frac{1}{2}\mu_0 n^2 i^2 = \frac{B^2}{2\mu_0} \qquad (u_E = \frac{1}{2}\varepsilon_0 E^2)$$

Ex.8 A long coaxial cable



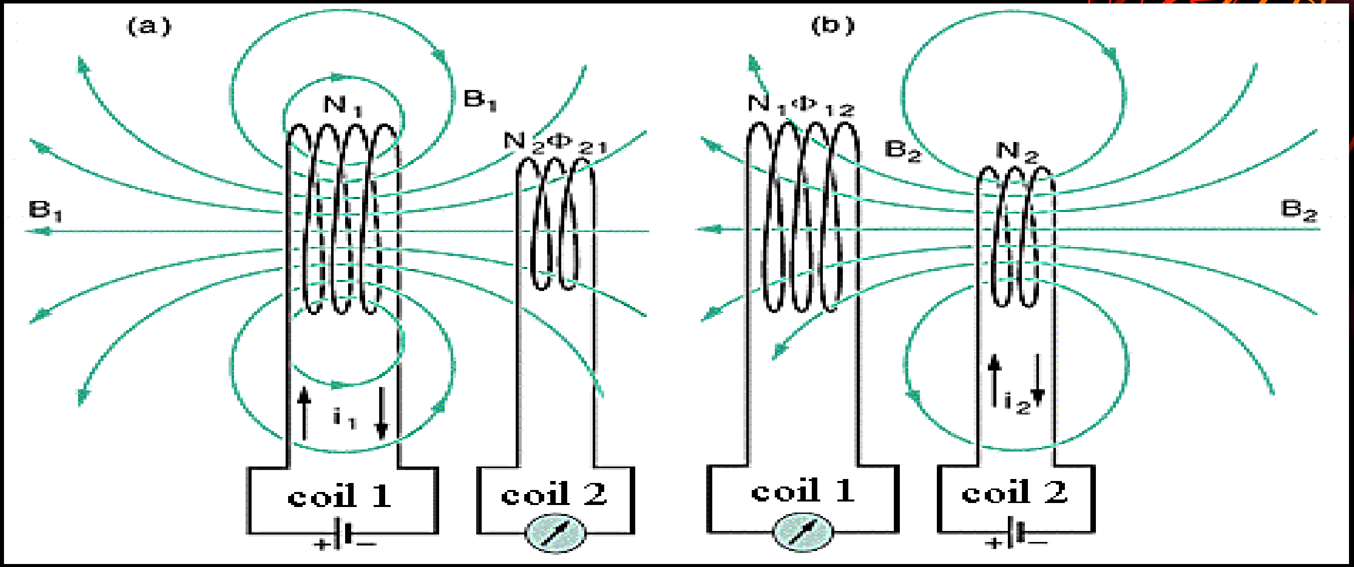
$$B=\frac{\mu_0 i}{2\pi r}, u_B=\frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2$$

$$dU=u_B dV$$

$$=\frac{\mu_0 i^2}{8\pi^2 r^2}(2\pi r l)(dr)$$

$$U=\int dU=\frac{\mu_0 i^2}{4\pi}\ln\frac{b}{a}$$

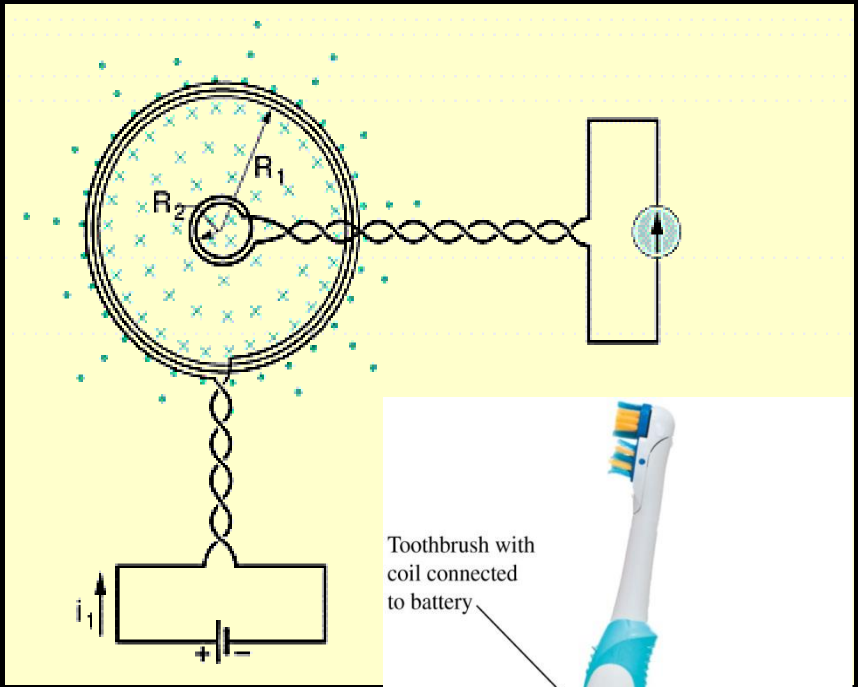
10-11 Mutual Induction

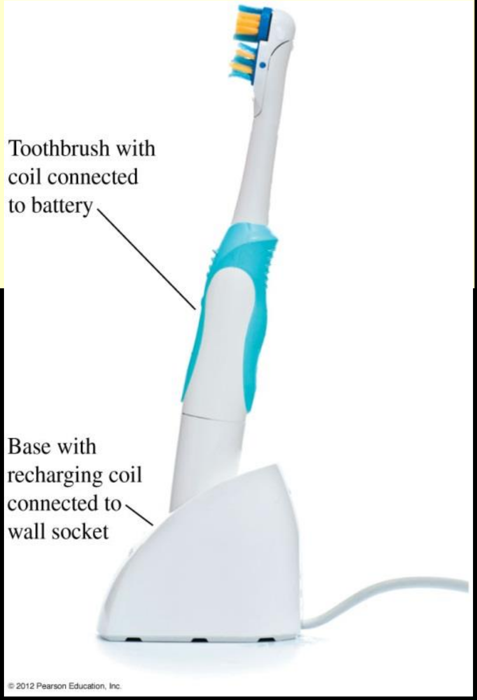


$$M_{21} = \frac{N_2\Phi_{21}}{i_1},\ M_{21}i_1 = N_2\Phi_{21},\ M_{21}\frac{di_1}{dt} = N_2\frac{d\Phi_{21}}{dt}$$

$$\rightarrow \mathcal{E} = -M_{21}\frac{di_1}{dt},\ \mathcal{E} = -M_{12}\frac{di_2}{dt},\ M_{21} = M_{12}$$

Ex.9 Two circular close-packed coils





$$B_1 = \frac{\mu_0 i_1 N_1}{2R_1}$$

$$N_2\Phi_{21} = N_2(B_1)(\pi R_2^2) = \frac{\pi\mu_0 N_1 N_2 R_2^2 i_1}{2R_1}$$

$$M = \frac{N_2\Phi_{21}}{i_1} = \frac{\pi\mu_0 N_1 N_2 R_2^2}{2R_1}$$

$$M = \frac{N_1\Phi_{12}}{i_2} = 2.3\text{mH}$$