Home Work Solutions 11

11-1 Using the loop rule, derive the differential equation for an *LC* circuit:

$$L\frac{d^2q}{dt^2}+\frac{1}{C}q=0$$

Sol: The loop rule, for just two devices in the loop, reduces to the statement that the magnitude of the voltage across one of them must equal the magnitude of the voltage across the other. Consider that the capacitor has charge q and a voltage (which we'll consider positive in this discussion) $V = q/C$. Consider at this moment that the current in the inductor at this moment is directed in such a way that the capacitor charge is increasing (so $i = +dq/dt$). Equation 30-35 then produces a positive result equal to the V across the capacitor: $V = -L(di/dt)$, and we interpret the fact that $-di/dt > 0$ in this discussion to mean that $d(dq/dt)/dt = d^2q/dt^2 < 0$ represents a "deceleration" of the charge-buildup process on the capacitor (since it is approaching its maximum value of charge). In this way we can "check" the signs in Eq. 31-11 (which states $q/C = -L\, d^2q/dt^2$) to make sure we have implemented the loop rule correctly.

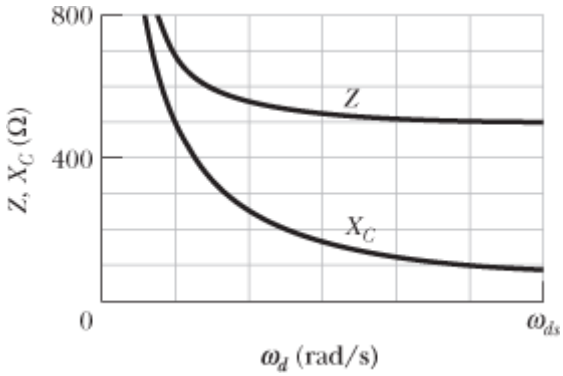
11-2 A series circuit containing inductance L_1 and capacitance C_1 oscillates at angular frequency ω. A second series circuit, containing inductance L_2 and capacitance C_2, oscillates at the same angular frequency. In terms of ω, what is the angular frequency of oscillation of a series circuit containing all four of these elements? Neglect resistance. (*Hint:* Use the formulas for equivalent capacitance and equivalent inductance; see Section 25-4 and Problem 47 in Chapter 30.)

Sol: For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\omega' = \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}}$$
$$= \frac{1}{\sqrt{L_1C_1}}\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega,$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

11-3 An alternating source with a variable frequency, a capacitor with capacitance C, and a resistor with resistance R are connected in series. The following figure gives the impedance Z of the circuit versus the driving angular frequency ω_d; the curve reaches an asymptote of 500 Ω, and the horizontal scale is set by ω_{ds} = 300 rad/s. The figure also gives the reactance X_C for the capacitor versus ω_d. What are (a) R and (b) C?



Sol: (a) The circuit has a resistor and a capacitor (but no inductor). Since the capacitive reactance decreases with frequency, then the asymptotic value of Z must be the resistance: R = 500 Ω.

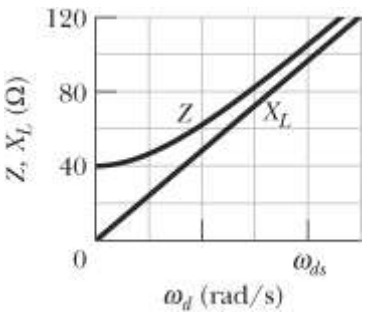
(b) We describe three methods here (each using information from different points on the graph):

<u>method 1</u>: At ω_d = 50 rad/s, we have $Z \approx 700\ \Omega$, which gives $C = (\omega_d\sqrt{Z^2 - R^2}\ )^{-1}$ = 41 μF.

<u>method 2</u>: At ω_d = 50 rad/s, we have $X_C \approx 500\ \Omega$, which gives $C = (\omega_d X_C)^{-1}$ = 40 μF.

<u>method 3</u>: At ω_d = 250 rad/s, we have $X_C \approx 100\ \Omega$, which gives $C = (\omega_d X_C)^{-1}$ = 40 μF.

11-4 An alternating source with a variable frequency, an inductor with inductance L, and a resistor with resistance R are connected in series. The following figure gives the impedance Z of the circuit versus the driving angular frequency ω_d, with the horizontal axis scale set by ω_{ds} = 1600 rad/s. The figure also gives the reactance X_L for the inductor versus ω_d. What are (a) R and (b) L?



Sol: (a) Since $Z = \sqrt{R^2 + X_L^2}$ and $X_L = \omega_d L$, then as $\omega_d \to 0$ we find $Z \to R = 40\ \Omega$.

(b) $L = X_L/\omega_d = slope = 60$ mH.