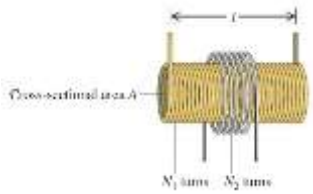
『北區高中物理科學人才培育』計畫高二物理期末考試卷

1 The electric potential between the electrodes of a vacuum tube diode is given by $V(x) = cx^3$, where x is the distance from the cathode and C is a constant. Assume that the distance between the cathode and anode is 13.0 mm and the potential difference between electrodes is 240V (a) Determine the value of C. (10%) (b) Obtain a formula for the electric field between the electrodes as a function of *x*. (10%)

2 A source with emf $\mathscr{E}$ and internal resistance *r* is connected to an external circuit. (a) Show that the power output of the source is maximum when the current in the circuit is one-half the short-circuit current of the source. (10%) (b) If the external circuit consists of a resistance *R*, show that the power output is maximum when $R = r$ and that the maximum power is $\mathscr{E}^2/4r$. (10%)

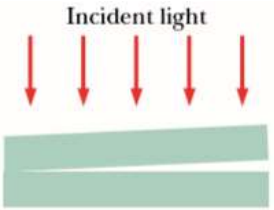
3 An electron follows a helical path in a uniform magnetic field given by **B** = (20**i** - 50**j** - 30**k**) mT. At time t = 0, the electron's velocity is given by **v** = (20**i** - 30**j** + 50**k**) m/s. (a) What is the angle φ between **v** and **B** ? What is the (b) radius, (c) pitch(螺距) of the helical path? (20%)

4 二線圈配置如右圖，其自感分別為 L_1 及 L_2、互感為 M；試證當其為串聯時之等效電感為 (20%)



$$L_{eq} = L_1 + L_2 + 2M.$$

5 二玻璃板於一端接觸，另端分離如右圖。當波長 600nm 的光垂直入設於上板時，你會看到上板上有 9 條暗紋和 8 條亮紋。如果你把二板在分離端的距離再增加 600nm，此時上板上會有幾條暗紋? (20%)



1 *Sol*:

(a) $V = Cx^{4/3}$

$$C = V/x^{4/3} = 240\ \text{V}/(13.0\times10^{-3}\ \text{m})^{4/3} = 7.85\times10^{4}\ \text{V/m}^{4/3}$$

(b) $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{4}{3}Cx^{1/3} = -(1.05\times10^{5}\ \text{V/m}^{4/3})x^{1/3}$

The minus sign means that E_x is in the $-x$-direction, which says that $\vec{E}$ points from the positive anode toward the negative cathode.

2 *Sol*:

(a) $P = \mathcal{E}I - I^2r$, so $\dfrac{dP}{dI} = \mathcal{E} - 2Ir = 0$ for maximum power output and $I_{P\ \max} = \dfrac{1}{2}\dfrac{\mathcal{E}}{r} = \dfrac{1}{2}I_{\text{short circuit}}$.

(b) For the maximum power output of part (a), $I = \dfrac{\mathcal{E}}{r+R} = \dfrac{1}{2}\dfrac{\mathcal{E}}{r}$. $r + R = 2r$ and $R = r$.

Then, $P = I^2R = \left(\dfrac{\mathcal{E}}{2r}\right)^2 r = \dfrac{\mathcal{E}^2}{4r}$.

3 *Sol*:

(a) $\mathbf{v}\cdot\mathbf{B} = vB\cos\phi$, $a = \phi = \cos^{-1}(2/19) = 84°$.

(b) We find $v_\perp = v\sin\phi = 61.3$ m/s, so $r = mv_\perp/eB = 5.7$ nm.

(c) $v_\parallel = v\cos\phi = 6.5$ m/s, so $p = 2\pi mv_\parallel/eB = 40.7$ nm.

4 *Sol*:

$$L_1\frac{di_1}{dt} + L_2\frac{di_2}{dt} + M_{21}\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} \equiv L_{\text{eq}}\frac{di}{dt}.$$

But $i = i_1 + i_2 \Rightarrow \dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$ and $M_{12} = M_{21} \equiv M$,

so $(L_1 + L_2 + 2M)\dfrac{di}{dt} = L_{\text{eq}}\dfrac{di}{dt}$ and $L_{\text{eq}} = L_1 + L_2 + 2M$.

5 *Sol*:

By the condition $m\lambda = 2y$ where y is the thickness of the air-film between the plates directly underneath the middle of a dark band), the edge of the plates (the edge where they are not touching) are $y = 8\lambda/2 = 2400$ nm apart (where we have assumed that the *middle* of the ninth dark band is at the edge). Increasing that to $y' = 3000$ nm would correspond to $m' = 2y'/\lambda = 10$ (counted as the eleventh dark band, since the first one corresponds to $m = 0$). There are thus 11 dark fringes along the top plate.