4 電容 Capacitance

National Ignition Facility

Explosions in Airborne Dust

Flour explosion sugar

4-1 電容與電容器

- 電容（Capacitance）
- 平行板電容器 ⊣⊢

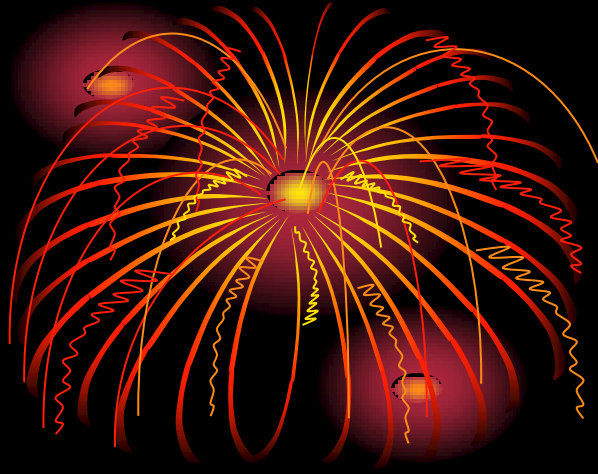
$$C = \frac{q(\text{電荷})}{V(\text{電位差})}$$

- $C = \dfrac{K\varepsilon_0 A}{d}$（F 法拉）

K: 介電常數，真空 = 1

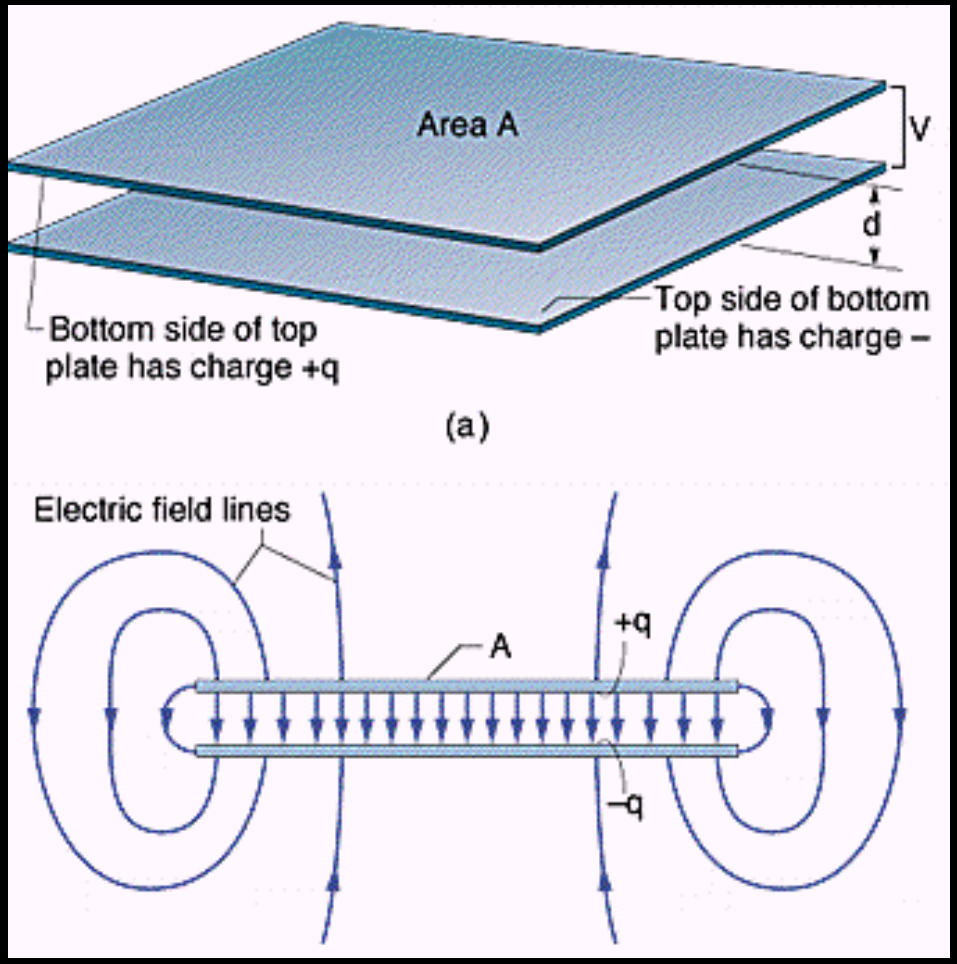
ε_0: 真空電容率 A: 板面積 d：兩板距離

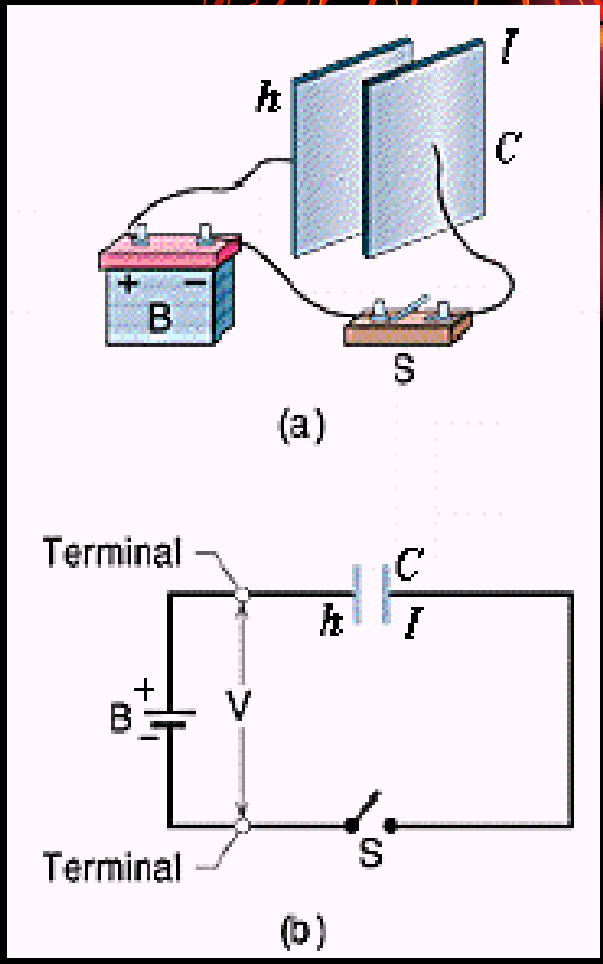
The Uses of Capacitors



- As storehouses of electric potential energy
- As vital elements in tuned circuits (radio and TV) and RAM chips

Charging a capacitor



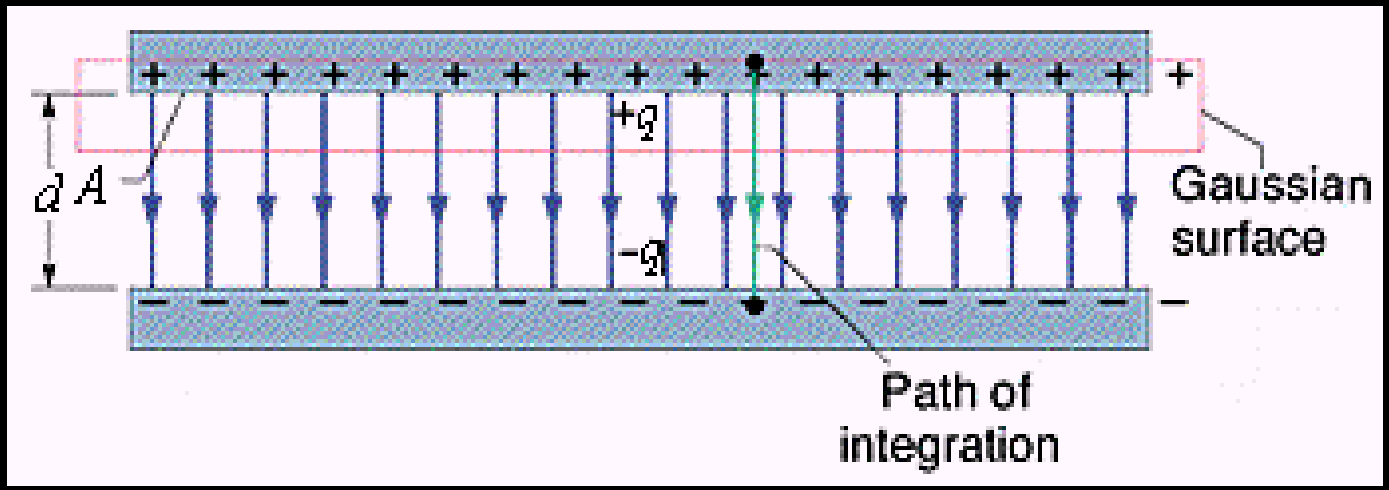


4-2 Calculating the Capacitance

1) Put a charge q on the plates
2) Find E using Gauss's Law
3) Find V between the plates
4) Calculate C by its definition

Calculating the electric field

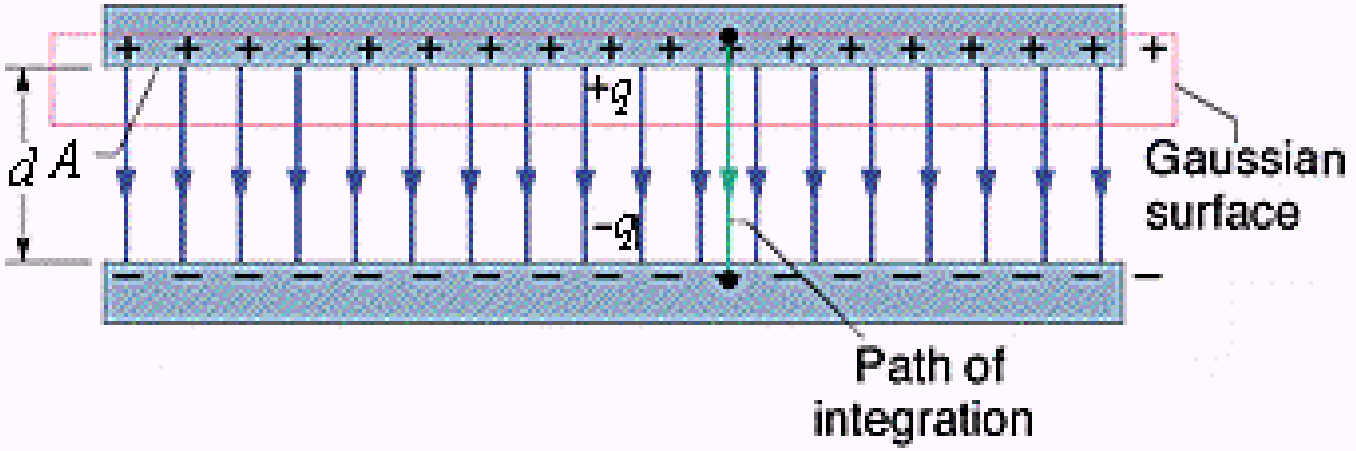
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q \rightarrow q = \varepsilon_0 EA$$



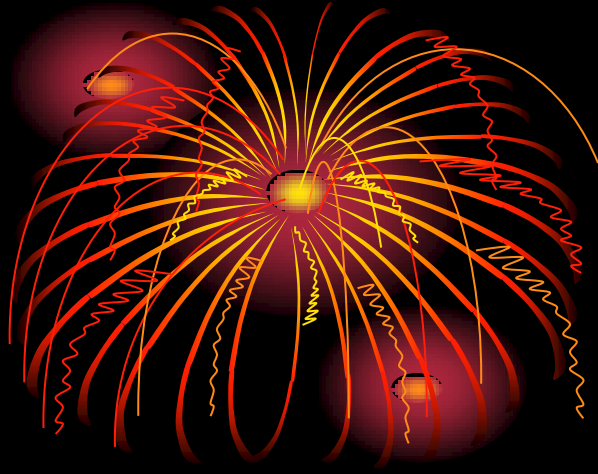
Calculating the potential difference

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$V = \int_-^+ E ds \quad (V_f - V_i = -V)$$



A parallel-plate capacitor

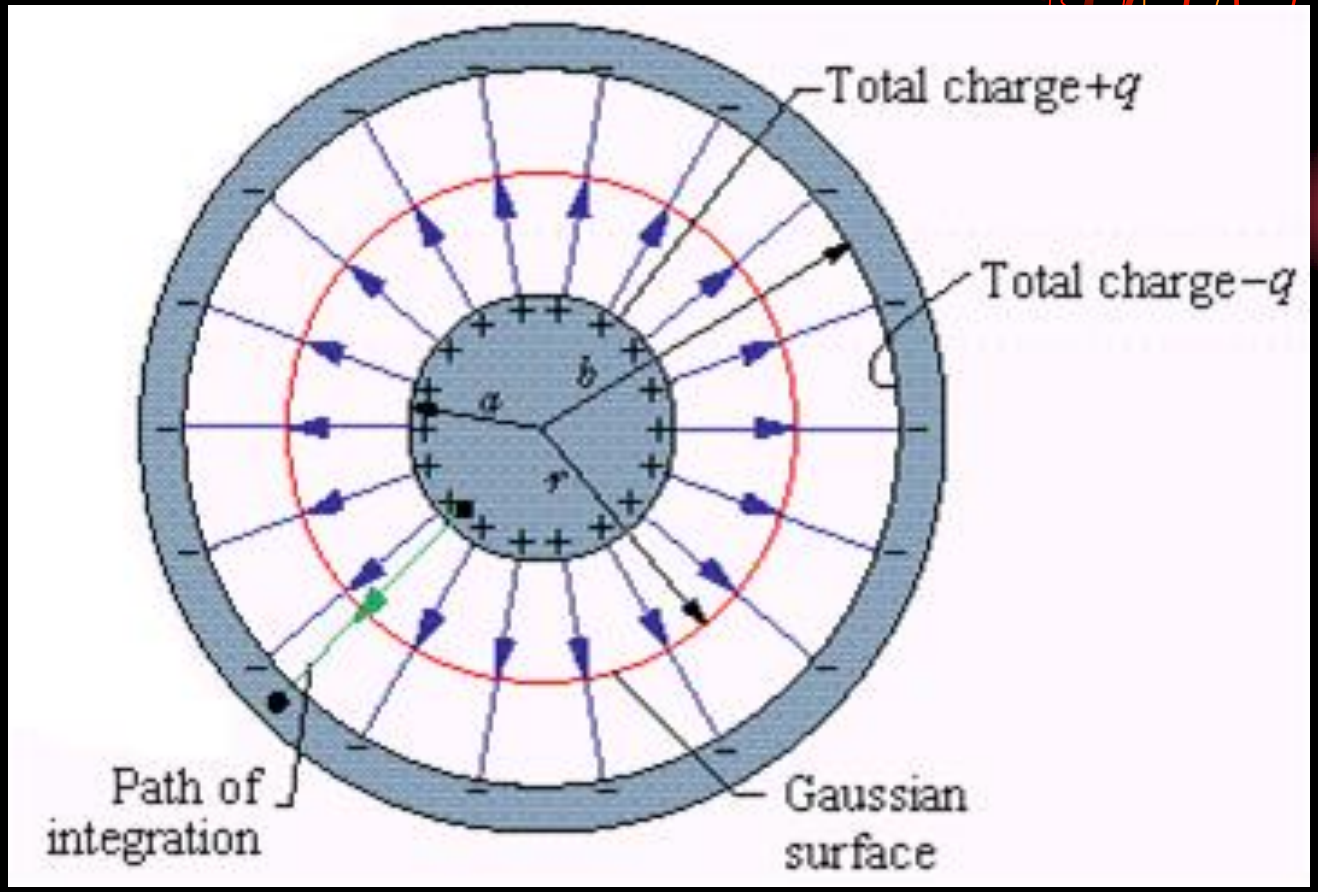


$$q = \varepsilon_0 EA$$

$$V = \int_{+}^{-} E\,ds = E\int_{0}^{d} ds = Ed$$

$$C = \frac{q}{V} = \frac{\varepsilon_0 A}{d}$$

A cylindrical capacitor



$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

$$E = \frac{q}{2\pi\varepsilon_0 Lr}$$

$$V = \int_+^- E ds = \frac{q}{2\pi\varepsilon_0 L}\int_a^b \frac{dr}{r}$$

$$= \frac{q}{2\pi\varepsilon_0 L}\ln(\frac{b}{a})$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \text{ or } \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

A spherical capacitor

$$q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$V = \int_+^- E ds = \frac{q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab}$$

$$C = \frac{q}{V} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

An Isolated Sphere – the missing plate is a conducting sphere of infinite radius

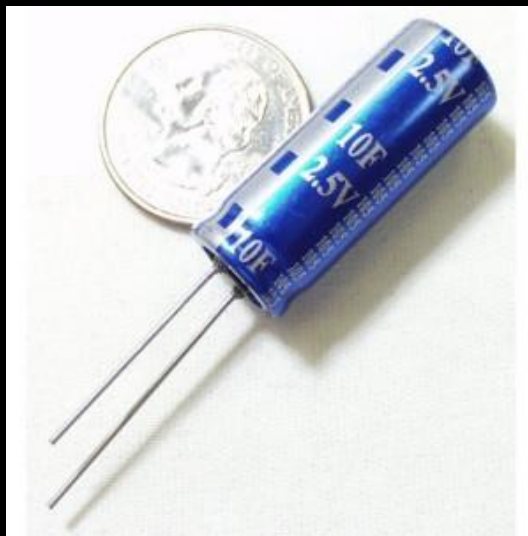
$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b} \rightarrow 4\pi\varepsilon_0 R \,(\mathrm{b} \rightarrow \infty)$$

Ex.1 1.0 F 電容器

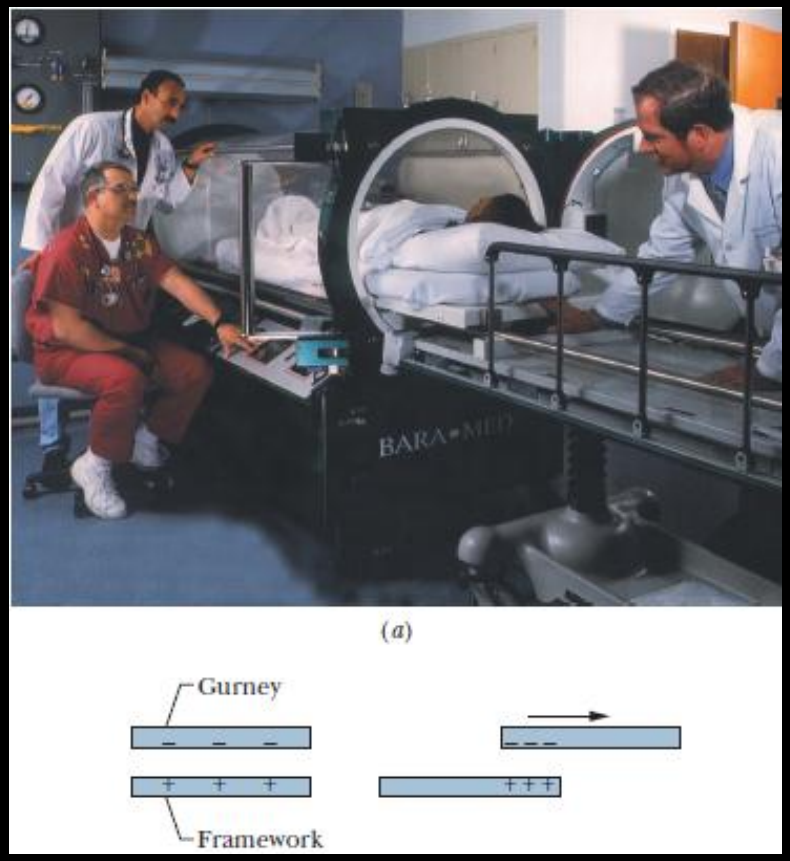
- d = 1.0mm

$$A = \frac{Cd}{\varepsilon_0} = \frac{(1.0F)(1.0\times10^{-3}m)}{8.85\times10^{-12}F/m}$$

$$= 1.1\times10^{8}m^2 \sim 100km^2$$



Ex.2 a hyperbaric chamber



- Spark : > 2000V
- fire : > 0.20 mJ
- $V_f = 600V$

→

- V_f : > 6000V
- U_f : > 0.45 mJ

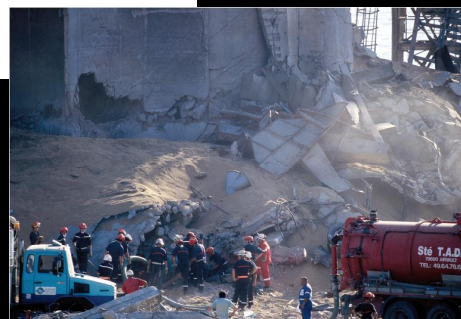
$$q = C_i V_i = C_f V_f,\ U = 1/2 CV^2$$

Ex.3 the threshold value U_t (150 mJ) required to ignite airborne grains.

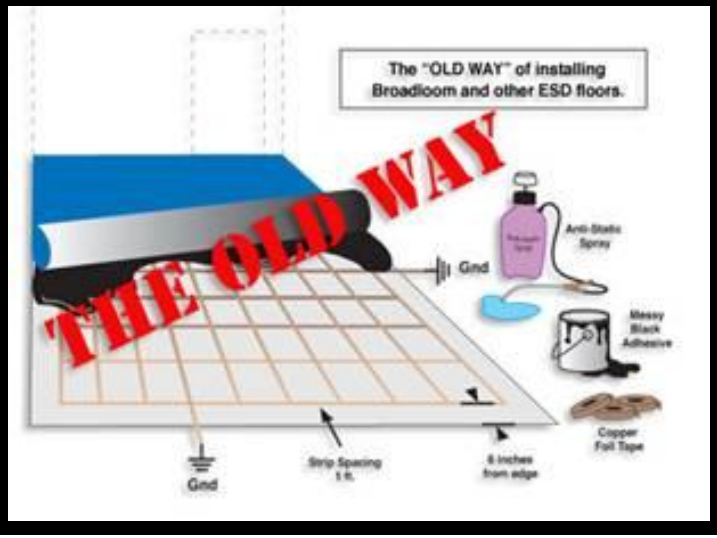
$$C = 4\pi\varepsilon_0 R \ , \quad U = 1/2 CV^2$$

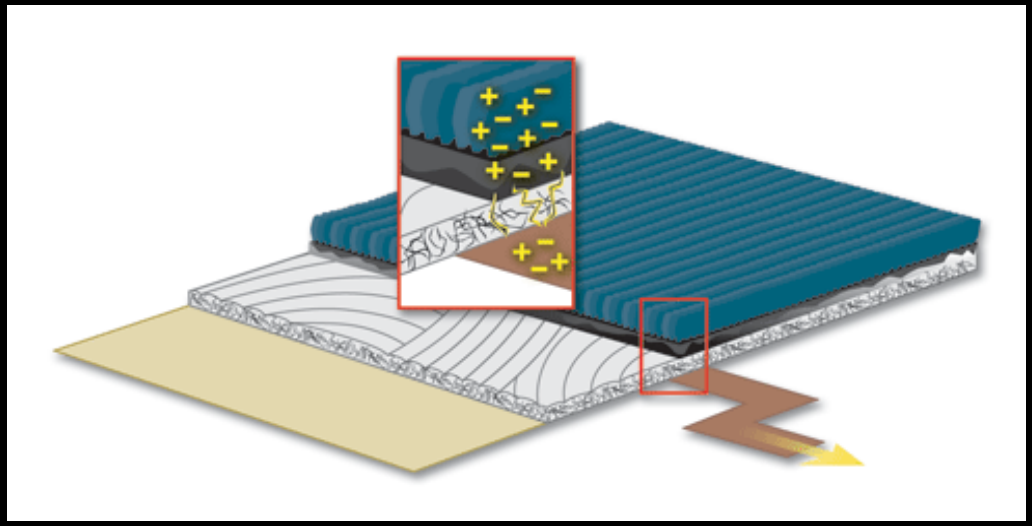
$$\rightarrow V = \sqrt{\frac{2U_t}{4\pi\varepsilon_0 R}} = \sqrt{\frac{2(150\times10^{-3})}{4\pi\varepsilon_0(1.8\text{m})}}$$

$$= 3.9\times10^4\,\text{V}$$

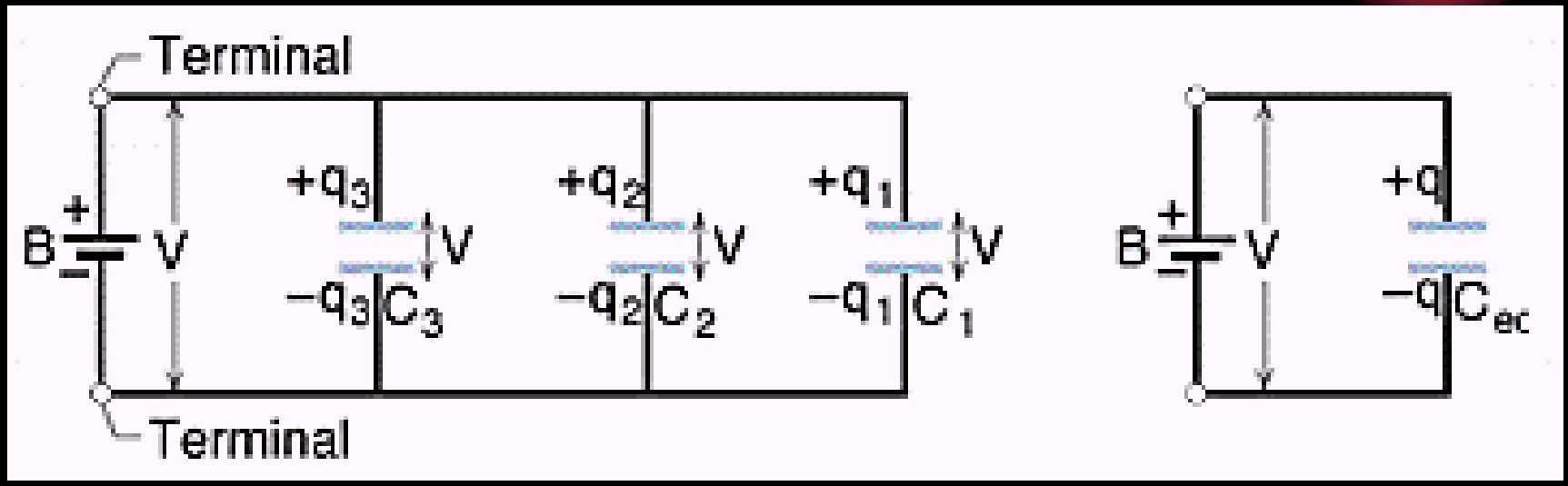


Conducting floor

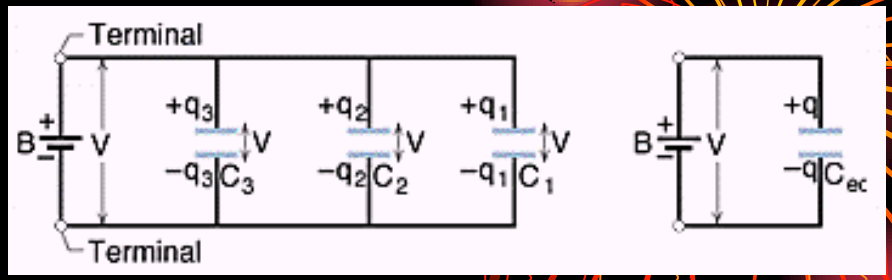




4-3 Capacitors in parallel



等效電容



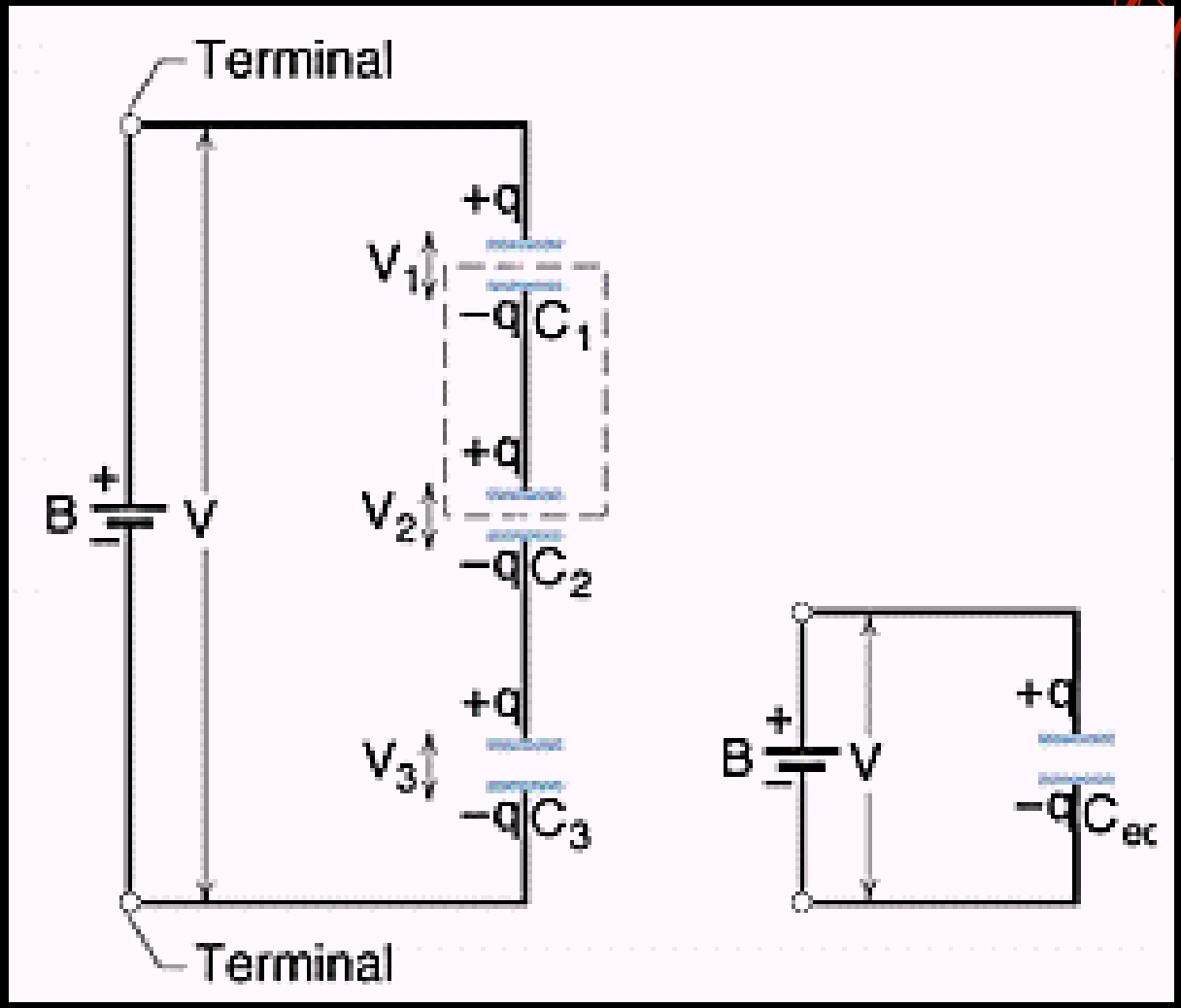
$$q_1 = C_1 V, q_2 = C_2 V, q_3 = C_3 V$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

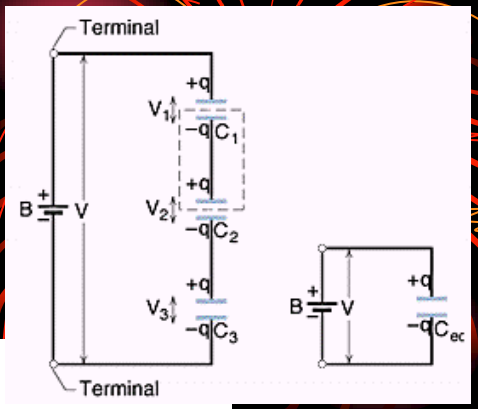
$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$$

$$\rightarrow C_{eq} = \sum_{j=1}^{n} C_j$$

4-4 Capacitors in series



等效電容



$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$$

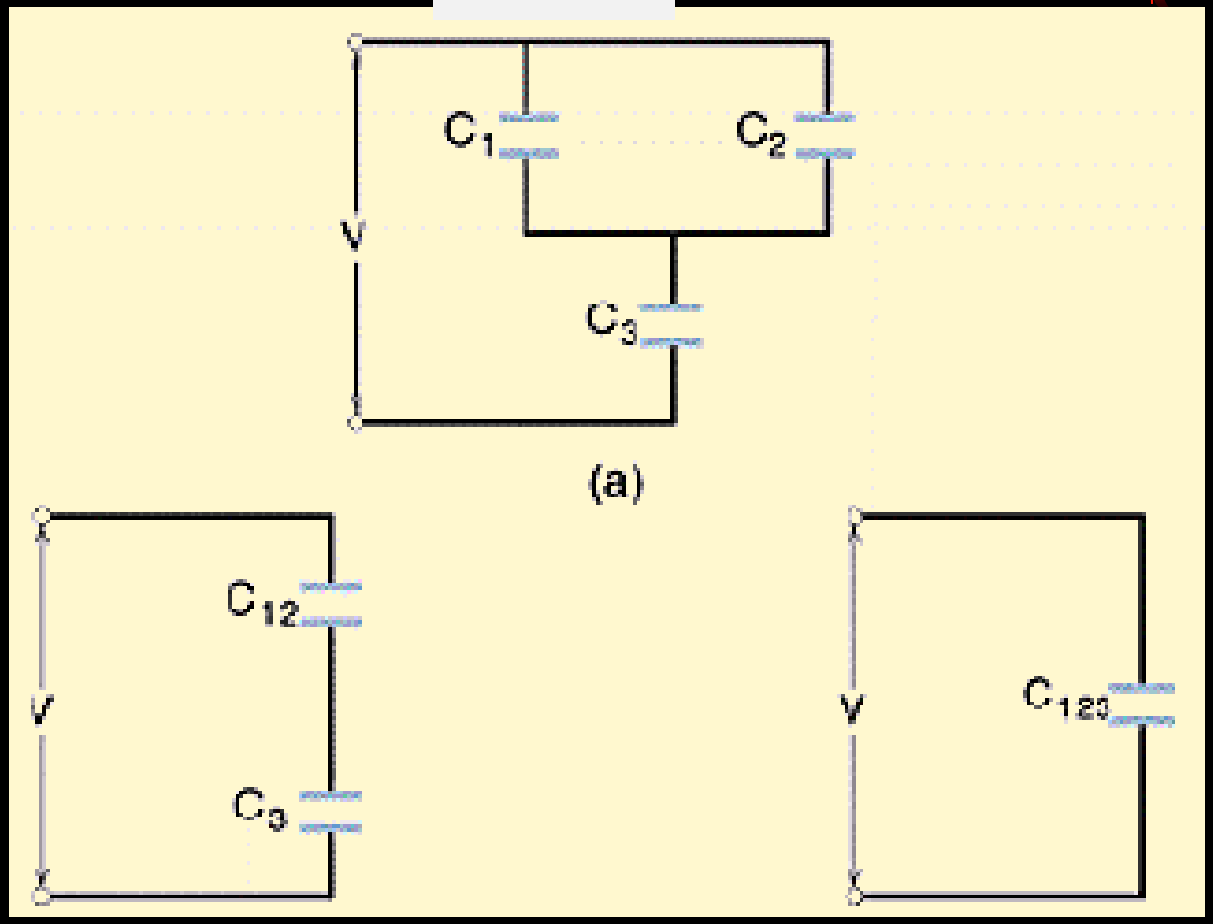
$$V = V_1 + V_2 + V_3 = q(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3})$$

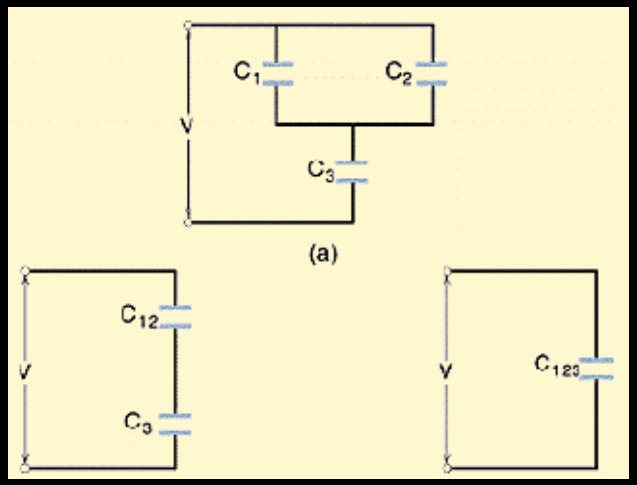
$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Ex.4 Finding eq. capacitor

$q_1 = ?$





$$C_{12} = C_1 + C_2, \frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$q_{123} = C_{123}V \rightarrow V_{12} = \frac{q_{12}}{C_{12}} (q_{12} = q_{123})$$

$$\rightarrow q_1 = C_1 V_1 (V_1 = V_{12}) = 31.0 \mu C$$

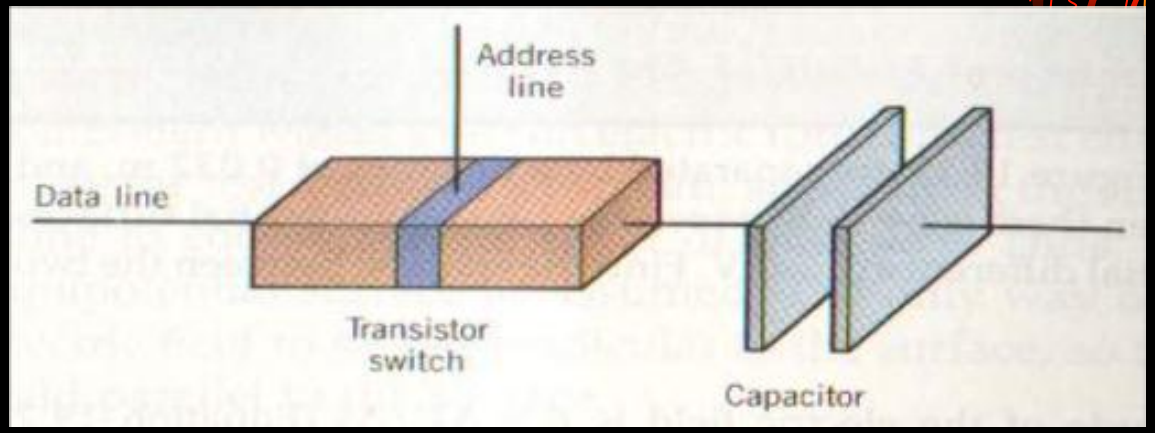
4-5 Storing Energy in an Electric Field

$$dW = V'dq' = \frac{q'}{C}dq'$$

$$W = \int dW = \frac{1}{C}\int_0^q q'dq' = \frac{q^2}{2C}$$

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2$$

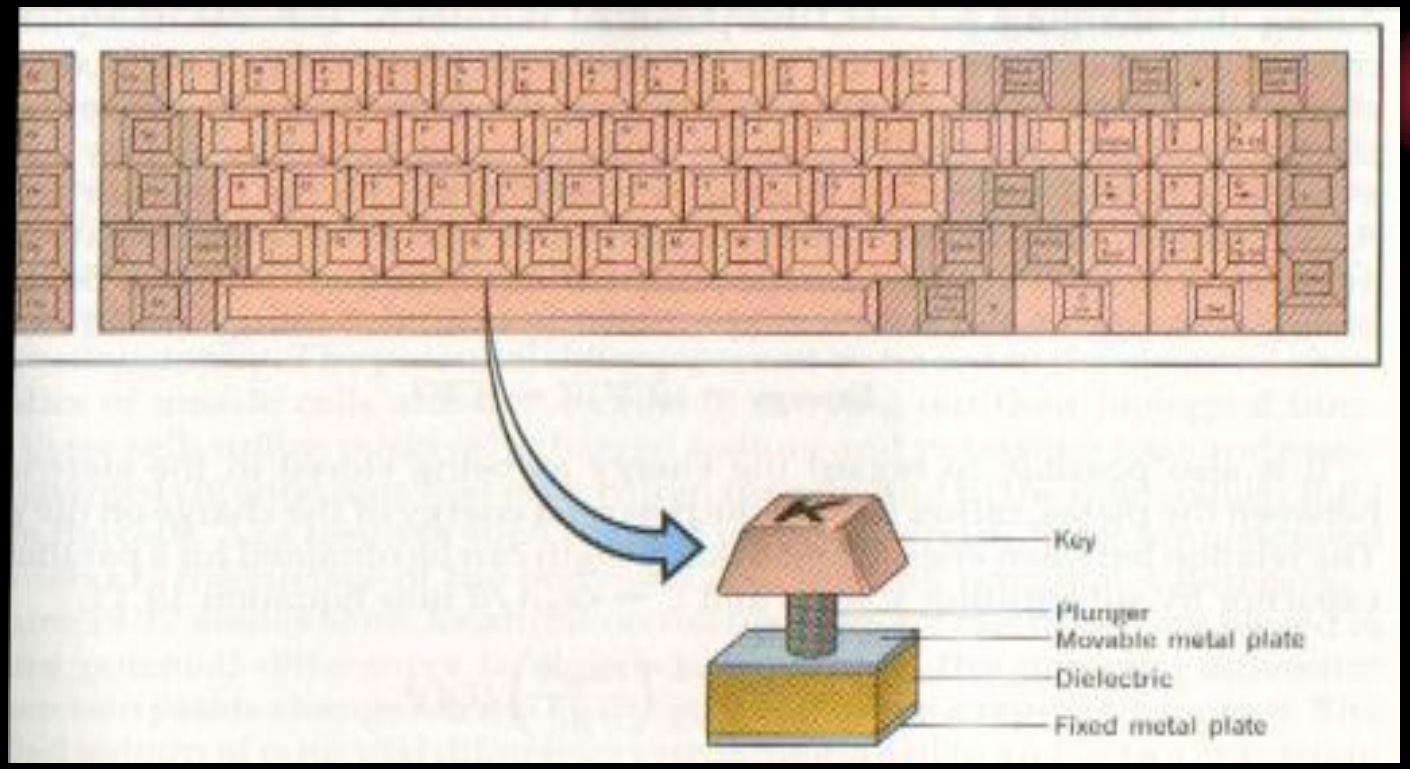
Ex.4 A RAM Chip



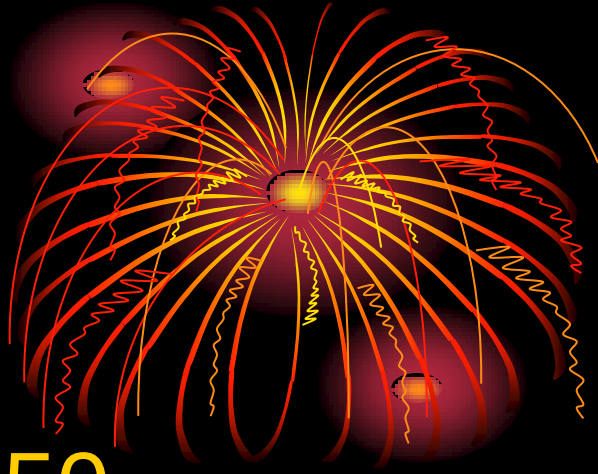
- C = 55fF（10^{-15}） V = 5.3 volt

$$n = \frac{q}{e} = \frac{CV}{e} = \frac{(55\times10^{-15}F)(5.3V)}{1.60\times10^{-19}C} = 1.8\times10^{6}\,\text{electrons}$$

Ex.5 電容式鍵盤


Key
Plunger
Movable metal plate
Dielectric
Fixed metal plate

Ex.5 (cont)



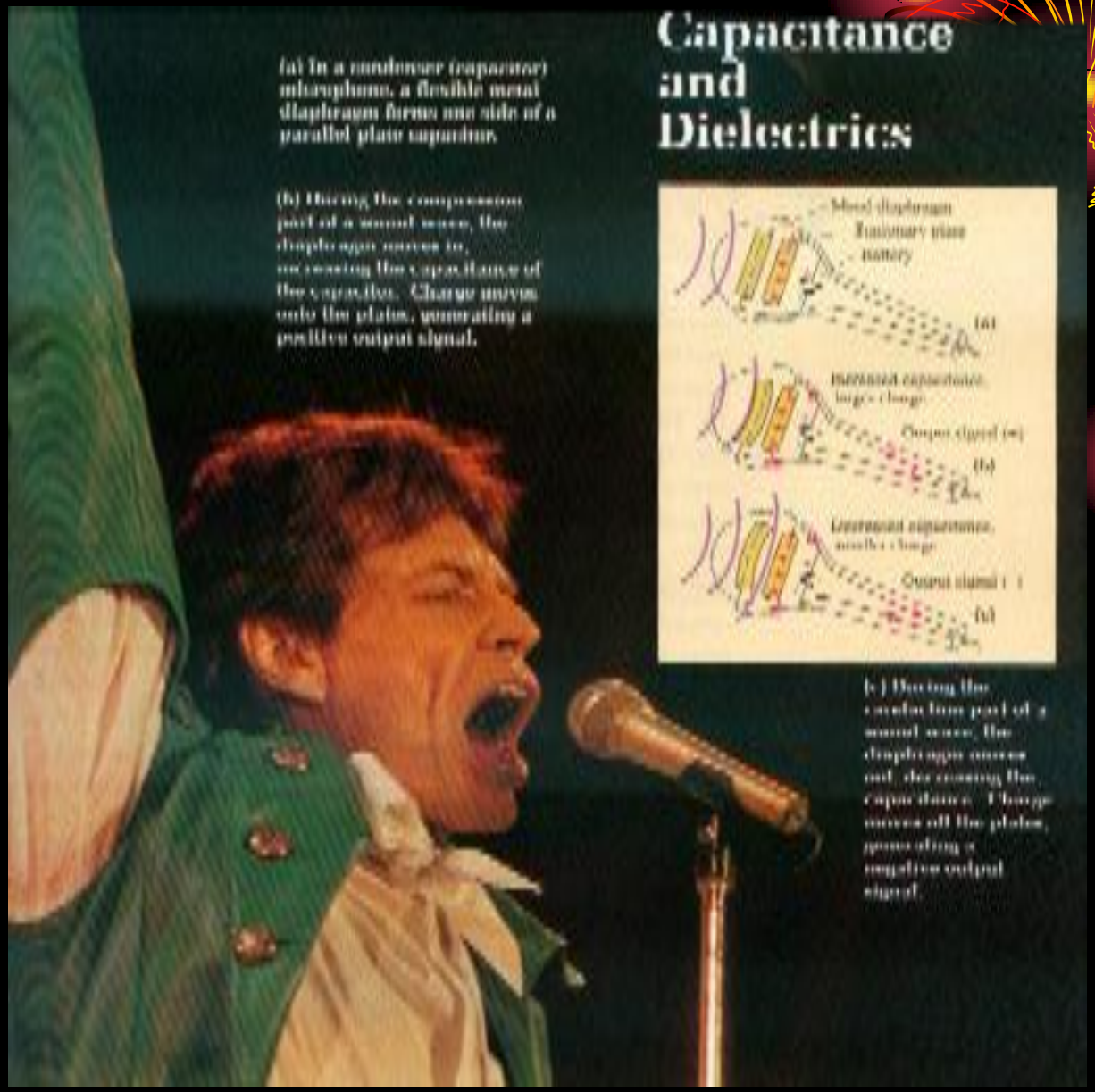
- A = 9.50 × $10^{-5}$$m^2$; k = 3.50
- 按鍵後d由5.00mm減為0.150mm

$$\frac{k\varepsilon_0 \mathrm{A}}{d} = 19.6PF\left(10^{-12}F\right)$$

- 按鍵前 C = 0.589PF

$$\delta C = 19.0PF$$

電容式麥克風

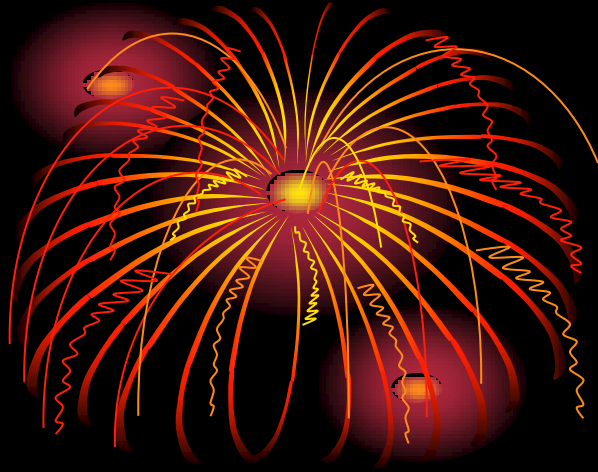

Capacitance and Dielectrics
(a) In a condenser (capacitor) microphone, a flexible metal diaphragm forms one side of a parallel plate capacitor.
(b) During the compression part of a sound wave, the diaphragm moves in, increasing the capacitance of the capacitor. Charge moves onto the plates, generating a positive output signal.
(c) During the rarefaction part of a sound wave, the diaphragm moves out, decreasing the capacitance. Charge moves off the plates, generating a negative output signal.

Ex.6 defibrillator (除纖顫機)

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(70\times10^{-6}\,\mathrm{F})(5000\,\mathrm{V})^2 = 875\,\mathrm{J}$$

$$P = \frac{U}{t} = \frac{200\,\mathrm{J}}{2.0\times10^{-3}\,\mathrm{s}} = 100\,\mathrm{kW}$$

心跳停止病人之急救



1. 盡快求助
2. 盡快給予先進的救命治療
3. 盡快實行心肺復甦術(CPR)
4. 盡快除纖顫