$$\lim_{x\to\infty} \frac{1-\sqrt{\frac{x}{x+1}}}{2-\sqrt{\frac{4x+1}{x+2}}} \longrightarrow \lim_{x\to\infty} \frac{1-(1+x^{-1})^{-1/2}}{2-(1+\frac{1}{4x})^{1/2}(1+\frac{2}{x})^{-1/2}} \to \frac{\frac{1}{2x}}{\frac{7}{4x}}$$

$$1-\left(1-\frac{1}{2}\frac{1}{x}-\frac{\frac{1}{2}(-\frac{3}{2})}{2}\frac{1}{x^2}+\cdots\right)$$

$$= \lim_{x\to\infty} \frac{0+\frac{1}{2}\sqrt{\frac{x+1}{x}}\left(\frac{1}{x+1}-\frac{x}{(x+1)^2}\right)}{0+\frac{1}{2}\sqrt{\frac{x+2}{4x+1}}\left(\frac{4}{4x+1}-\frac{4x+1}{(4x+1)^2}\right)}$$

$$2-2\left\{\left(1+\frac{1}{2}\frac{1}{4x}+\cdots\right)\left(1-\frac{1}{2}\frac{2}{x}+\cdots\right)\right\}$$

$$= 2-2\left\{1+\left(\frac{1}{8}-\frac{1}{2}\right)+\cdots\right\}$$

$$\frac{\cos x-1}{x\sin x}$$

$$\downarrow x\to 0$$

$$\frac{\left(1-\frac{x^2}{2}+\cdots\right)-1}{x\left(x-\frac{x^3}{6}+\cdots\right)}$$

$$\xrightarrow[x\to 0]{} \frac{-\frac{x^2}{2}}{x^2} \to -\frac{1}{2}$$

$$f(x)=\cos x = 1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\cdots$$

$$\sin x = 0+\frac{1}{1!}x-0-\frac{1}{3!}x^3$$

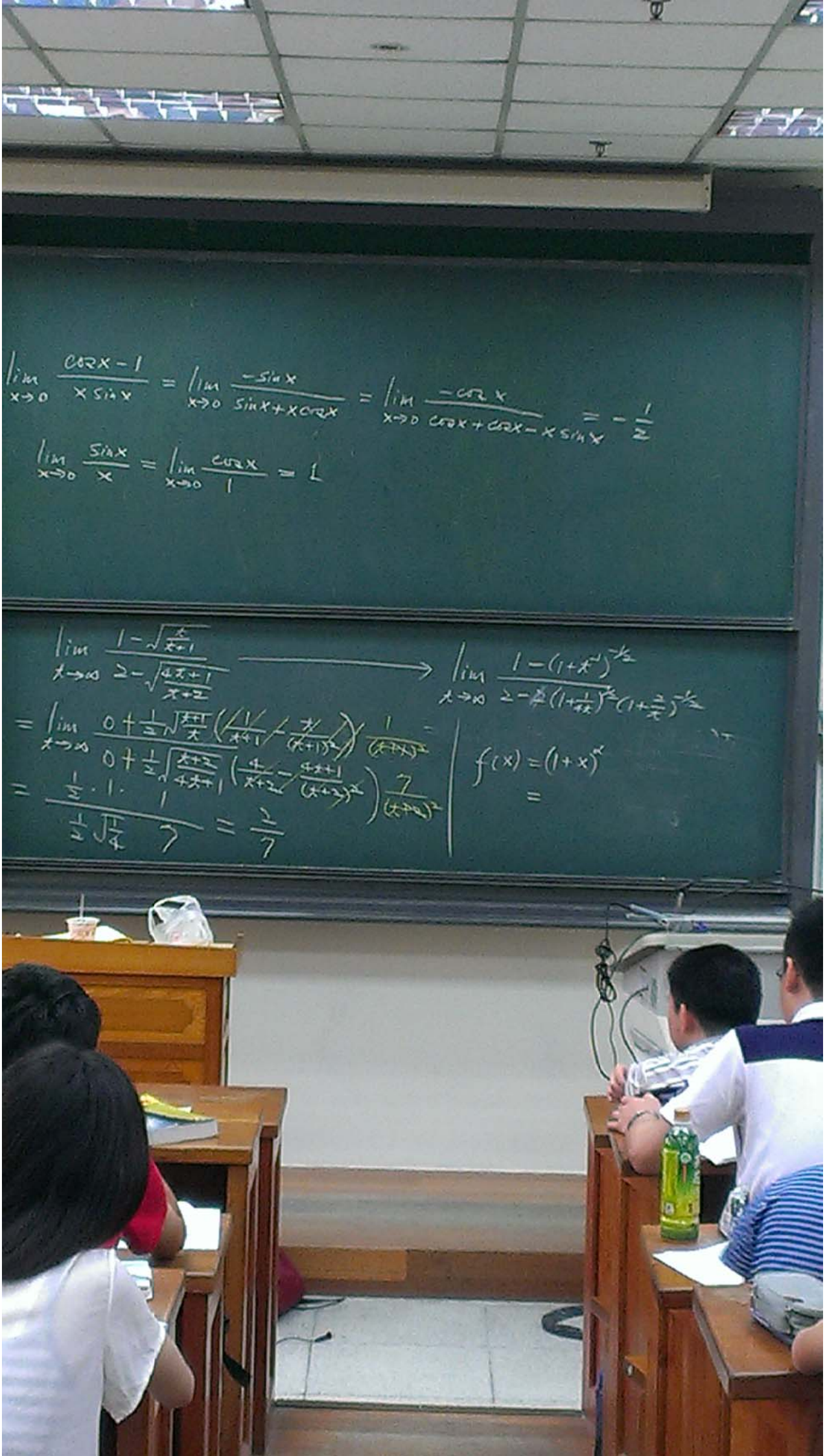
$$f'(x)=-\sin x \qquad (\sin x)'=\cos x$$

$$f''(x)=-\cos x \qquad (\sin x)''=-\sin x$$

$$f'''(x)=\sin x \qquad (\sin x)'''=-\cos x$$

$$f^{(4)}(x)=\cos x \qquad (\sin x)^{(4)}=+\sin x$$

$$\sin x = \frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$


$\lim_{x\to 0} \frac{\cos x - 1}{x \sin x} = \lim_{x\to 0} \frac{-\sin x}{\sin x + x\cos x} = \lim_{x\to 0} \frac{-\cos x}{\cos x + \cos x - x\sin x} = -\frac{1}{2}$
$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1$
$\lim_{x\to\infty} \frac{1-\sqrt{\frac{x}{x+1}}}{2-\sqrt{\frac{4x+1}{x+2}}} \longrightarrow \lim_{x\to\infty} \frac{1-(1+x^{-1})^{-1/2}}{2-(1+\frac{1}{4x})^{1/2}(1+\frac{2}{x})^{-1/2}}$
$= \lim_{x\to\infty} \frac{0+\frac{1}{2}\sqrt{\frac{x+1}{x}}\left(\frac{1}{x+1}-\frac{x}{(x+1)^2}\right)\frac{1}{(x+1)^2}}{0+\frac{1}{2}\sqrt{\frac{x+2}{4x+1}}\left(\frac{4}{x+2}-\frac{4x+1}{(x+2)^2}\right)\frac{7}{(x+2)^2}} =$
$= \frac{\frac{1}{2}\cdot 1\cdot \frac{1}{1}}{\frac{1}{2}\sqrt{\frac{1}{4}}\ \frac{7}{1}} = \frac{2}{7}$
$f(x) = (1+x)^{\alpha}$
$=$

$$\lim_{x\to 0}\frac{\cos x-1}{x\sin x}=\lim_{x\to 0}\frac{-\sin x}{\sin x+x\cos x}=\lim_{x\to 0}\frac{-\cos x}{\cos x+\cos x-x\sin x}=-\frac{1}{2}$$

$$\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\cos x}{1}=1$$

$$\lim_{x\to\infty}\frac{1-\sqrt{\frac{x}{x+1}}}{2-\sqrt{\frac{4x+1}{x+2}}}\longrightarrow\lim_{x\to\infty}\frac{1-(1+x^{-1})^{-1/2}}{2-\cancel{x}(1+\frac{1}{4x})^{1/2}(1+\frac{2}{x})^{-1/2}}$$

$$=\frac{0+\frac{1}{2}\sqrt{\frac{x+1}{x}}\left(\cancel{\frac{1}{x+1}}-\cancel{\frac{x}{(x+1)^2}}\right)\frac{1}{(x+1)^2}}{0+\frac{1}{2}\sqrt{\frac{x+2}{4x+1}}\left(\cancel{\frac{4}{x+2}}-\cancel{\frac{4x+1}{(x+2)^2}}\right)\frac{7}{(x+2)^2}}$$

$$=\frac{1}{7}=\frac{2}{7}$$

$$f(x)=(1+x)^{\alpha}$$

$$=$$

L'Hospital's rule

$f(a)=g(a)=0$, $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{g(x)-g(a)}\dfrac{(x-a)}{(x-a)}=\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}$

$f(x)$, $g(x)$ 可微分

$f''(x)=2\cdot1\cdot b_2+3\cdot2\cdot b_3(x-a)+\cdots$

$f''(a)=2\cdot1\cdot b_2$

$f^{(n)}(a)=n\cdot(n-1)\cdot(n-2)\cdots2\cdot1\, b_n$

$f(x)=b_0+b_1(x-a)+b_2(x-a)^2+\cdots$

$f(x)=f(a)+\dfrac{f'(a)}{1!}(x-a)+\dfrac{f''(a)}{2!}(x-a)^2+\cdots+\dfrac{f^{(n)}(a)}{1\cdot n!}(x-a)^n+\cdots$

$\lim\limits_{x\to0}\dfrac{\cos x-1}{x\sin x}=\lim\limits_{x\to0}\dfrac{-\sin x}{\sin x+x\cos x}=$

$\lim\limits_{x\to0}\dfrac{\sin x}{x}=\lim\limits_{x\to0}\dfrac{\cos x}{1}=1$

$\lim\limits_{x\to\infty}\dfrac{1-\sqrt{\frac{x}{x+1}}}{2-\sqrt{\frac{4x+1}{x+2}}}$

$=\lim\limits_{x\to\infty}\dfrac{0+\frac{1}{2}\sqrt{\frac{x+1}{x}}\left(\frac{1}{x+1}-\frac{x}{(x+1)^2}\right)}{0+\frac{1}{2}\sqrt{\frac{x+2}{4x+1}}\left(\frac{4}{x+2}-\frac{4x+1}{(x+2)^2}\right)}$

$=\dfrac{\frac{1}{2}\cdot1\cdot1}{\frac{1}{2}\sqrt{\frac{1}{4}}\cdot7}=\dfrac{2}{7}$