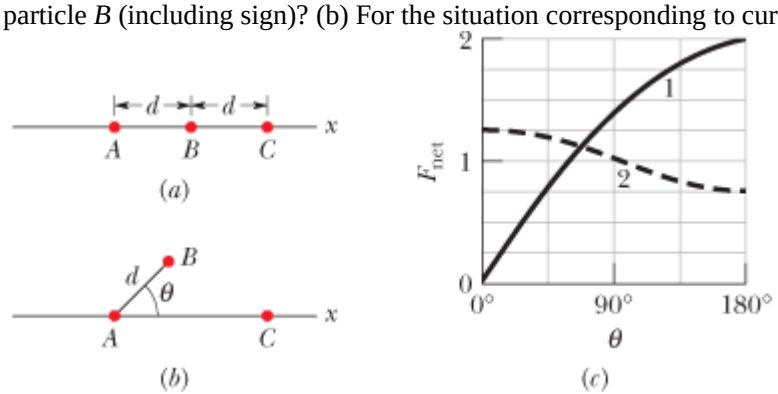
Home Work Solutions 1

1. Figure *a* shows an arrangement of three charged particles separated by distance *d*. Particles *A* and *C* are fixed on the *x* axis, but particle *B* can be moved along a circle centered on particle *A*. During the movement, a radial line between *A* and *B* makes an angle θ relative to the positive direction of the *x* axis (Fig. *b*). The curves in Fig. *c* give, for two situations, the magnitude F_{net} of the net electrostatic force on particle *A* due to the other particles. That net force is given as a function of angle θ and as a multiple of a basic amount F_0. For example on curve 1, at $\theta = 180°$, we see that $F_{net} = 2F_0$. (a) For the situation corresponding to curve 1, what is the ratio of the charge of particle *C* to that of particle *B* (including sign)? (b) For the situation corresponding to curve 2, what is that ratio?



Sol:

We note that the problem is examining the force <u>on</u> charge *A*, so that the respective distances (involved in the Coulomb force expressions) between *B* and *A*, and between *C* and *A*, do not change as particle *B* is moved along its circular path. We focus on the endpoints (θ = 0º and 180º) of each graph, since they represent cases where the forces (on *A*) due to *B* and *C* are either parallel or antiparallel (yielding maximum or minimum force magnitudes, respectively). We note, too, that since Coulomb's law is inversely proportional to r^2 then (if, say, the charges were all the same) the force due to *C* would be one-fourth as big as that due to *B* (since *C* is twice as far away from *A*). The charges, it turns out, are not the same, so there is also a factor of the charge ratio ξ (the charge of *C* divided by the charge of *B*), as well as the aforementioned ¼ factor. That is, the force exerted by *C* is, by Coulomb's law, equal to $\pm\frac{1}{4}\xi$ multiplied by the force exerted by *B*.

(a) The maximum force is $2F_0$ and occurs when θ = 180º (*B* is to the left of *A*, while *C* is the right of *A*). We choose the minus sign and write

$$2\,F_0 = (1 - \tfrac{1}{4}\xi)\,F_0 \qquad \Rightarrow \qquad \xi = -4\,.$$

One way to think of the minus sign choice is cos(180º) = –1. This is certainly consistent with the minimum force ratio (zero) at θ = 0º since that would also imply

$$0 = 1 + \tfrac{1}{4}\xi \qquad \Rightarrow \qquad \xi = -4\,.$$

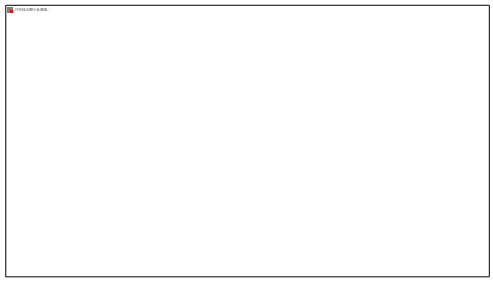
(b) The ratio of maximum to minimum forces is 1.25/0.75 = 5/3 in this case, which implies

$$\frac{5}{3} = \frac{1 + \frac{1}{4}\xi}{1 - \frac{1}{4}\xi} \qquad \Rightarrow \qquad \xi = 16\,.$$

Of course, this could also be figured as illustrated in part (a), looking at the maximum force

ratio by itself and solving, or looking at the minimum force ratio (¾) at θ = 180º and solving for ξ.

2. In crystals of the salt cesium chloride, cesium ions Cs^+ form the eight corners of a cube and a chlorine ion Cl^- is at the cube's center (see the following figure). The edge length of the cube is 0.40 nm. The Cs^+ ions are each deficient by one electron (and thus each has a charge of +*e*), and the Cl^- ion has one excess electron (and thus has a charge of -*e*). (a) What is the magnitude of the net electrostatic force exerted on the Cl^- ion by the eight Cs^+ ions at the corners of the cube? (b) If one of the Cs^+ ions is missing, the crystal is said to have a *defect*; what is the magnitude of the net electrostatic force exerted on the Cl^- ion by the seven remaining Cs^+ ions?



Sol:

(a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) Rather than remove a cesium ion, we superpose charge –*e* at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where *a* is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = \left(\sqrt{3}/2\right)a$. The force has magnitude

$$F = k\frac{e^2}{d^2} = \frac{ke^2}{\left(\sqrt{3}/4\right)a^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{\left(\sqrt{3}/4\right)\left(0.40\times10^{-9}\ \mathrm{m}\right)^2} = 1.9\times10^{-9}\ \mathrm{N}.$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

3. (a) What equal positive charges would have to be placed on Earth and on the Moon to neutralize their gravitational attraction? (b) Why don't you need to know the lunar distance to solve this problem? (c) How many kilograms of hydrogen ions (that is, protons) would be needed to provide the positive charge calculated in (a)?

Sol:

(a) The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = G\frac{mM}{r^2}$$

where q is the charge on either body, r is the center-to-center separation of Earth and Moon, G is the universal gravitational constant, M is the mass of Earth, and m is the mass of the Moon. We solve for q:

$$q = \sqrt{4\pi\varepsilon_0 GmM}.$$

According to Appendix C of the text, $M = 5.98 \times 10^{24}$ kg, and $m = 7.36 \times 10^{22}$ kg, so (using $4\pi\varepsilon_0 = 1/k$) the charge is

$$q = \sqrt{\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(7.36\times10^{22}\ \mathrm{kg}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}{8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}}} = 5.7\times10^{13}\ \mathrm{C}.$$

(b) The distance r cancels because both the electric and gravitational forces are proportional to $1/r^2$.

(c) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19}$ C, so there must be

$$n = \frac{q}{e} = \frac{5.7\times10^{13}\ \mathrm{C}}{1.6\times10^{-19}\ \mathrm{C}} = 3.6\times10^{32}\ \text{ions}.$$

Each ion has a mass of $m_i = 1.67 \times 10^{-27}$ kg, so the total mass needed is

$$m = nm_i = \left(3.6\times10^{32}\right)\left(1.67\times10^{-27}\ \mathrm{kg}\right) = 6.0\times10^{5}\ \mathrm{kg}.$$

4. A nonconducting spherical shell, with an inner radius of 4.0 cm and an outer radius of 6.0 cm, has charge spread nonuniformly through its volume between its inner and outer surfaces. The *volume charge density* ρ is the charge per unit volume, with the unit coulomb per cubic meter. For this shell $\rho = b/r$, where r is the distance in meters from the center of the shell and $b = 3.0\ \mu\mathrm{C/m^2}$. What is the net charge in the shell?

Sol:

The charge dq within a thin shell of thickness dr is $dq = \rho dV = \rho A dr$ where $A = 4\pi r^2$. Thus, with $\rho = b/r$, we have

$$q = \int dq = 4\pi b\int_{r_1}^{r_2} r\,dr = 2\pi b\left(r_2^2 - r_1^2\right).$$

With $b = 3.0\ \mu\mathrm{C/m^2}$, $r_2 = 0.06$ m, and $r_1 = 0.04$ m, we obtain $q = 0.038\ \mu\mathrm{C} = 3.8 \times 10^{-8}$ C.

5. Earth's atmosphere is constantly bombarded by *cosmic ray protons* that originate somewhere in space. If the

protons all passed through the atmosphere, each square meter of Earth's surface would intercept protons at the average rate of 1500 protons per second. What would be the electric current intercepted by the total surface area of the planet?

Sol:

The unit ampere is discussed in Section 21-4. The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi(6.37 \times 10^6\ \text{m})^2 = 5.1 \times 10^{14}\ \text{m}^2$ would be

$$i = \left(5.1\times10^{14}\,\text{m}^2\right)\left(1500\,\frac{\text{protons}}{\text{s}\cdot\text{m}^2}\right)\left(1.6\times10^{-19}\,\text{C/proton}\right) = 0.122\,\text{A}.$$