Home Work Solutions 7

1. A 10-km-long underground cable extends east to west and consists of two parallel wires, each of which has resistance $13 \, \Omega/km$. An electrical short develops at distance $x$ from the west end when a conducting path of resistance $R$ connects the wires (Fig. 27-31). The resistance of the wires and the short is then 100 $\Omega$ when measured from the east end and 200 $\Omega$ when measured from the west end. What are (a) $x$ and (b) $R$?

![Figure 27-31 Problem 13.](image)

Sol
(a) We denote $L = 10$ km and $\alpha = 13 \, \Omega/km$. Measured from the east end we have

$$R_1 = 100 \, \Omega = 2\alpha(L - x) + R,$$

and measured from the west end $R_2 = 200 \, \Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\Omega - 100\Omega}{4 \times 13 \, \Omega/km} + \frac{10 \text{ km}}{2} = 6.9 \text{ km}.$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\Omega + 200\Omega}{2} - \frac{13 \, \Omega/km \times 10 \text{ km}}{2} = 20 \, \Omega.$$

2. A solar cell generates a potential difference of 0.10 V when a 500 $\Omega$ resistor is connected across it, and a potential difference of 0.15 V when a 1000 $\Omega$ resistor is substituted. What are the (a) internal resistance and (b) emf of the solar cell? (c) The area of the cell is 5.0 cm$^2$, and the rate per unit area at which it receives energy from light is 2.0 mW/cm$^2$. What is the efficiency of the cell for converting light energy to thermal energy in the 1000 $\Omega$ external resistor?

Sol
Let the emf of the solar cell be $\varepsilon$ and the output voltage be $V$. Thus,

$$V = \varepsilon - ir = \varepsilon - \frac{\varepsilon}{R},$$

for both cases. Numerically, we get

$$0.10 \, \text{V} = \varepsilon - (0.10 \, \text{V/500} \, \Omega)r,$$

$$0.15 \, \text{V} = \varepsilon - (0.15 \, \text{V/1000} \, \Omega)r.$$

We solve for $\varepsilon$ and $r$.
(a) $r = 1.0 \times 10^3 \, \Omega$.
(b) $\varepsilon = 0.30 \, \text{V}$.
(c) The efficiency is

$$\frac{V^2 / R}{P_{\text{received}}} = \frac{0.15 \, \text{V}}{(1000\Omega) \left( 5.0 \, \text{cm}^2 \right) \left( 2.0 \times 10^{-3} \, \text{W/cm}^2 \right)} = 2.3 \times 10^{-3} = 0.23\%.$$
3. In Fig. 27-46, \( V = 12.0 \text{ V} \), \( R_1 = 2000 \Omega \), \( R_2 = 3000 \Omega \), and \( R_3 = 4000 \Omega \). What are the potential differences (a) \( V_A - V_B \), (b) \( V_B - V_C \), (c) \( V_C - V_D \), and (d) \( V_A - V_C \)?

**Sol**

(a) The symmetry of the problem allows us to use \( i_2 \) as the current in both of the \( R_2 \) resistors and \( i_1 \) for the \( R_1 \) resistors. We see from the junction rule that \( i_3 = i_1 - i_2 \). There are only two independent loop rule equations:

\[
\begin{align*}
\varepsilon - i_2 R_2 - i_1 R_1 &= 0 \\
\varepsilon - 2i_1 R_1 - (i_1 - i_2) R_3 &= 0
\end{align*}
\]

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find \( i_1 = 0.002625 \text{ A} \), \( i_2 = 0.00225 \text{ A} \) and \( i_3 = i_1 - i_2 = 0.000375 \text{ A} \). Therefore, \( V_A - V_B = i_1 R_1 = 5.25 \text{ V} \).

(b) It follows also that \( V_B - V_C = i_3 R_3 = 1.50 \text{ V} \).

(c) We find \( V_C - V_D = i_1 R_1 = 5.25 \text{ V} \).

(d) Finally, \( V_A - V_C = i_2 R_2 = 6.75 \text{ V} \).

4. In Fig. 27-52, an array of \( n \) parallel resistors is connected in series to a resistor and an ideal battery. All the resistors have the same resistance. If an identical resistor were added in parallel to the parallel array, the current through the battery would change by 1.25%. What is the value of \( n \)?  

**Hint:** \( 0.0125 = 1/80 \)  

**Sol**

The equivalent resistance in Fig. 27-52 (with \( n \) parallel resistors) is

\[
R_{\text{eq}} = R + \frac{R}{n} = \left( \frac{n+1}{n} \right) R.
\]

The current in the battery in this case should be
\[ i_n = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n}{n+1} \frac{V_{\text{battery}}}{R} . \]

If there were \( n + 1 \) parallel resistors, then
\[ i_{n+1} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n + 1}{n + 2} \frac{V_{\text{battery}}}{R} . \]

For the relative increase to be 0.0125 \((= 1/80)\), we require
\[ \frac{i_{n+1} - i_n}{i_n} = \frac{i_{n+1}}{i_n} - 1 = \frac{(n + 1)/(n + 2)}{n/(n + 1)} - 1 = \frac{1}{80} . \]

This leads to the second-degree equation
\[ n^2 + 2n - 80 = (n + 10)(n - 8) = 0. \]

Clearly the only physically interesting solution to this is \( n = 8 \). Thus, there are eight resistors in parallel (as well as that resistor in series shown toward the bottom) in Fig. 27-52.

5. In Fig. 27-61, \( R_s \) is to be adjusted in value by moving the sliding contact across it until points \( a \) and \( b \) are brought to the same potential. (One tests for this condition by momentarily connecting a sensitive ammeter between \( a \) and \( b \); if these points are at the same potential, the ammeter will not deflect.) Show that when this adjustment is made, the following relation holds: \( R_s = R_1R_2/R_1 \). An unknown resistance \((R_x)\) can be measured in terms of a standard \((R_s)\) using this device, which is called a Wheatstone bridge.

\begin{center}
\textbf{Figure 27-61} Problem 55.
\end{center}

\textbf{Sol}
Let \( i_1 \) be the current in \( R_1 \) and \( R_2 \), and take it to be positive if it is toward point \( a \) in \( R_1 \). Let \( i_2 \) be the current in \( R_s \) and \( R_x \), and take it to be positive if it is toward \( b \) in \( R_s \). The loop rule yields \((R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0\). Since points \( a \) and \( b \) are at the same potential, \( i_1R_1 = i_2R_s \). The second equation gives \( i_2 = i_1R_1/R_s \), which is substituted into the first equation to obtain
\[ (R_1 + R_2)i_1 = (R_s + R_x) \frac{R_s}{R_x} i_1 \implies R_s = \frac{R_xR_s}{R_1}. \]