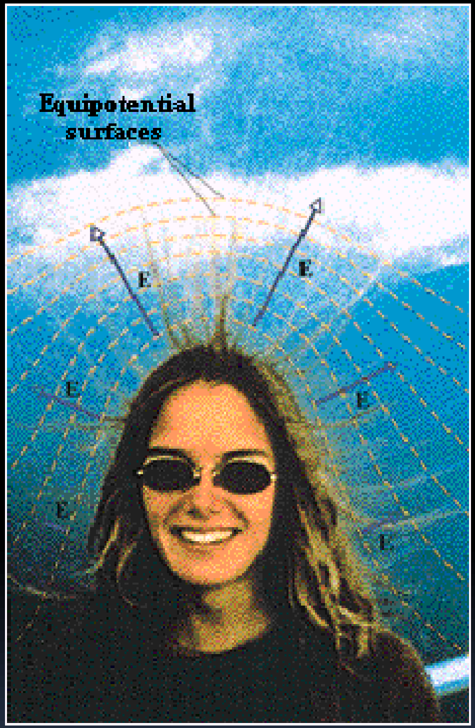
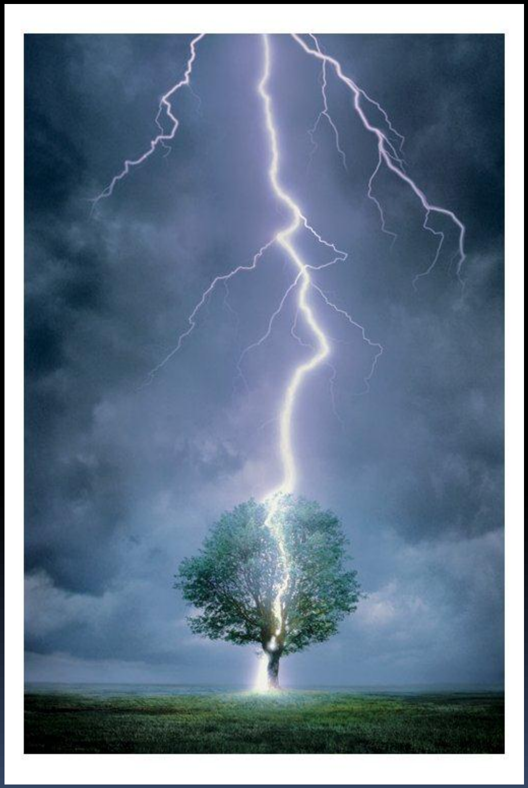
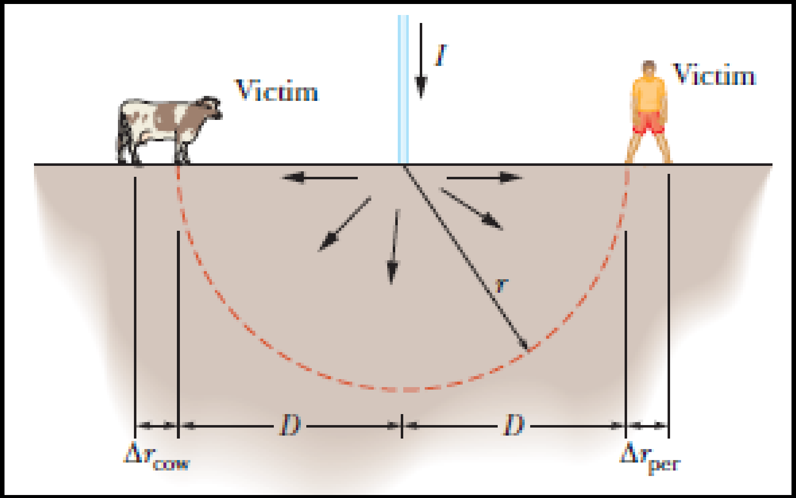
3 電位



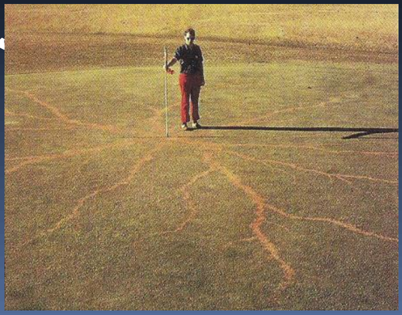
What is the danger if your hair suddenly stands up?

Lightning bolt and ground current



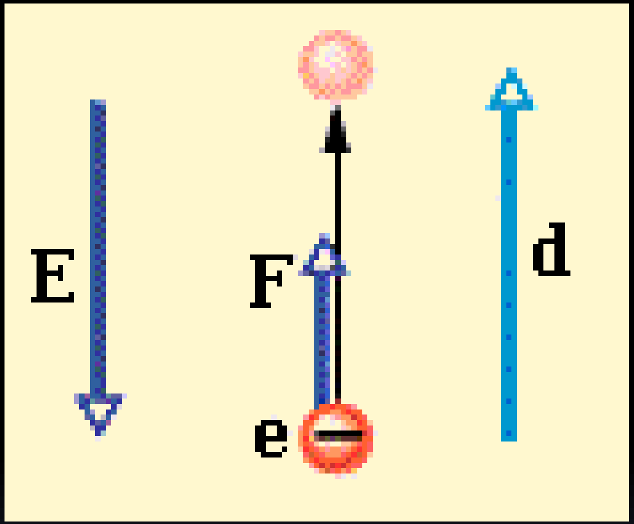


How can you reduce your risk from ground curr



3-1 電位能

- F_e and F_g are mathematically identical
- F_e is a conservative force



$$\Delta U = U_f - U_i = -W = -\int \vec{F}_e \cdot d\vec{r}$$

3-2 電位

- U_e depends on q, but V_e does not

$$V = \frac{U}{q}$$

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q}$$

$$= \frac{\Delta U}{q} = -\frac{W}{q}$$

3-3 等位面

- Four Equipotential Surfaces

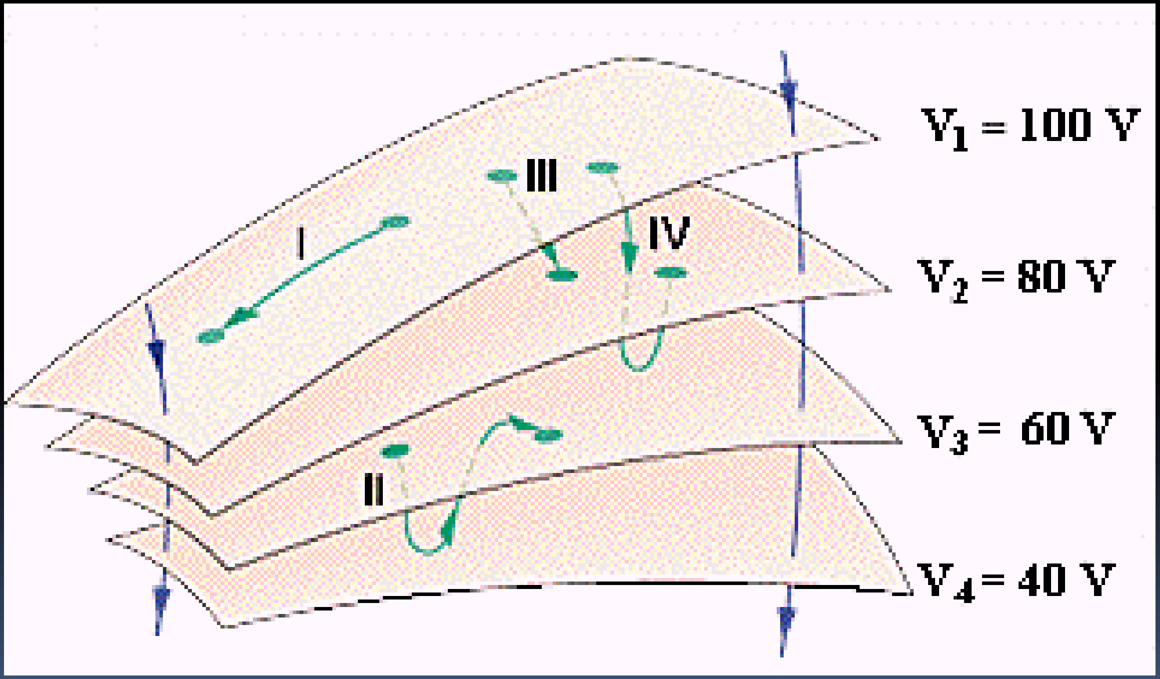
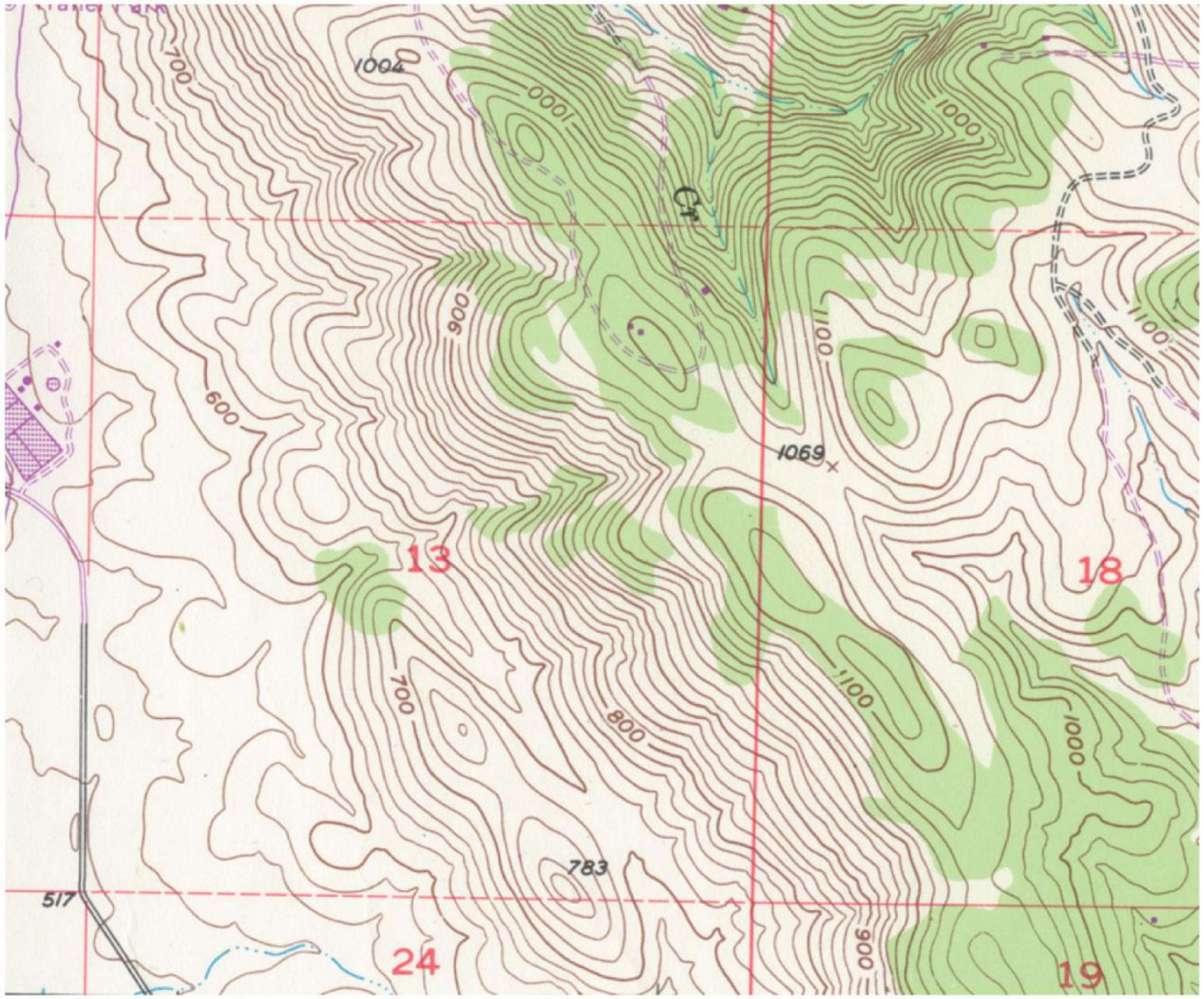
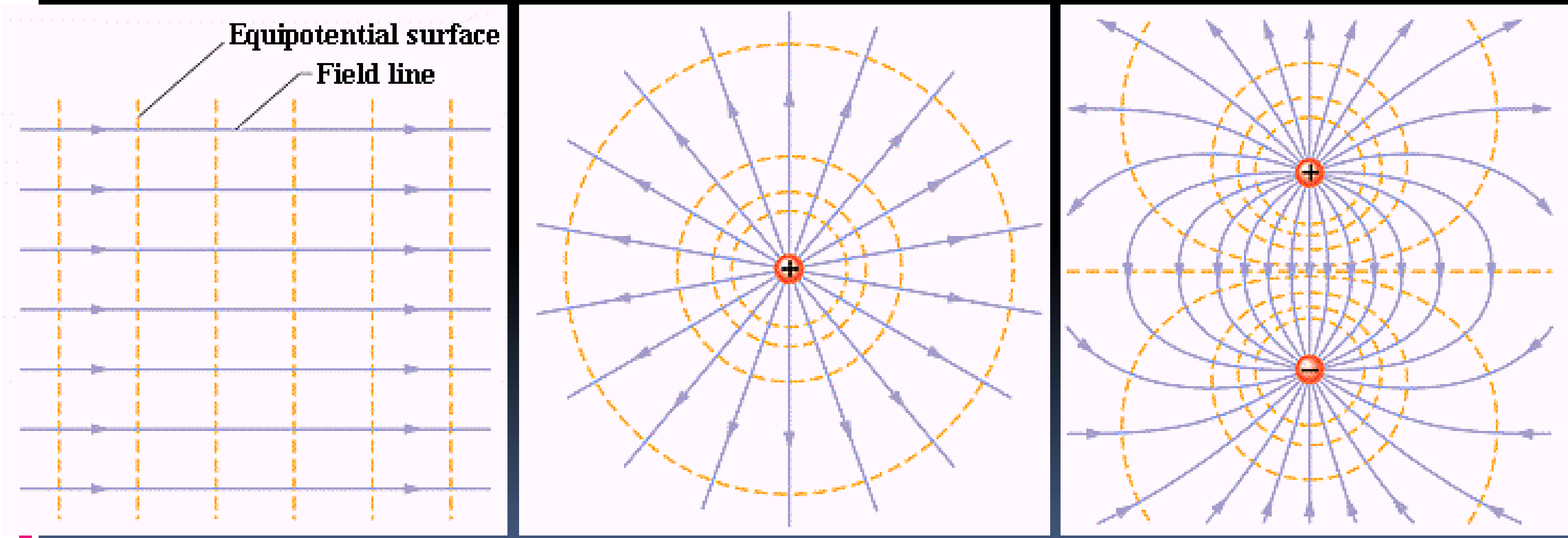


FIGURE 23.22 CONTOUR LINES, CURVES OF CONSTANT ELEVATION



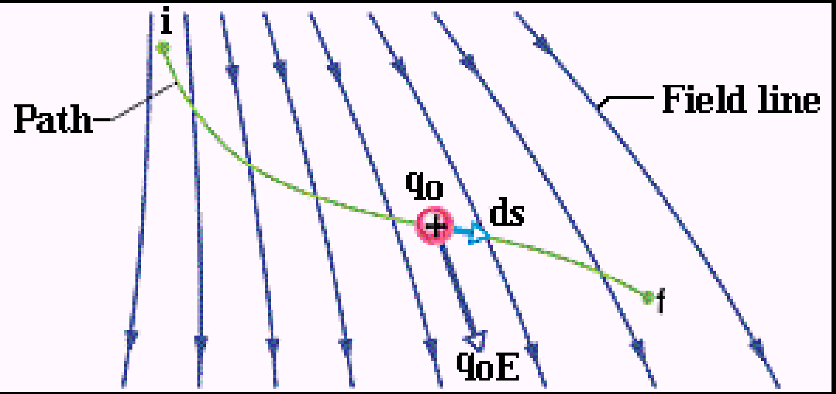
等位面3例

- E.S. for a uniform field, a point charge, and an electric dipole



3-4 由電場算電位

- The differential work done by **F**

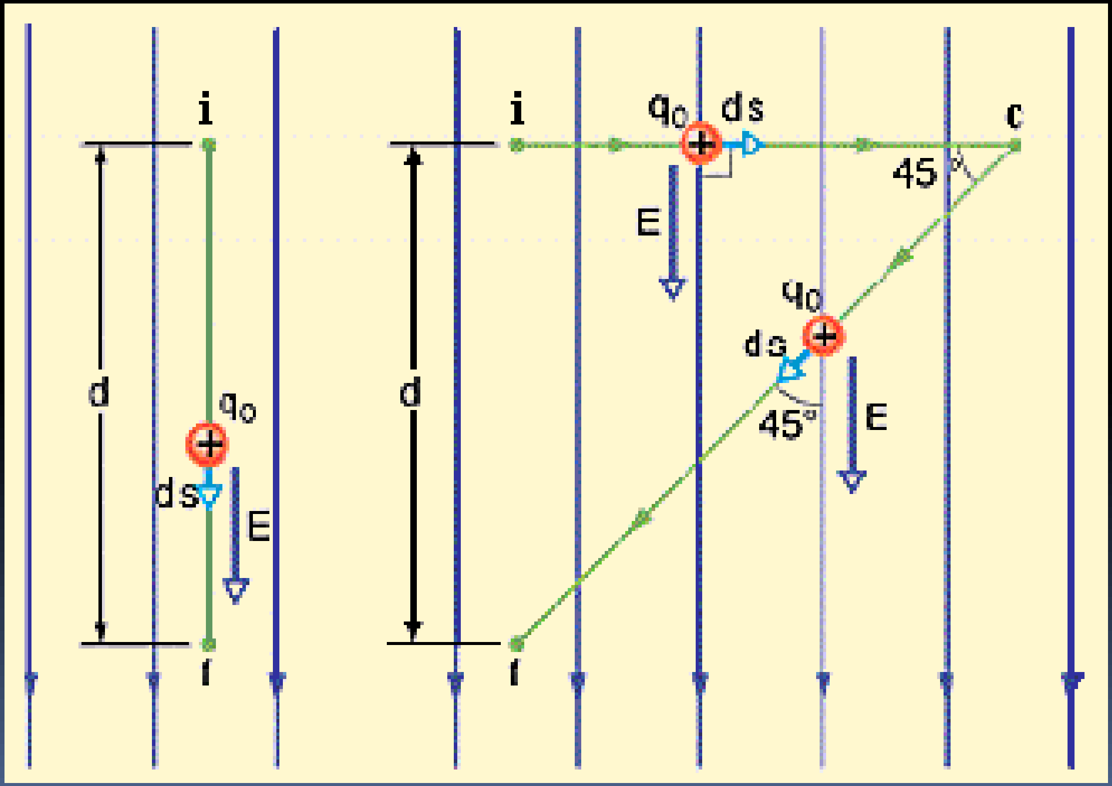


$$dW = \vec{F} \cdot d\vec{s}, dW = q_0 \vec{E} \cdot d\vec{s}$$

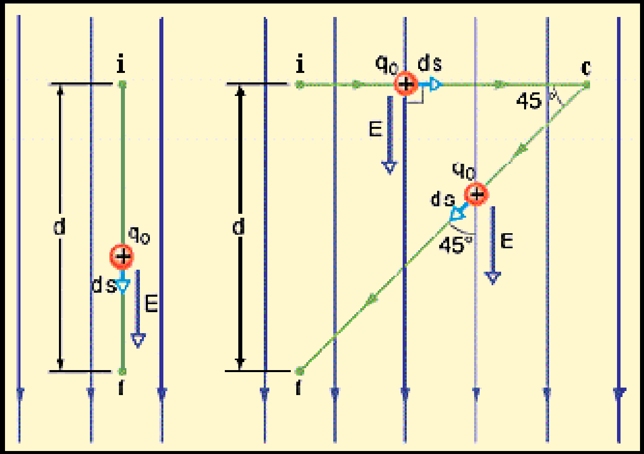
$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}$$

Ex.1 均勻電場

(a) Along path (a)



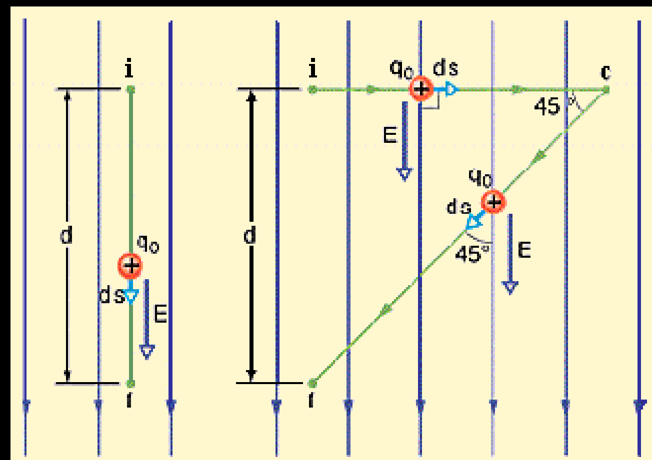
Ex. 1 均勻電場：路徑(a)



$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f E(\cos 0)ds$$

$$= -\int_i^f Eds = -E\int_i^f ds = -Ed$$

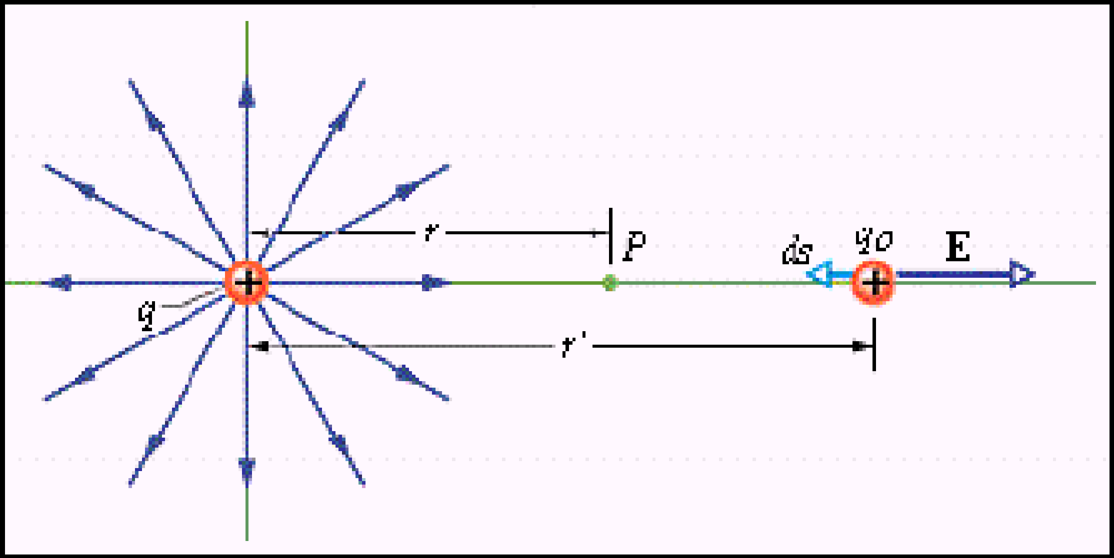
Ex.1 均勻電場：路徑(b)



$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f E(\cos 45^\circ)ds$$

$$= -\frac{E}{\sqrt{2}}\int_c^f ds = -\frac{E}{\sqrt{2}}\sqrt{2}d = -Ed$$

3-5 點電荷的電位



$$\vec{E}\cdot d\vec{s} = |E||ds|\cos 180^\circ = |E|dr'$$

$$V = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_\infty^f |E|dr'$$

點電荷的電位

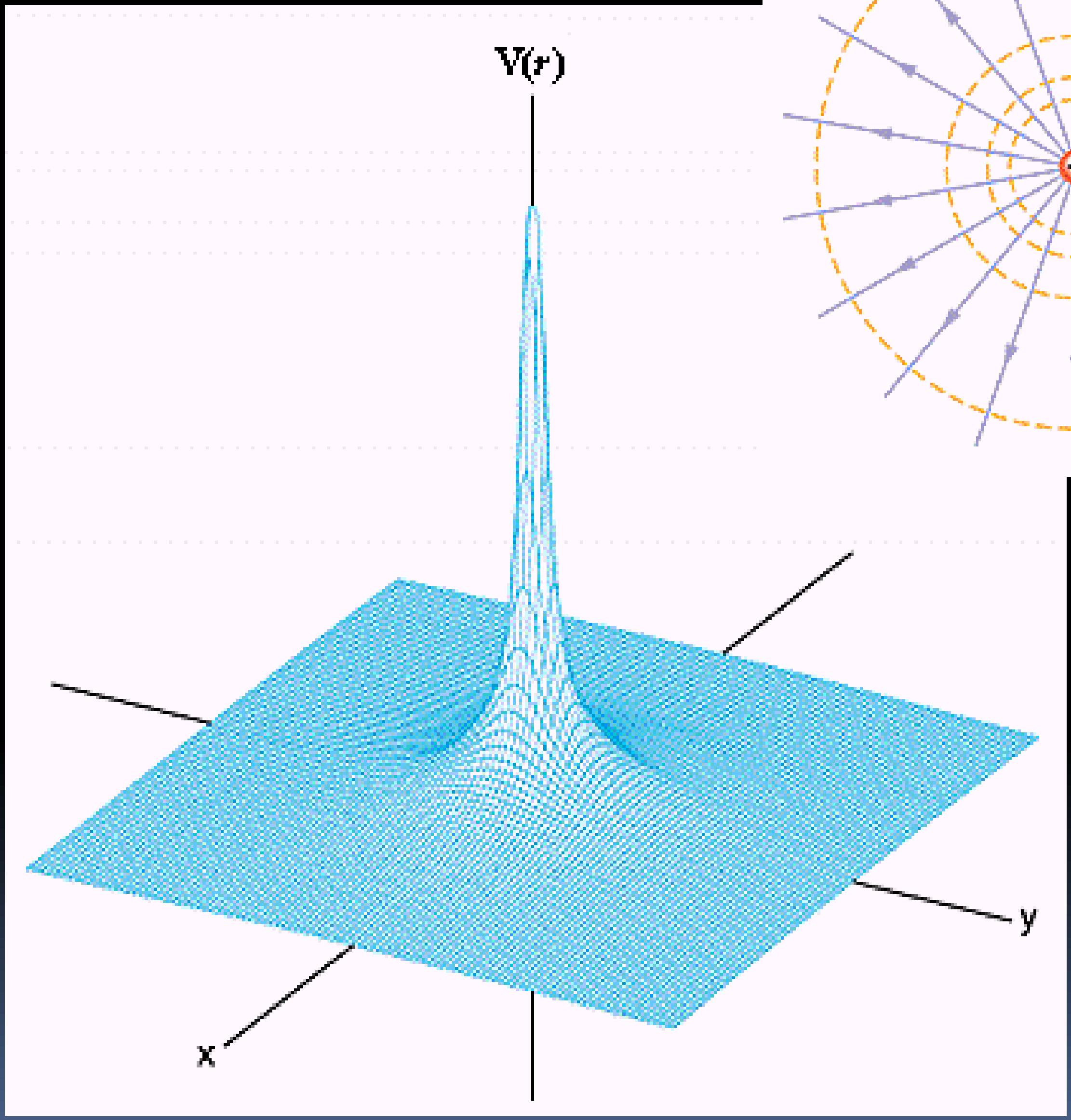
$$V = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_\infty^f |E| dr'$$

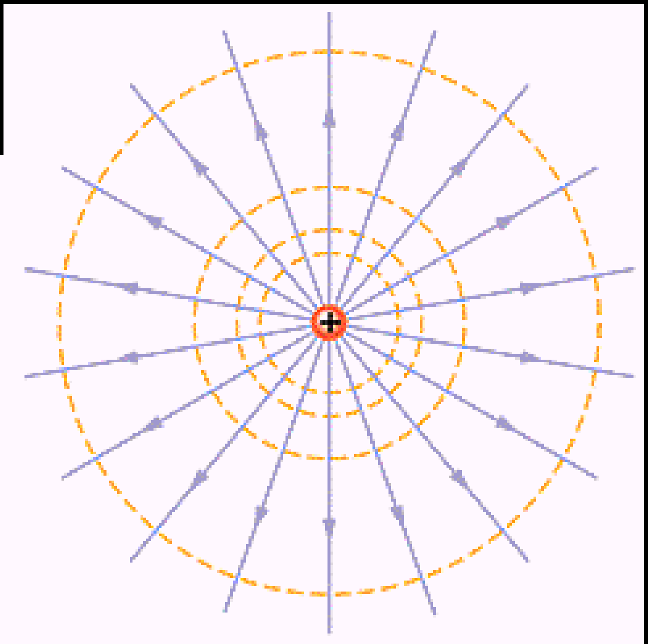
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r'^2}$$

$$V = -\frac{q}{4\pi\varepsilon_0} \int_\infty^f \frac{1}{r'^2} dr' = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r'} \right]_\infty^f$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

點電荷的電位圖



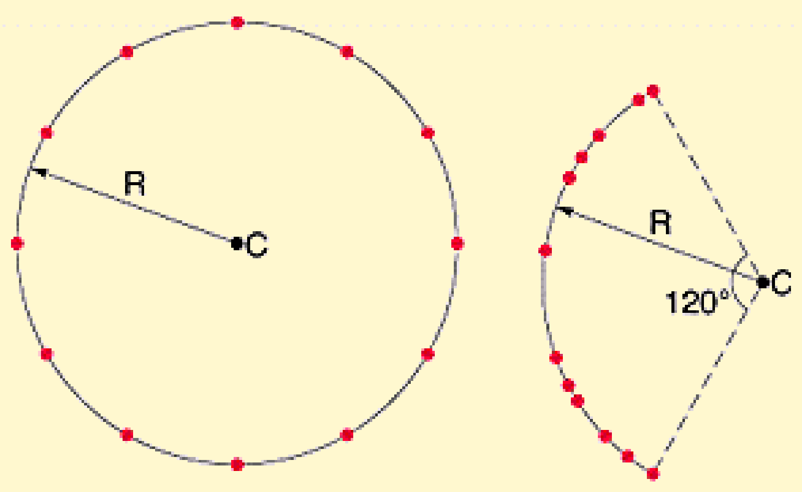


3-6 一組點電荷的電位

- **The principle of superposition**

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

Ex.2 V_c and E_c for 12 electrons



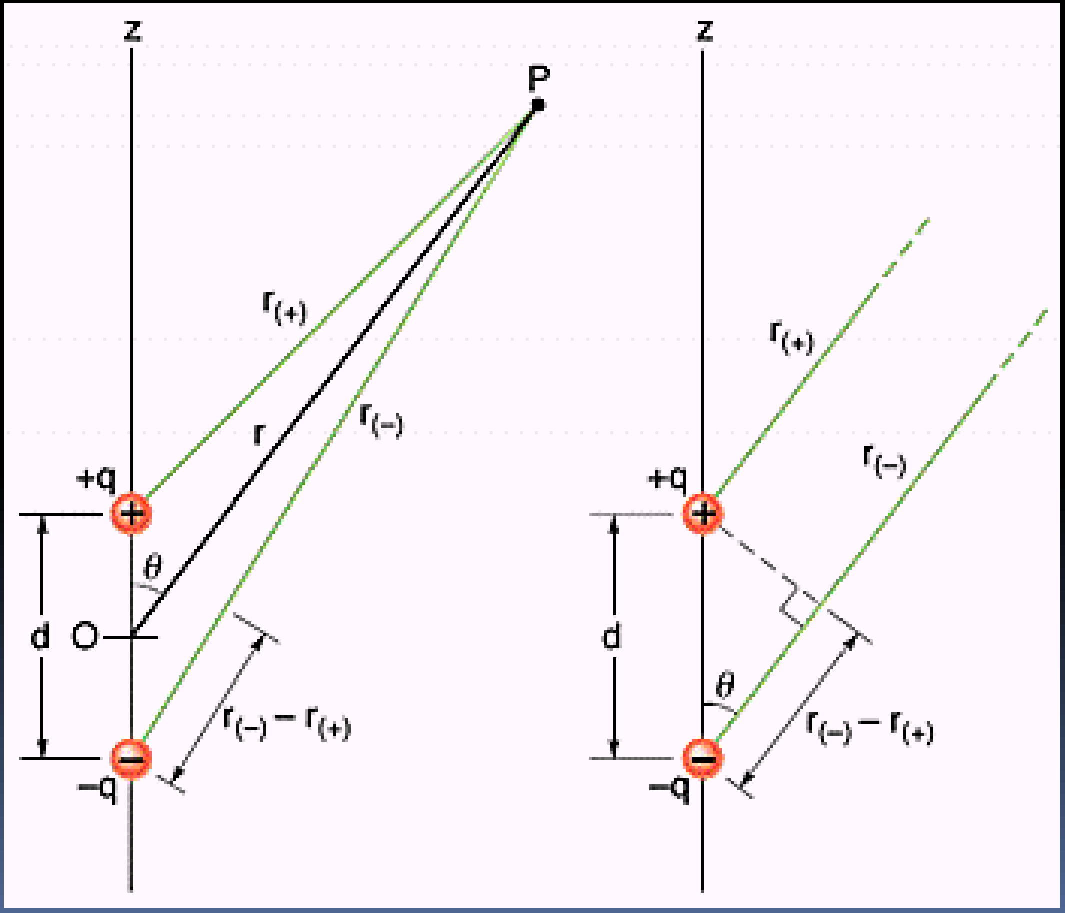
(a) on a circle

$$V = -12\frac{1}{4\pi\varepsilon_0}\frac{e}{R},\ E = 0$$

(b) on an arc

$$V = -12\frac{1}{4\pi\varepsilon_0}\frac{e}{R},\ E \neq 0$$

3-7 電雙極的電位



雙極矩 $p = qd$

$$V = \sum_{i=1}^{2} V_i = V_+ + V_- = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} + \frac{-q}{r_-}\right)$$

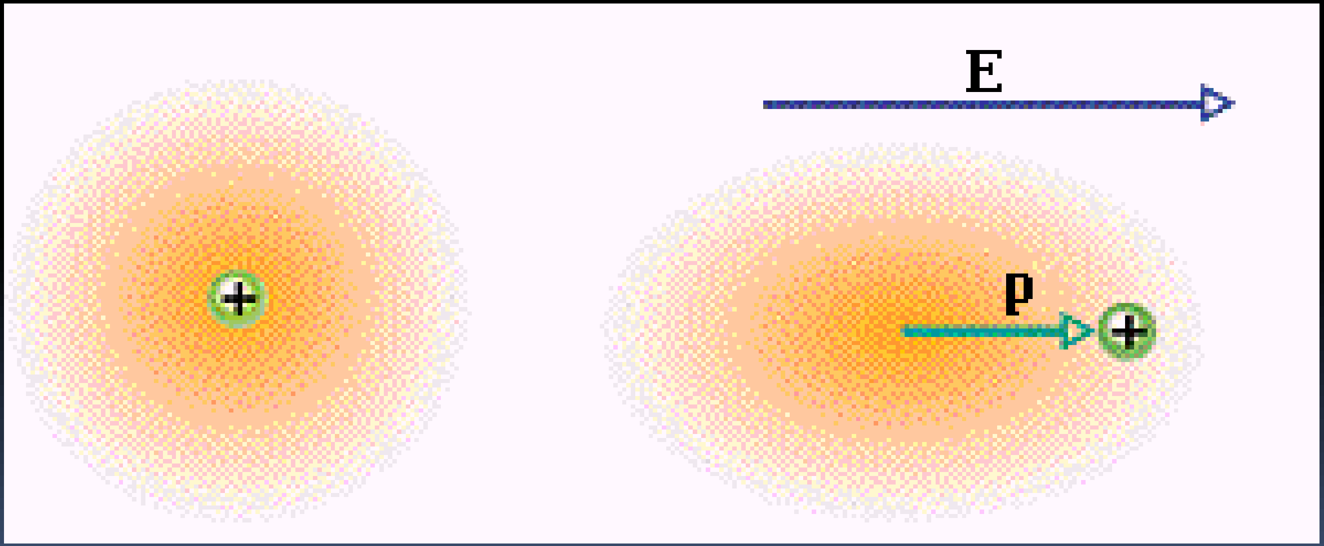
$$= \frac{q}{4\pi\varepsilon_0}\frac{r_- - r_+}{r_- r_+} \approx \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$(r_- - r_+ \approx d\cos\theta,\ r_- r \approx r^2)$$

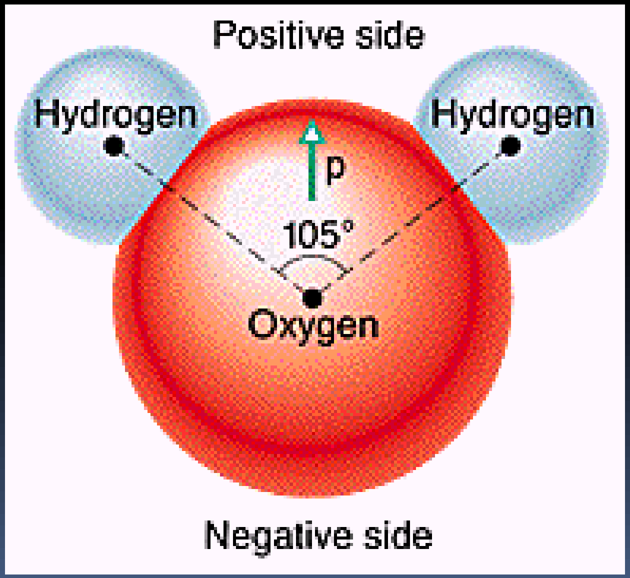
- $\vec{p}$ 由負電荷指向正電荷

Induced Dipole Moment

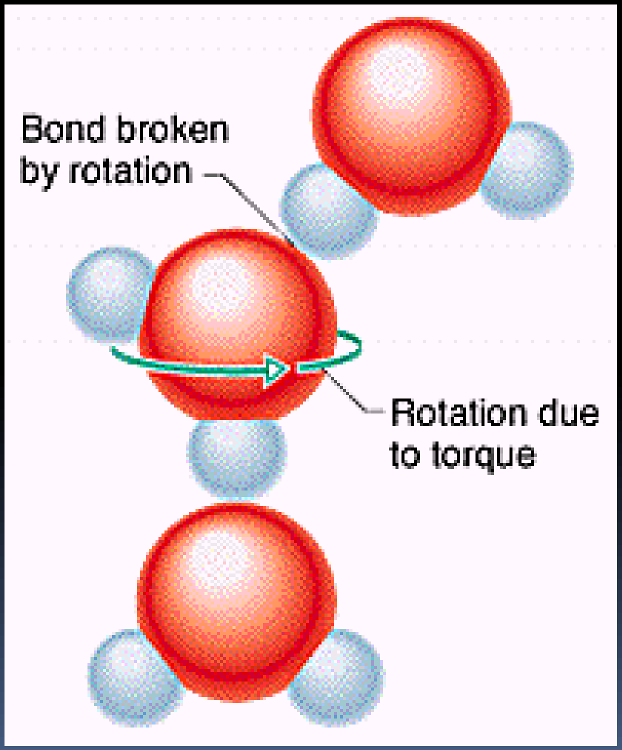
- 感應雙極矩
- 極化 (*polarization*)



水分子與微波爐



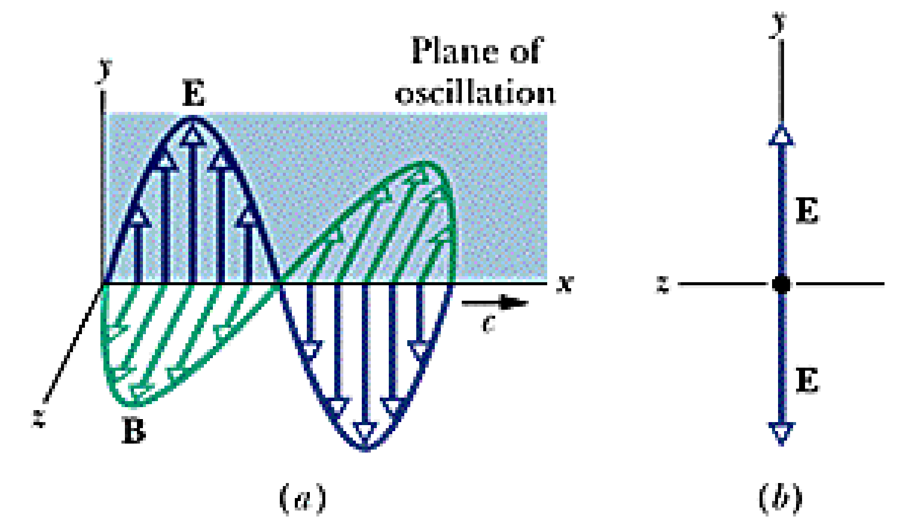
Positive side
Hydrogen
Hydrogen
p
105°
Oxygen
Negative side

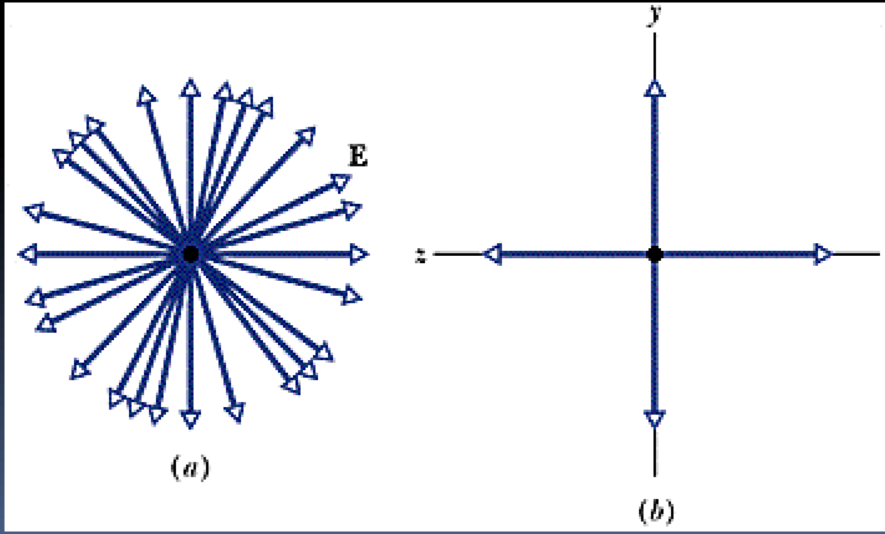


Bond broken
by rotation
Rotation due
to torque

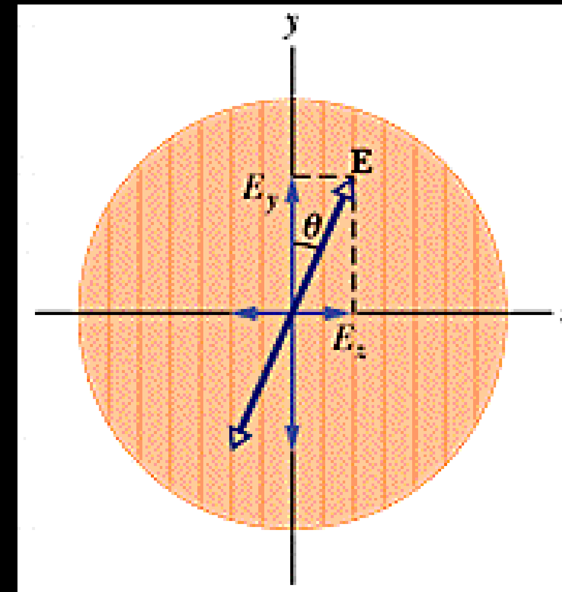
3-8 Polarization (偏振)

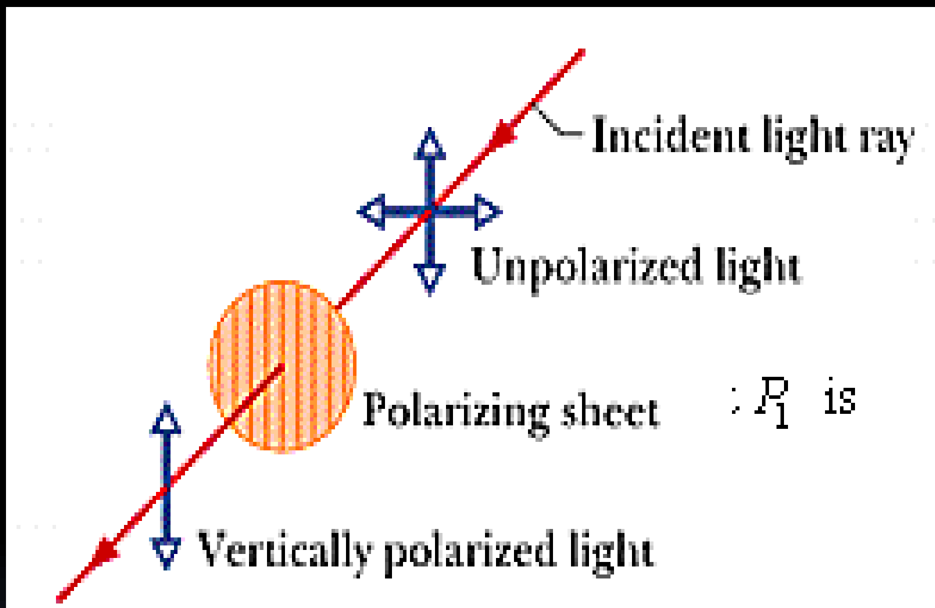
- Polarized Light
- The Plane of Polarization
- Linear, Circular and Elliptical polarization

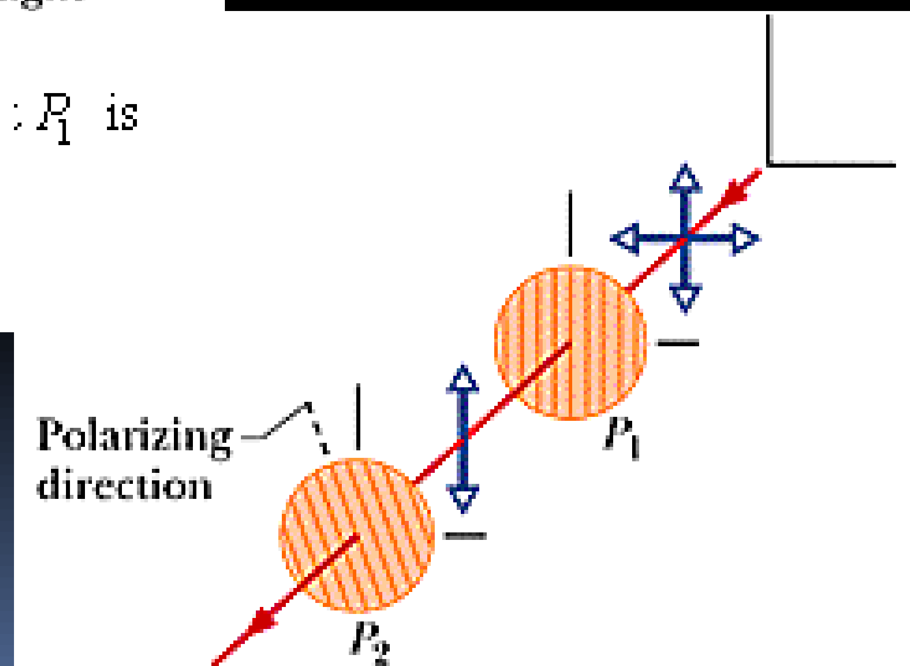




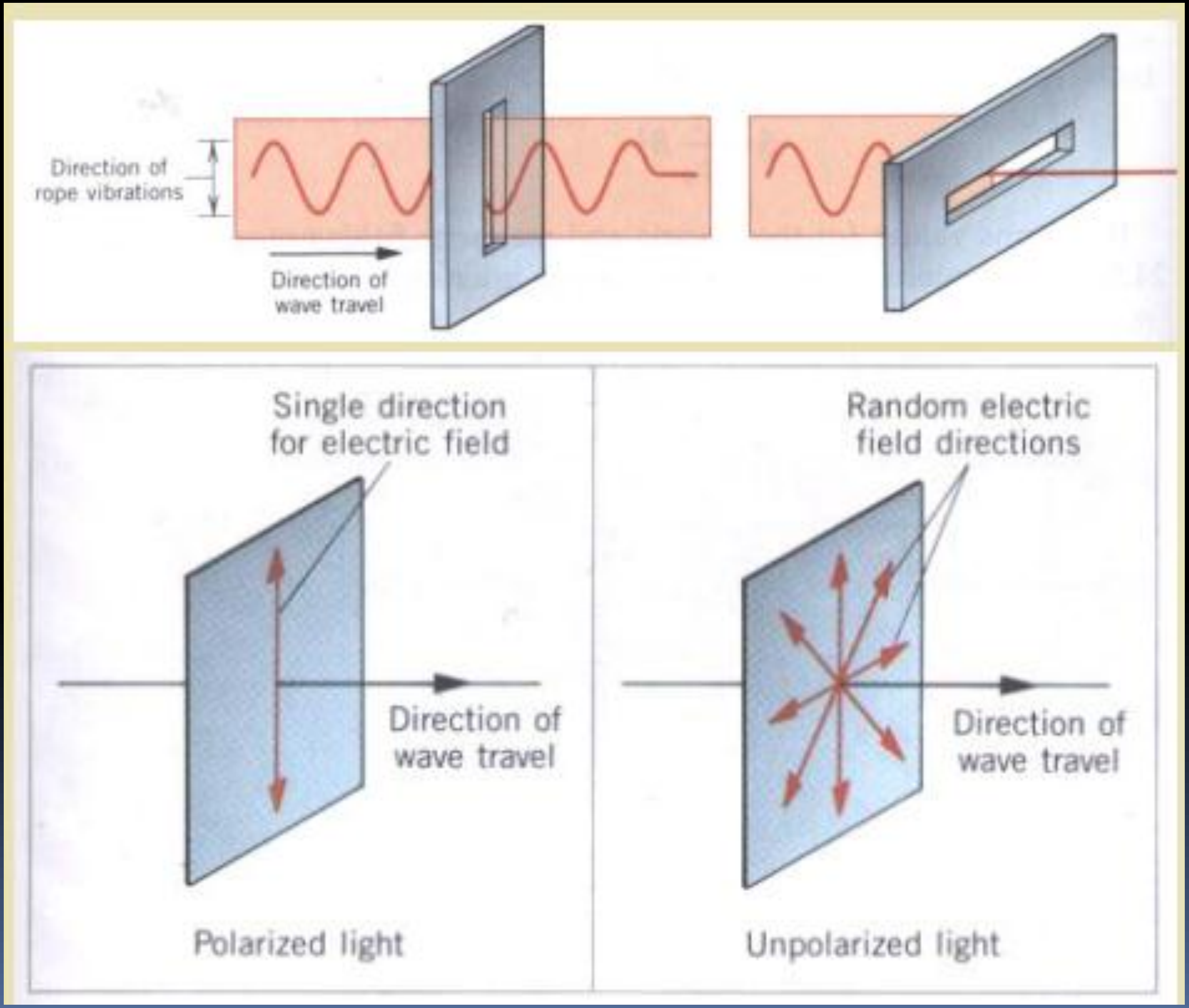
偏振片


y
E
E_y
θ
z
E_z

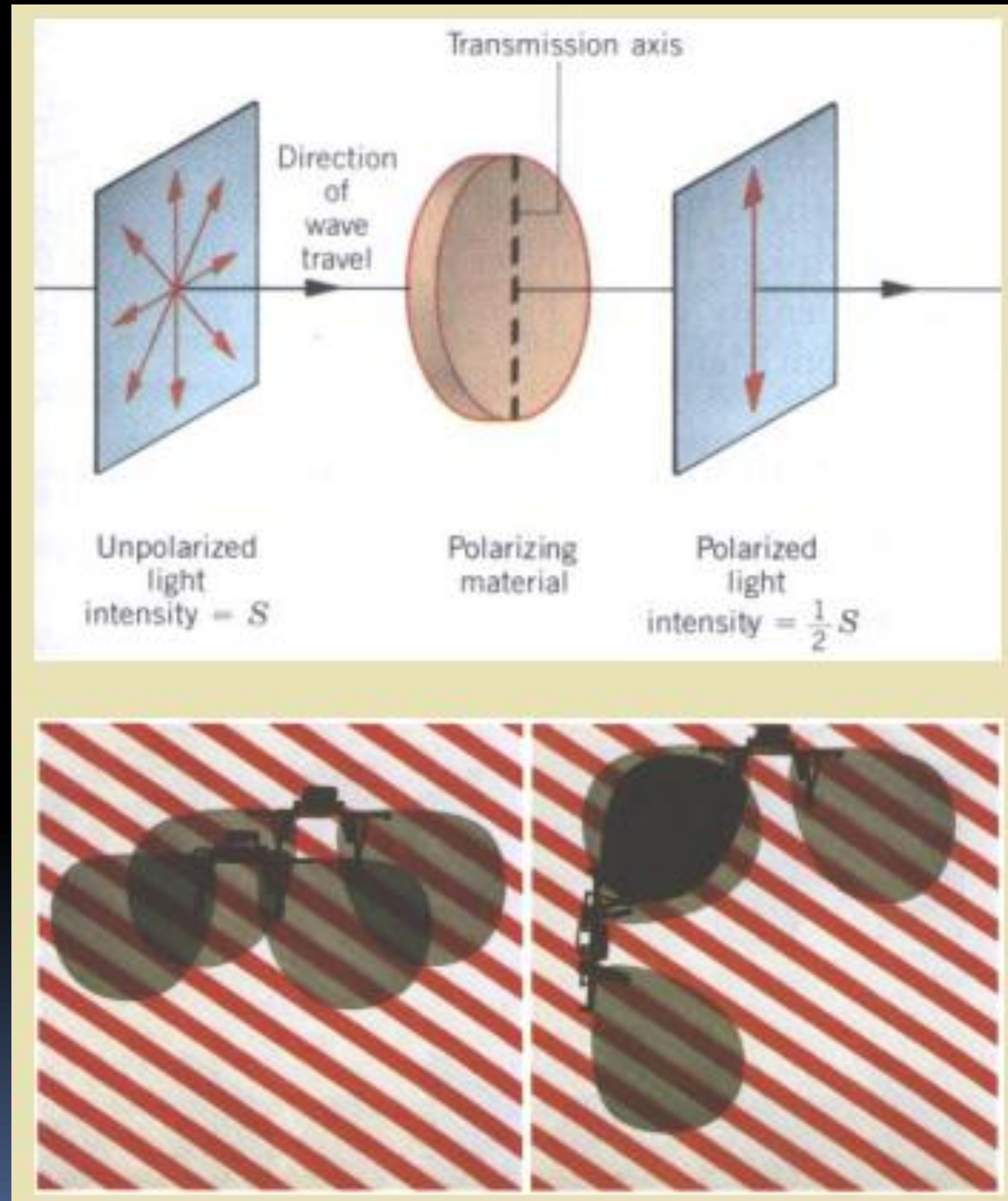

Incident light ray
Unpolarized light
Polarizing sheet
Vertically polarized light


Polarizing direction
P_1
P_2

Polarization of Light



偏振與太陽眼鏡

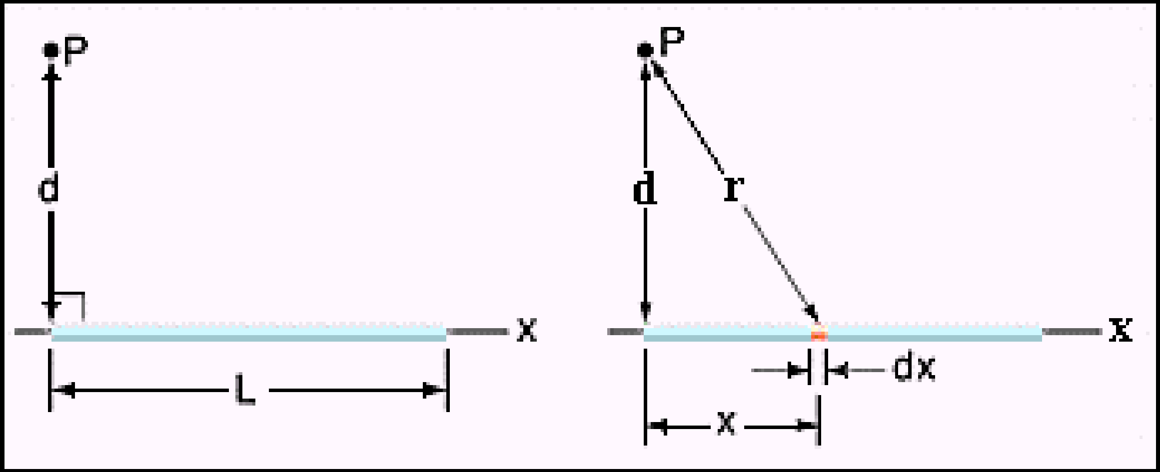


Transmission axis
Direction of wave travel
Unpolarized light intensity = S
Polarizing material
Polarized light intensity = $\frac{1}{2}S$

3-9 連續電荷分佈的電位

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \rightarrow \int dV = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$

Line of Charge



$$dq = \lambda dx,$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(x^2 + d^2)^{1/2}}$$

計算

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{(x^2 + d^2)^{1/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}$$

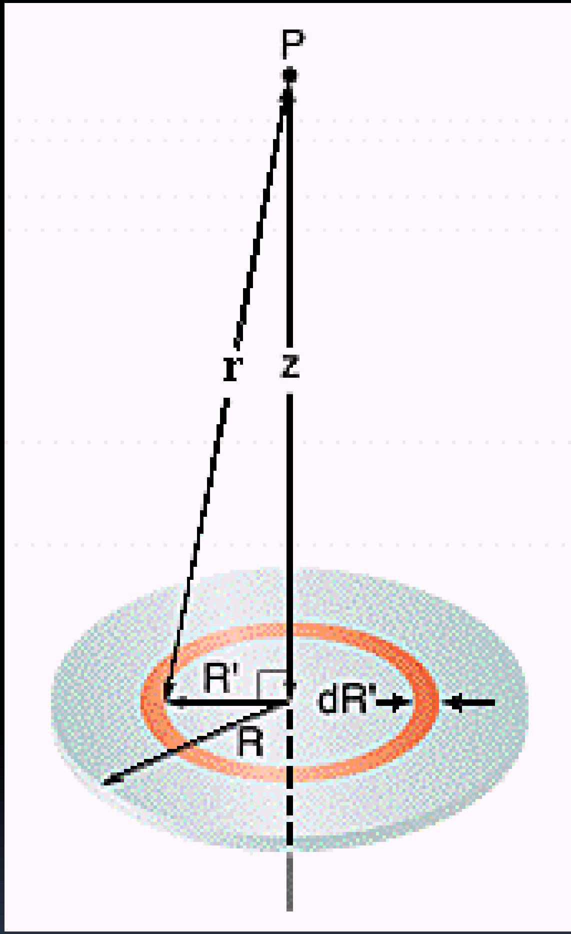
$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln(x + (x^2 + d^2)^{1/2} \right]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \left[\ln[L + (L^2 + d^2)^{1/2}] - \ln d \right]$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{L + (L^2 + d^2)^{1/2}]}{d} \right]$$

Charged Disk

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$



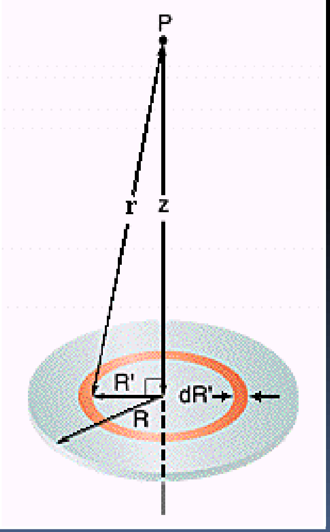
計算

$$dq = \sigma(2\pi R')(dR')$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{(z^2 + R^2)^{1/2}}$$

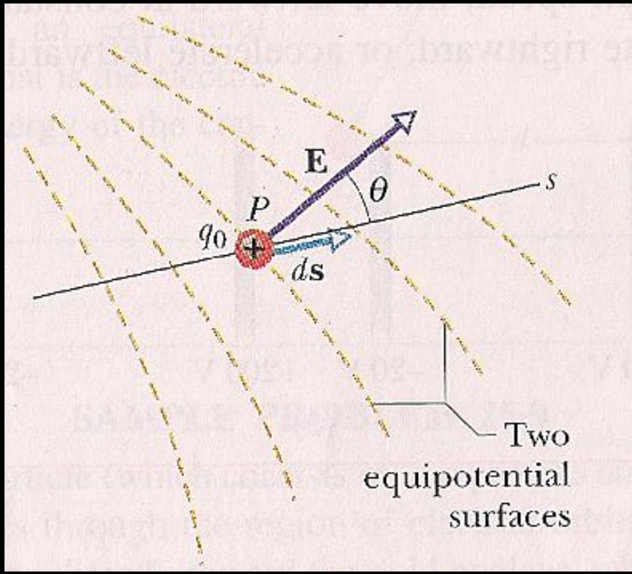
$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'dR'}{(z^2 + R'^2)^{1/2}}$$

$$= \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z)$$



3-10 由電位算電場

- The work done by the electric field

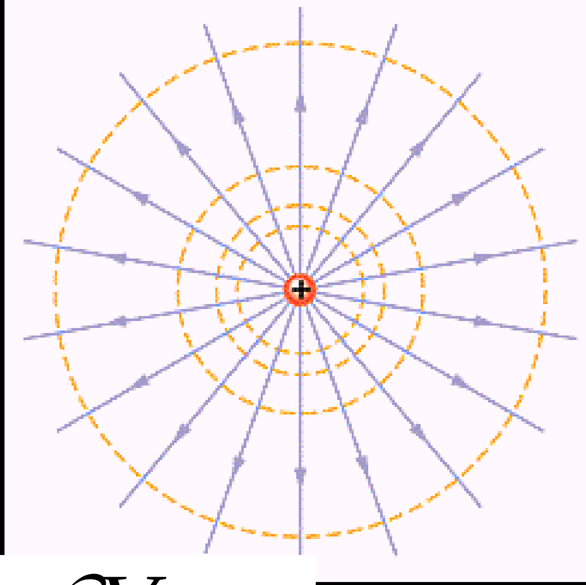


$$dW = -q_0 dV = q_0 E(\cos\theta)ds$$

$$E\cos\theta = -\frac{dV}{ds} \rightarrow E_s = -\frac{\partial V}{\partial s}$$

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

3-11 The gradient operator 梯度運算子



$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$= -\frac{\partial V}{\partial x}\hat{i} + -\frac{\partial V}{\partial y}\hat{j} + -\frac{\partial V}{\partial z}\hat{k}$$

$$= -(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})V \quad = -\vec{\nabla}V$$

The Laplacian operator

$$\vec{E} = -\vec{\nabla} V \qquad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot (-\vec{\nabla} V) = -\nabla^2 V = \frac{\rho}{\varepsilon_0}$$

$$\nabla^2 = (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}) \cdot (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})V = -\frac{\rho}{\varepsilon_0}$$

The Laplace Equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)V=0$$

For 2 dimensional problems

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)V=0,\ \text{If } V(x,y)=X(x)Y(y)$$

$$\frac{\partial^2 X(x)Y(y)}{\partial x^2}+\frac{\partial^2 X(x)Y(y)}{\partial y^2}=0$$

$$e^{-x}\sin y$$

$$Y(y)\frac{\partial^2 X(x)}{\partial x^2}+X(x)\frac{\partial^2 Y(y)}{\partial y^2}=0$$

Solving Laplace Equation by separation of variables

$$Y(y)\frac{\partial^2 X(x)}{\partial x^2} + X(x)\frac{\partial^2 Y(y)}{\partial y^2} = 0$$

$$\frac{Y(y)}{X(x)Y(y)}\frac{\partial^2 X(x)}{\partial x^2} + \frac{X(x)}{X(x)Y(y)}\frac{\partial^2 Y(y)}{\partial y^2} = 0$$

$$\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2} + \frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2} = 0$$

$$\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2} = \lambda, \quad \frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2} = -\lambda$$

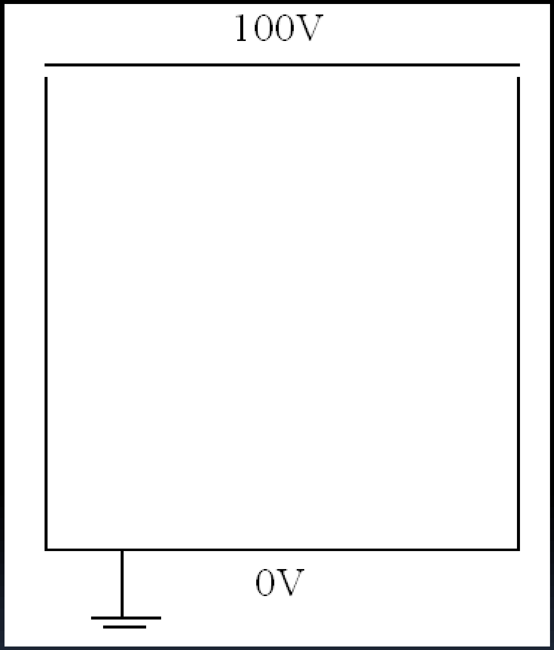
The Solution

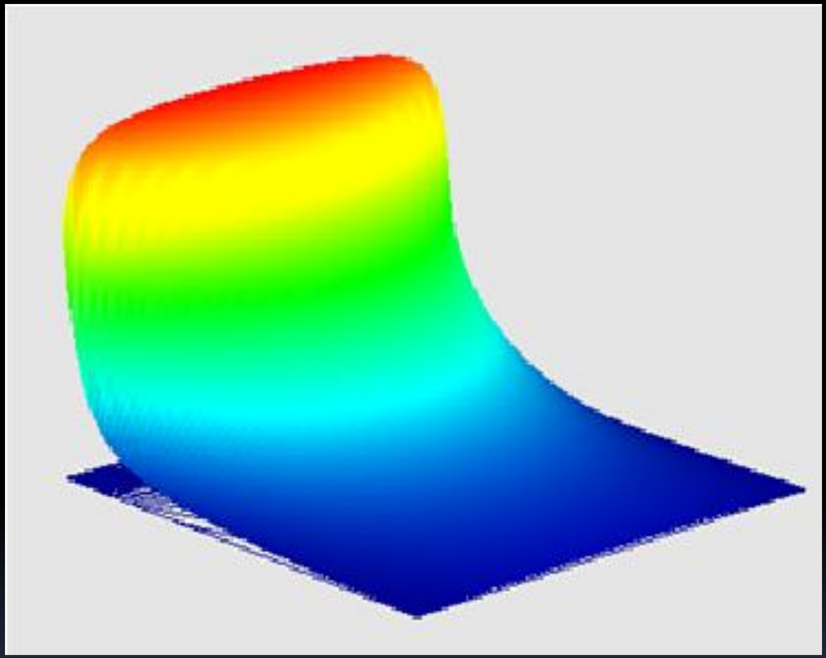
$$\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2} = \lambda^2, \quad \frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2} = -\lambda^2$$

$$\frac{\partial^2 X(x)}{\partial x^2} - \lambda^2 X(x) = 0 \rightarrow X(x) \square\; e^{-\lambda x}$$

$$\frac{\partial^2 Y(y)}{\partial y^2} + \lambda^2 Y(y) = 0 \rightarrow X(x) \square\; \sin \lambda y$$

A boundary value problem





Ex.3 The charged disk

$$V = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z)$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}(\sqrt{z^2 + R^2} - z)$$

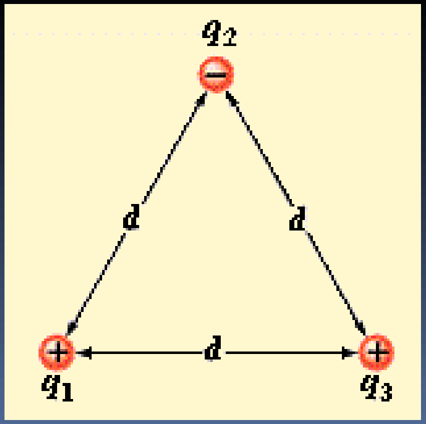
$$= \frac{\sigma}{2\varepsilon_0}(1 - \frac{z}{\sqrt{z^2 + R^2}})$$

3-12一組點電荷的電位能

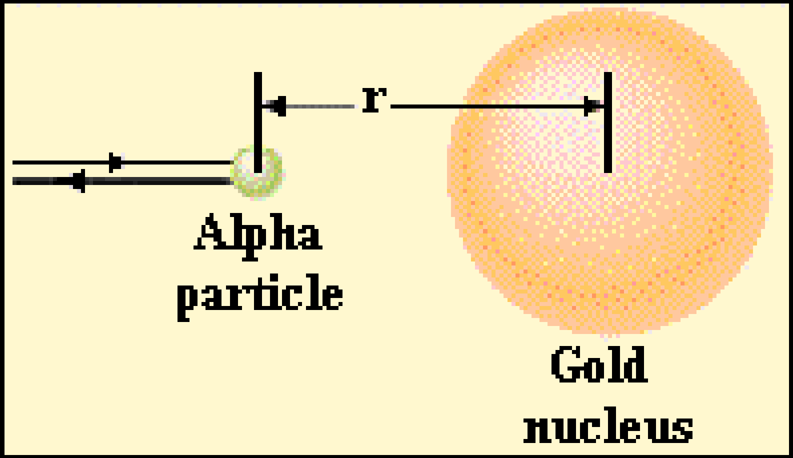
- For charges q_1 and q_2

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} \rightarrow U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

Ex.4 Three charges



Ex.4 An alpha particle and a gold nucleus



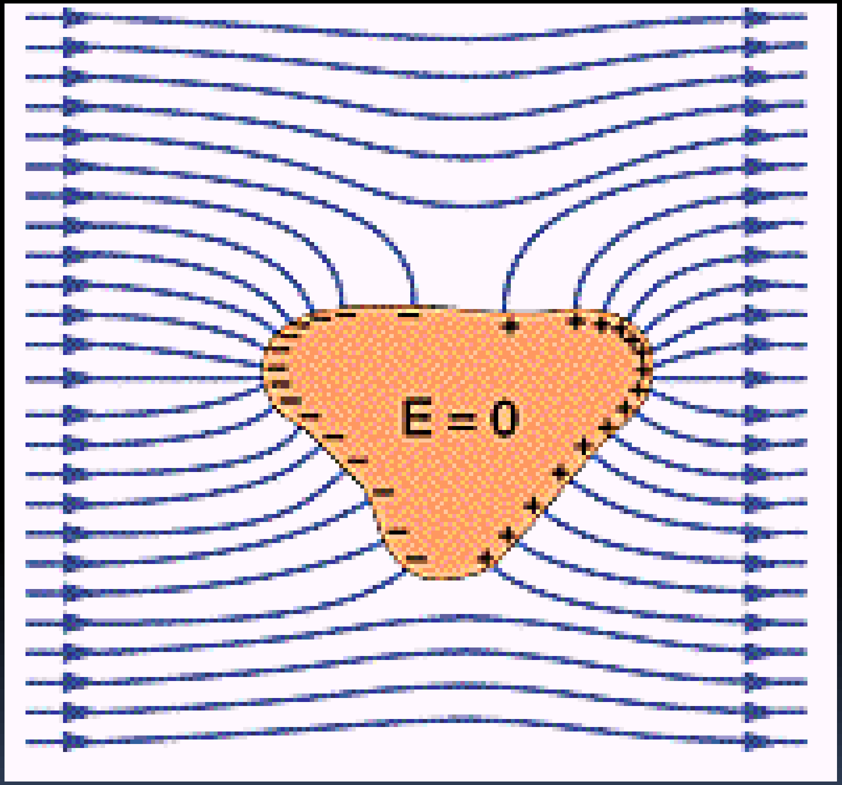
$$K = U \rightarrow K = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{9.23\,fm} = 24.6 Mev$$

3-13 Potential of a charged isolated conductor

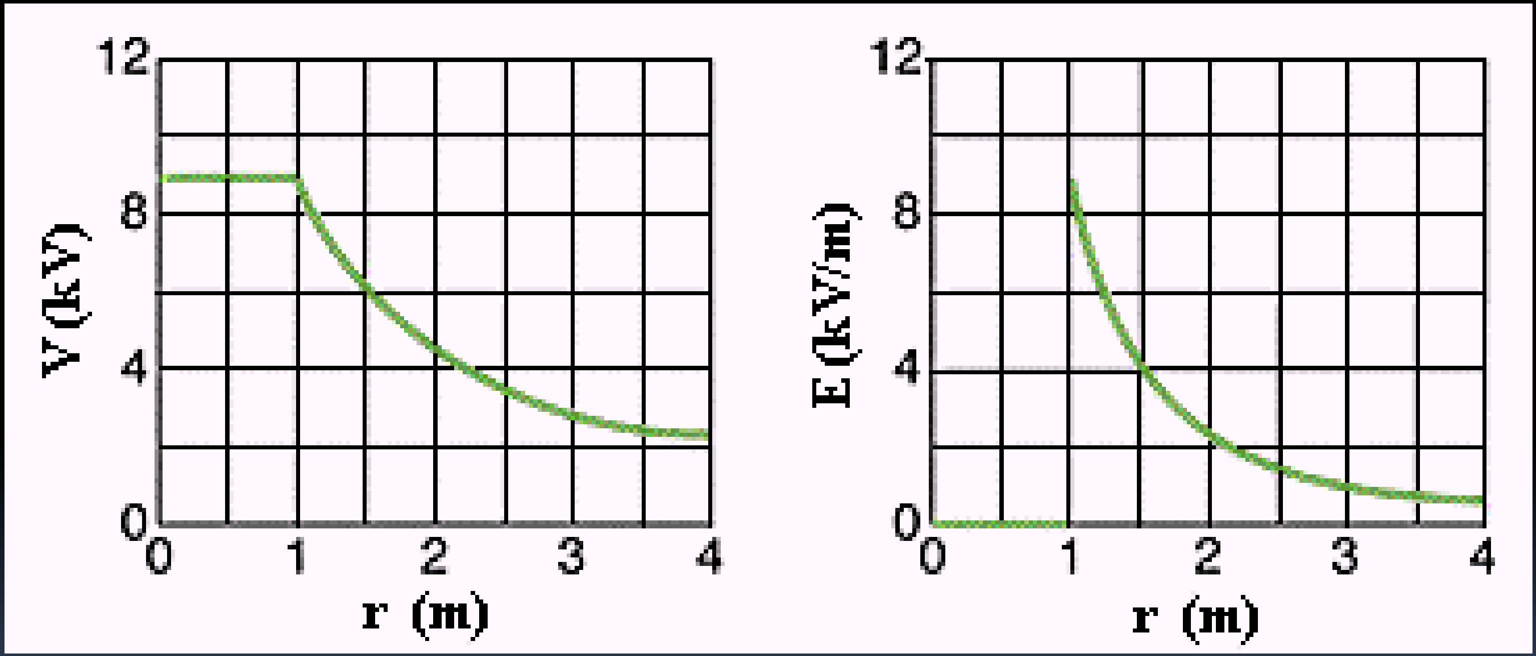
- 導體內部及表面為一等位面 (equipotentail surface)

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = 0 \;\; (E = 0)$$

Discharge

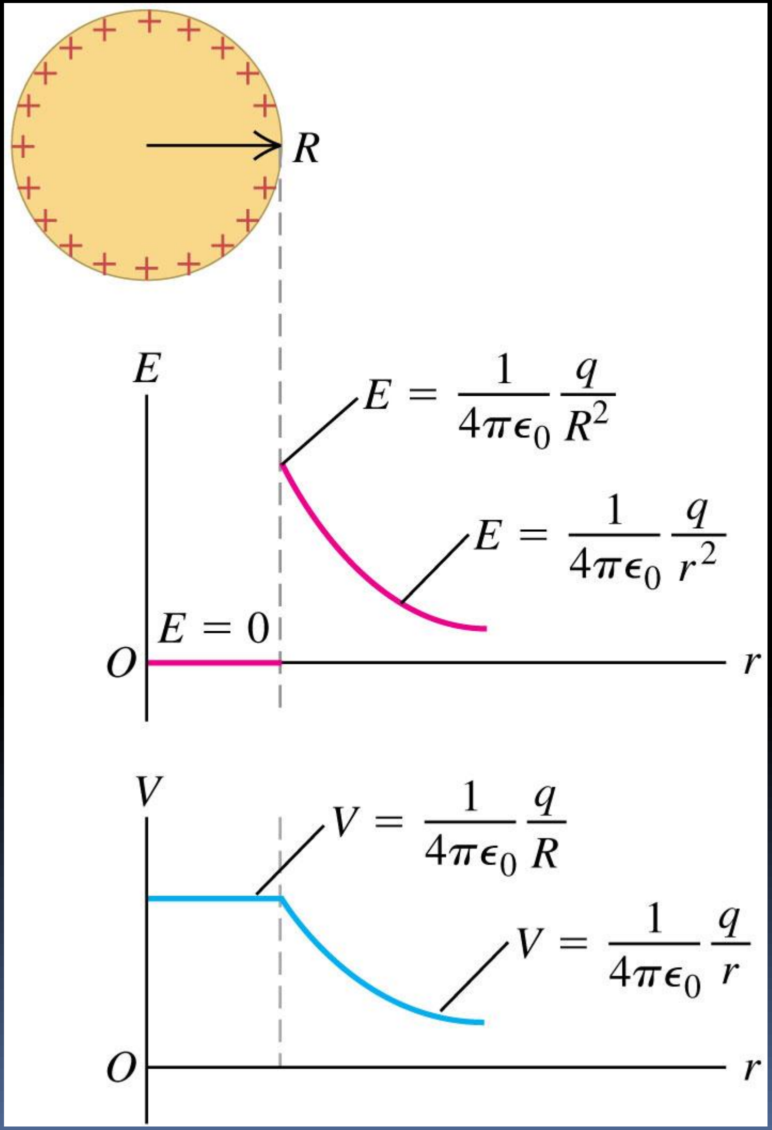


Plots of **E** (r) and V(r)



Calculating electric potential

Example 23.8 A charged conducting sphere



$$V_{surface} = E_{surface}R$$

$$V_{max} = E_{max}R$$

e.g. $E_{max} = 3\times10^6$ V/m

If R=1 cm V_{max} = 30,000V

If R=2 m V_{max} = 6MV