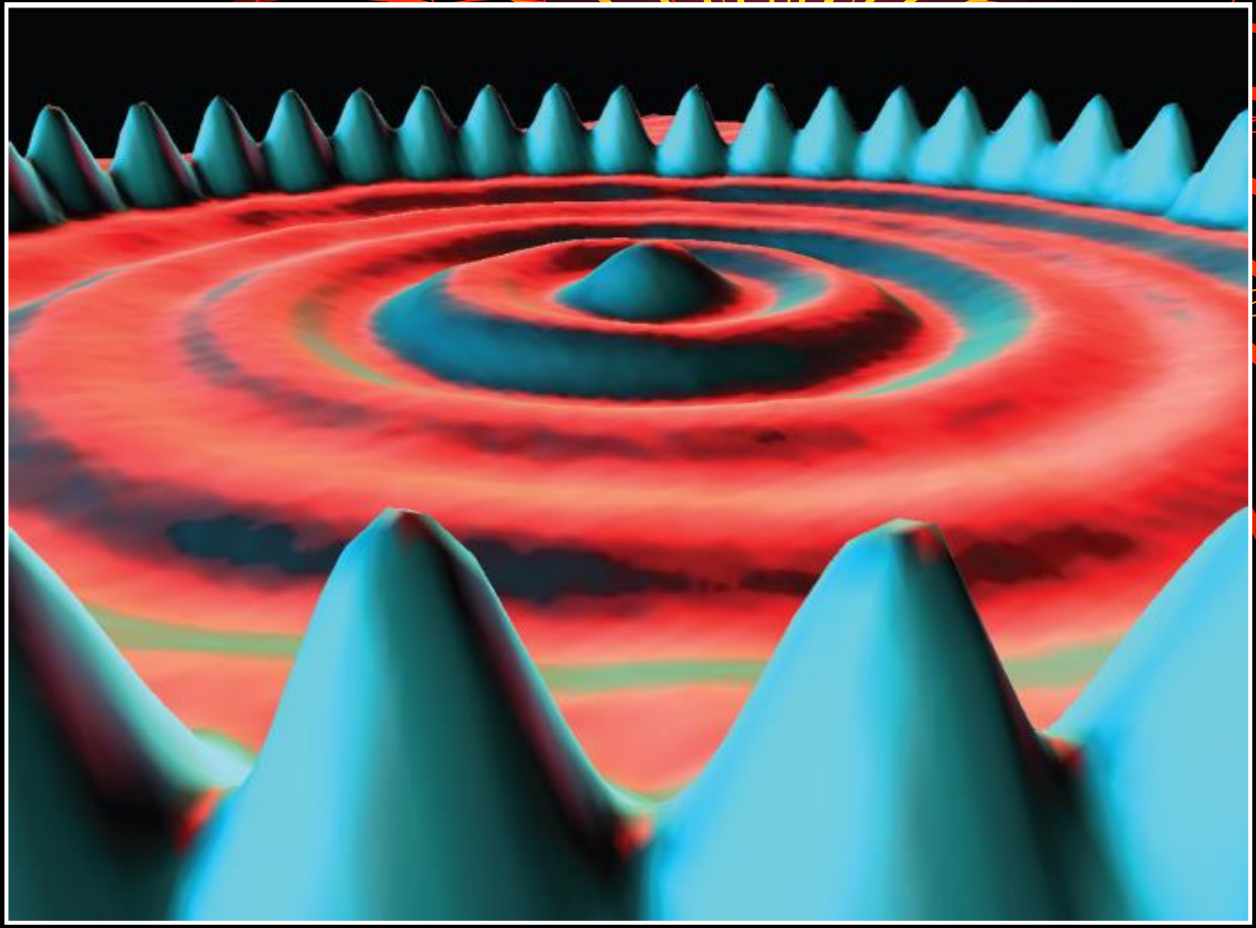
12 量子物理



Sections

1. Photon and Matter Waves
2. Compton Effect
3. Light as a Probability Wave
4. Electrons and Matter Waves
5. Schrodinger's Equation
6. Waves on Strings and Matter Waves
7. Trapping an Electron
8. Three Electron Traps
9. The Hydrogen Atom

12-1 Photon and Matter Waves (光子和物質波)

- Light Waves and Photons

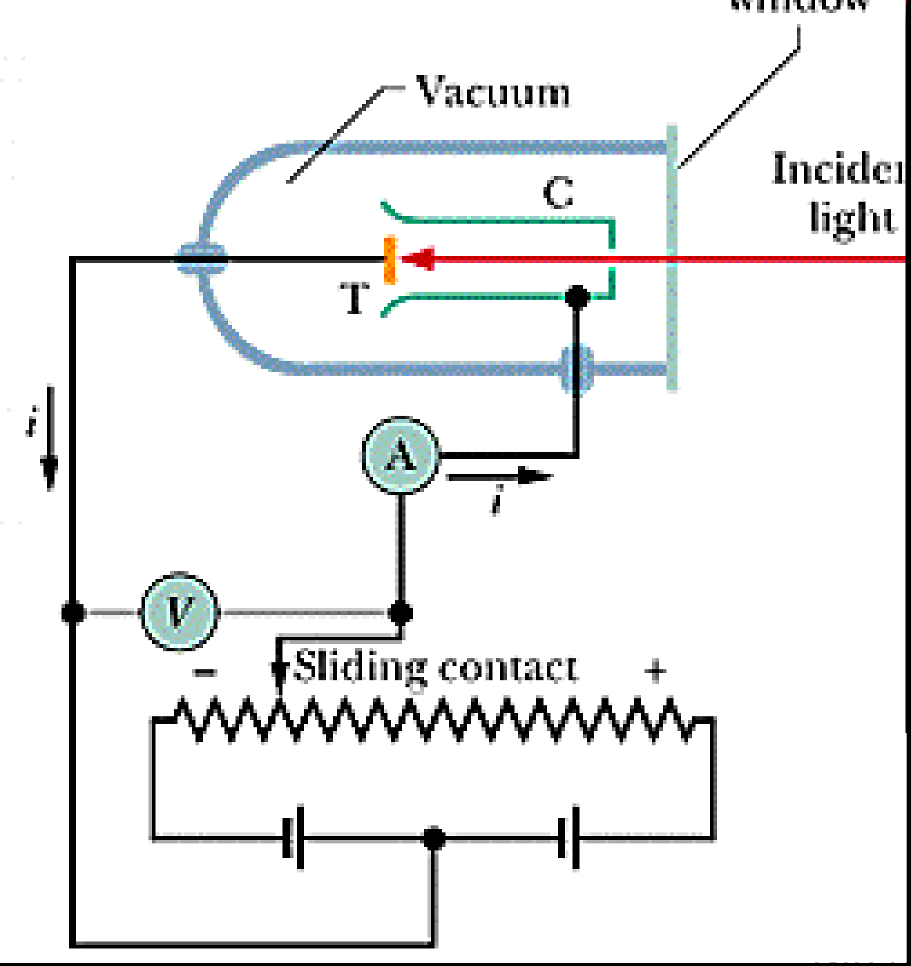
$$c = \lambda f$$

$$E = hf \quad (\text{photon energy})$$

$$h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$$

The Photoelectric Effect

光電效應



The experiment

- First Experiment (adjusting V) – the stopping potential V_{stop}

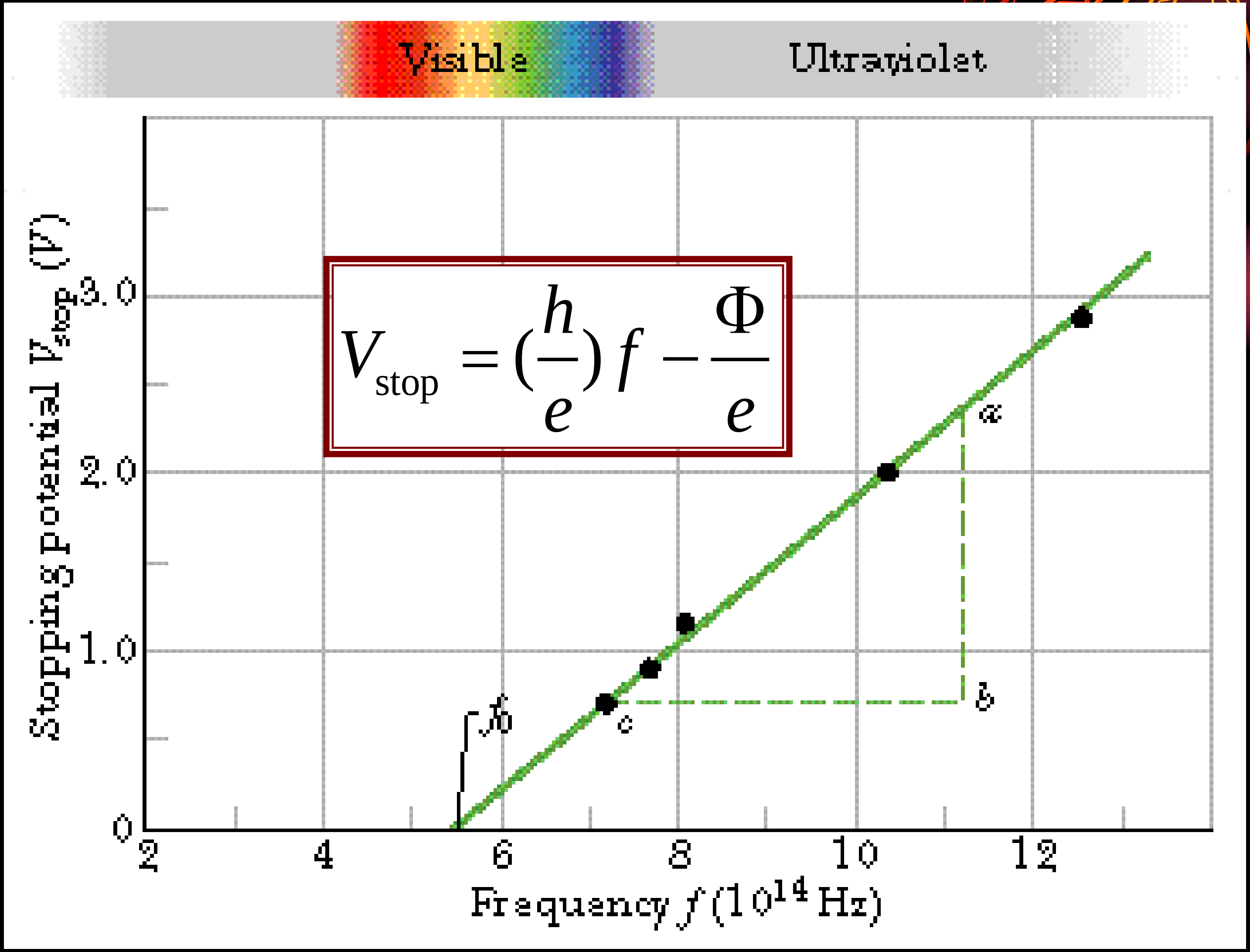
$$K_{\max} = eV_{\text{stop}}$$

光電子的最大動能與光強度無關

- Second Experiment (adjusting f) – the cutoff frequency f_o

低於截止頻率時即使光再強也不會有光電效應

The plot of V_{stop} against f



$$V_{stop} = \left(\frac{h}{e}\right) f - \frac{\Phi}{e}$$

The Photoelectric Equation

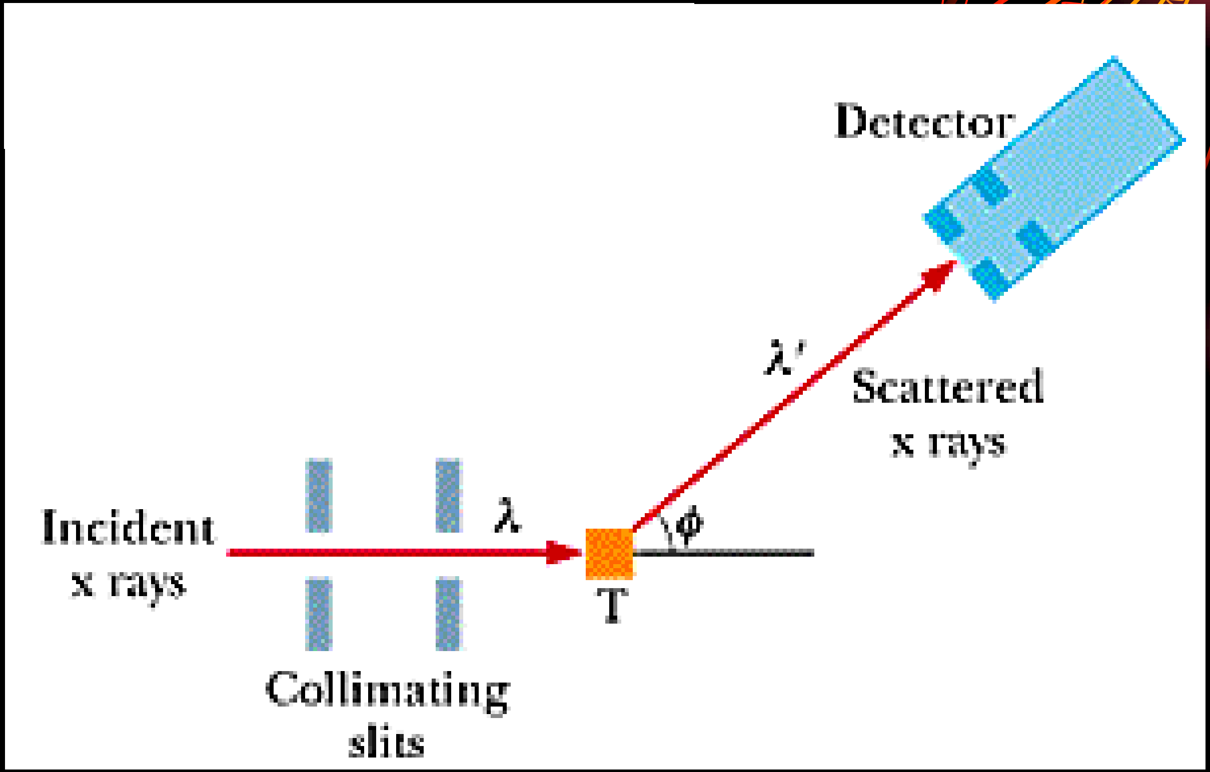
$$hf = K_{\max} + \Phi$$

$$V_{\text{stop}} = \left(\frac{h}{e}\right) f - \frac{\Phi}{e}$$

$$h = 6.6 \times 10^{-34}\ \text{J} \cdot \text{s}$$

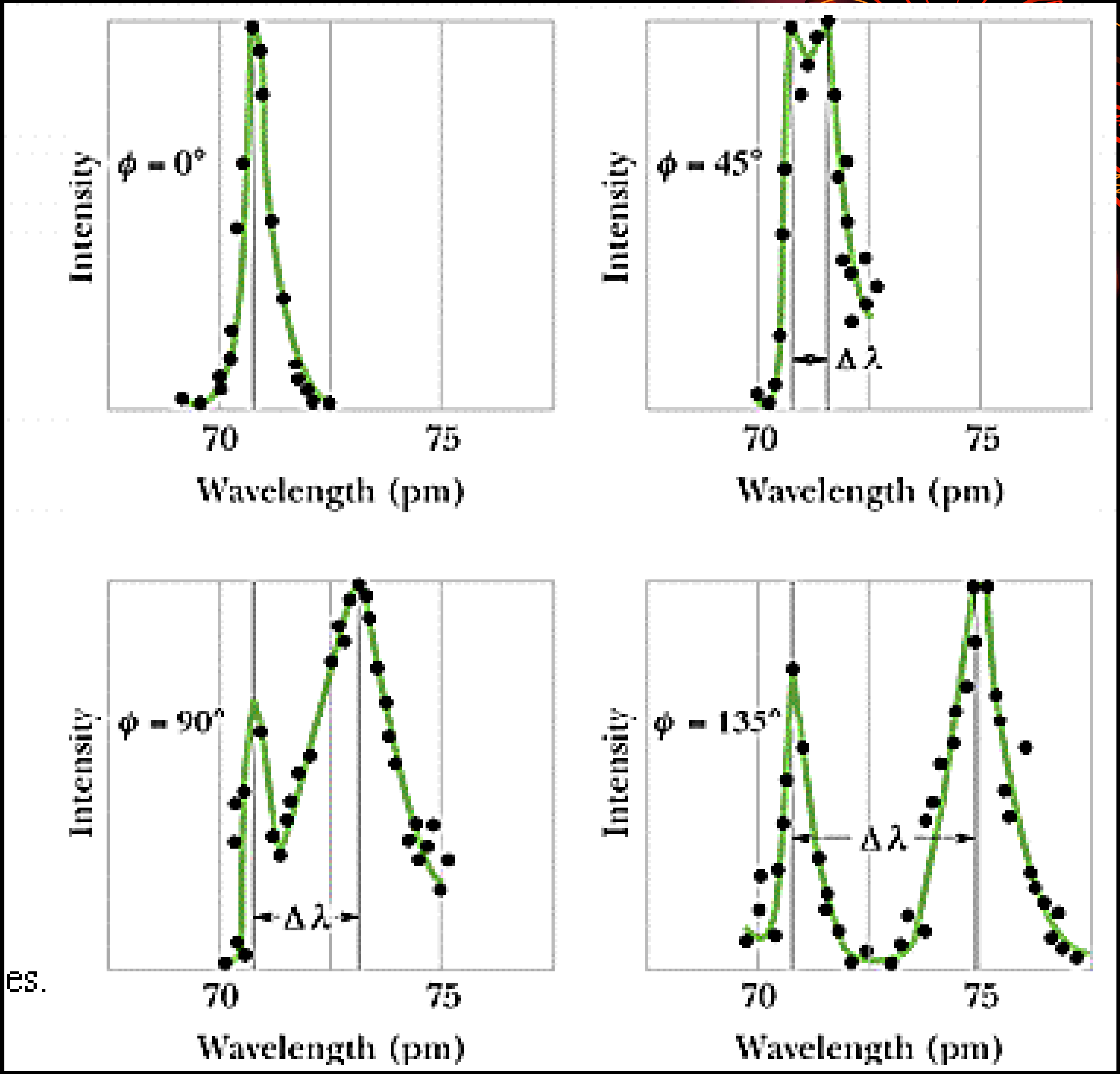
Work function

12-2 Compton Effect

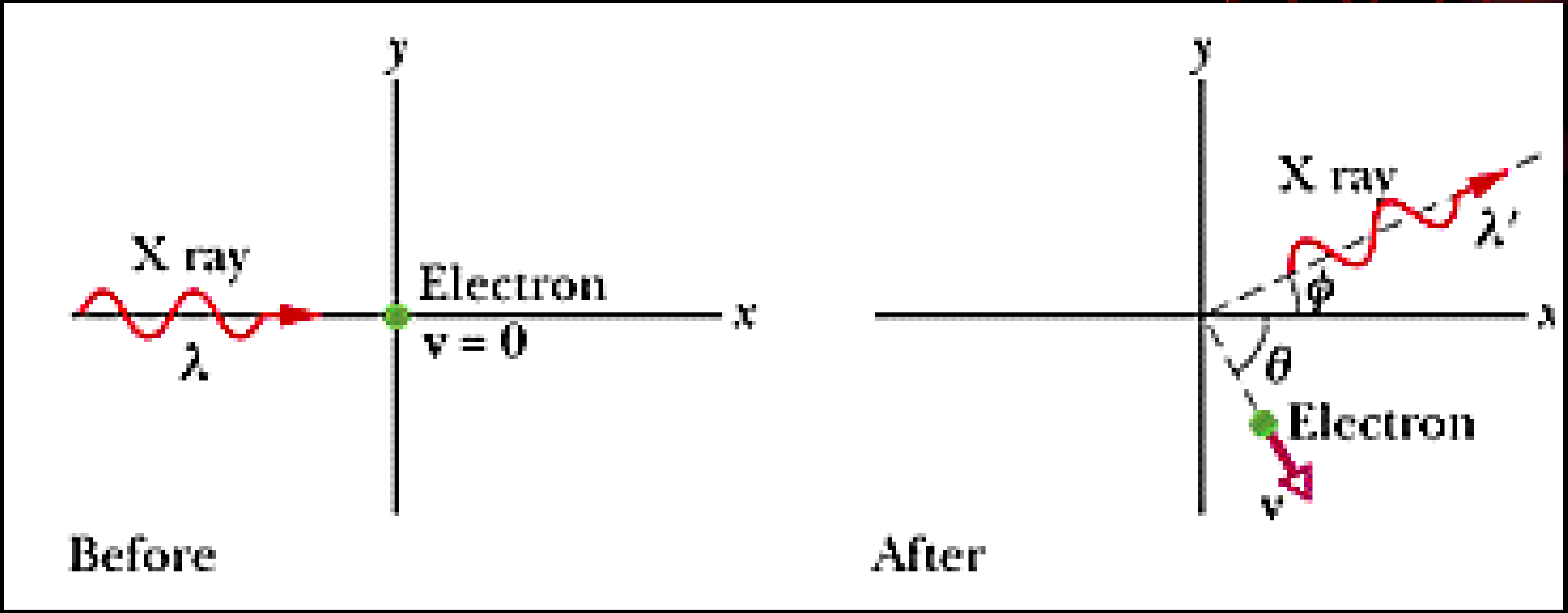


$$p = \frac{hf}{c} = \frac{h}{\lambda} \quad \text{(photon momentum)}$$

康普吞效應實驗圖表



康普吞效應圖示



Energy and momentum conservation

$$hf = hf' + K \qquad K = mc^2(\gamma - 1)$$

$$hf = hf' + mc^2(\gamma - 1)$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'} + mc(\gamma - 1)$$

$$p_X = h/\lambda \qquad p_e = \lambda mv$$

Frequency shift

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + \gamma m v \cos\theta$$

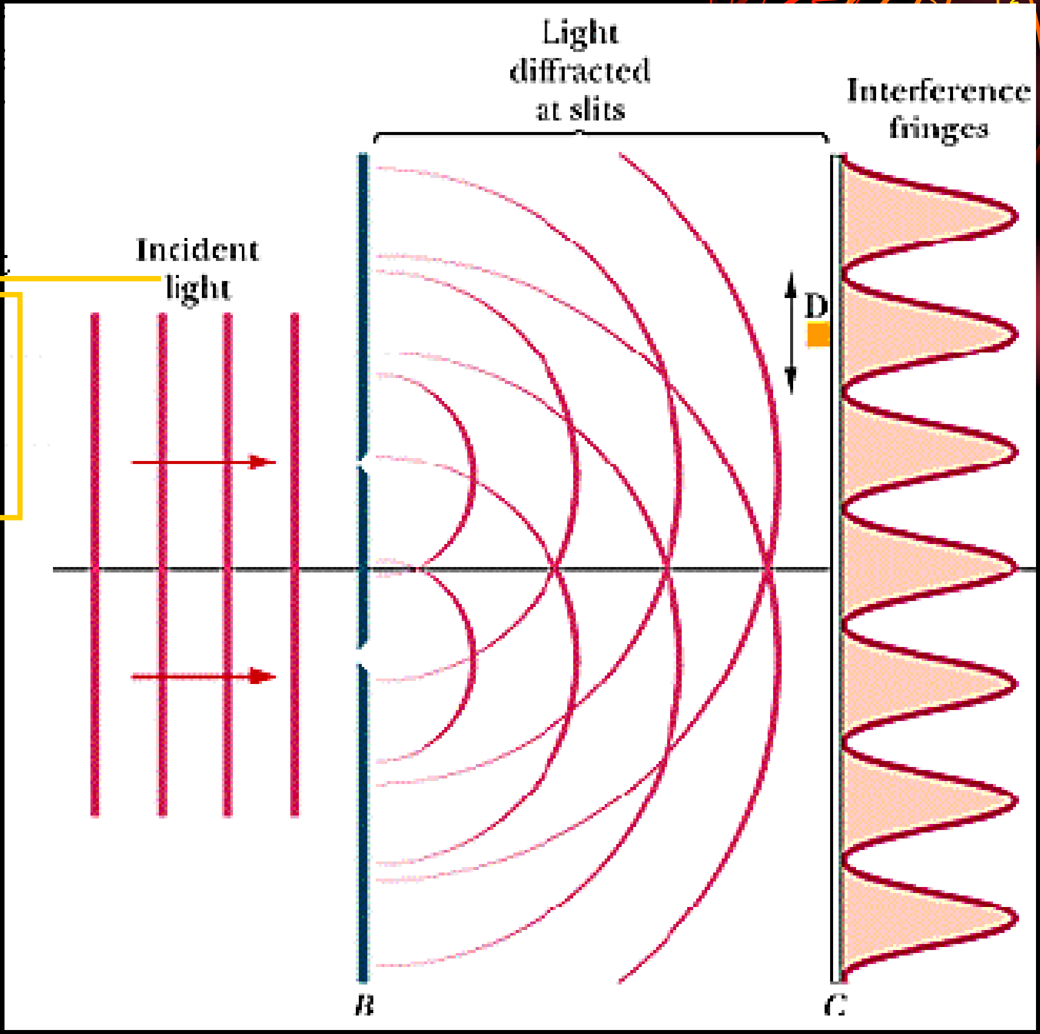
$$0 = \frac{h}{\lambda'}\sin\phi - \gamma m v \sin\theta$$

$$\Delta\lambda = \frac{h}{mc}(1-\cos\phi)$$

Compton wavelength

12-3 Light as a Probability Wave

The standard version

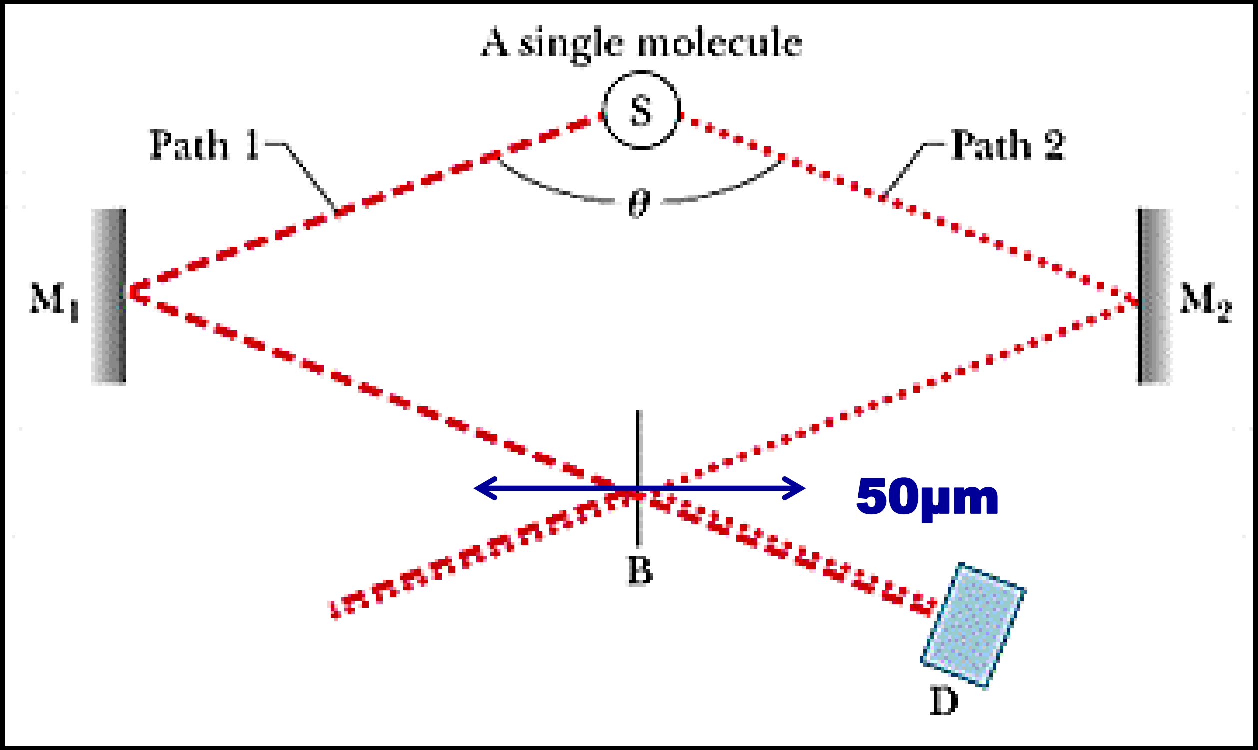


The Single-Photon Version

First by Taylor in 1909

<u>The single-photon, double-slit experiment is a phenomenon which is impossible, absolutely impossible to explain in any classical way, and which has in it the heart of quantum mechanics</u> - Richard Feynman

The Single-Photon, Wide-Angle Version (1992)



The postulate

- Light is generated in the source as photons
- Light is absorbed in the detector as photons
- Light travels between source and detector as a probability wave

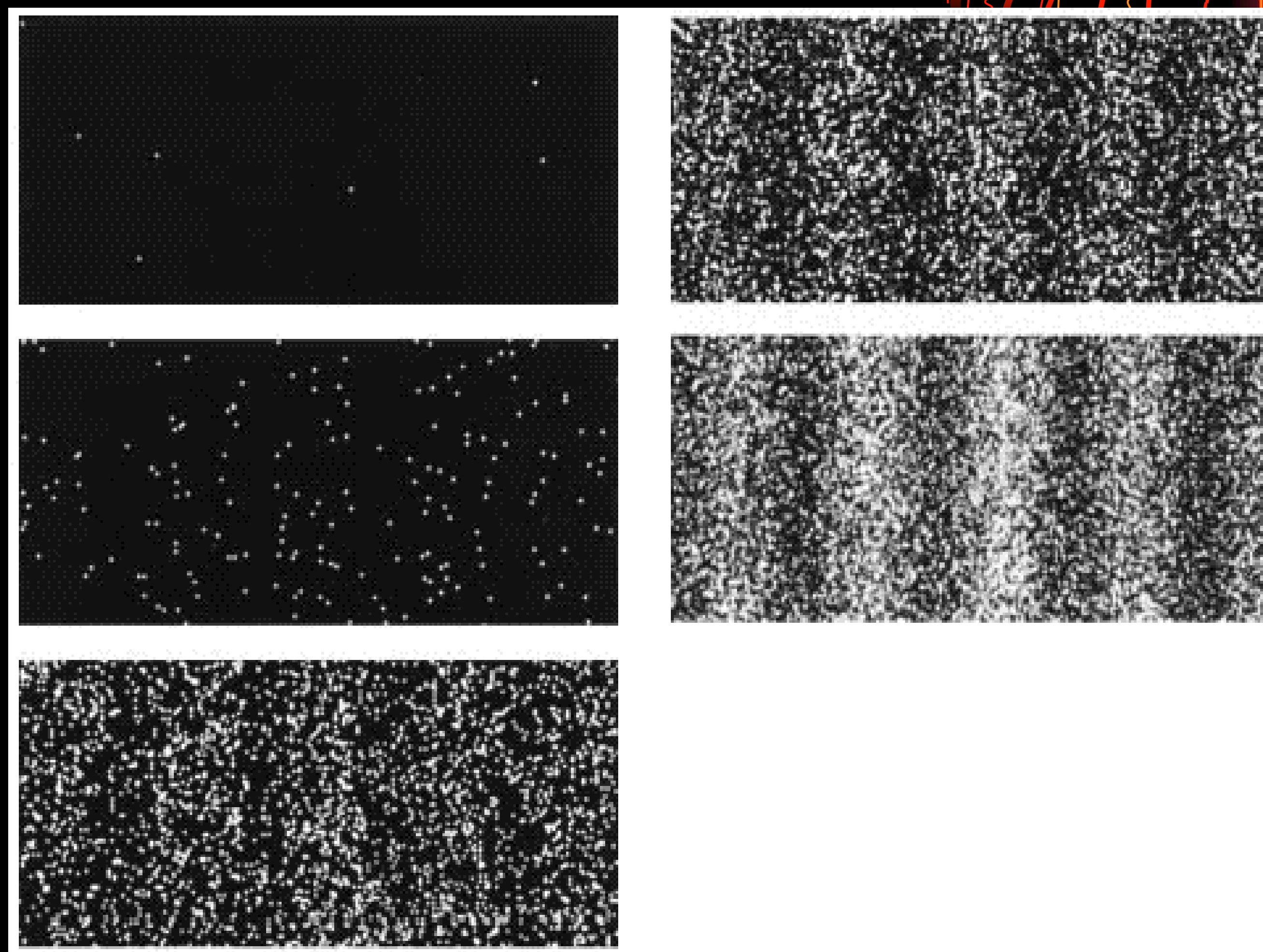
12-4 Electrons and Matter Waves

$$\lambda = \frac{h}{p}$$

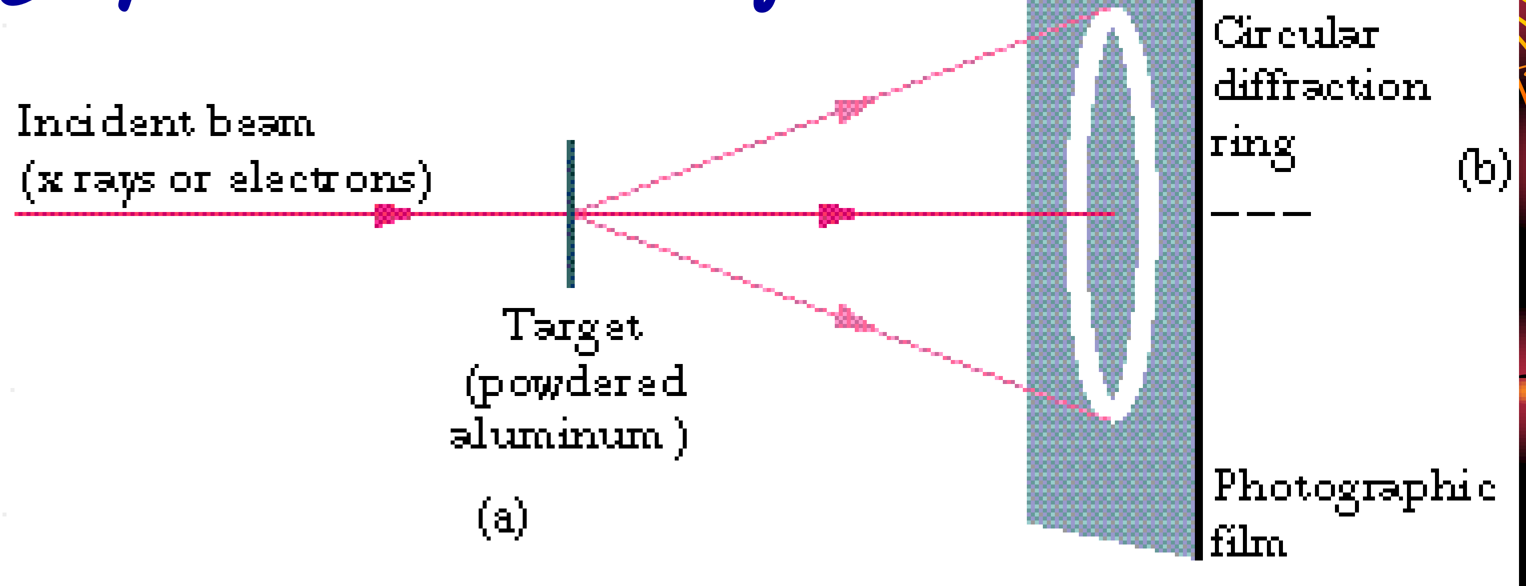
- The de Broglie wave length
- Experimental verification in 1927
- Iodine molecule beam in 1994

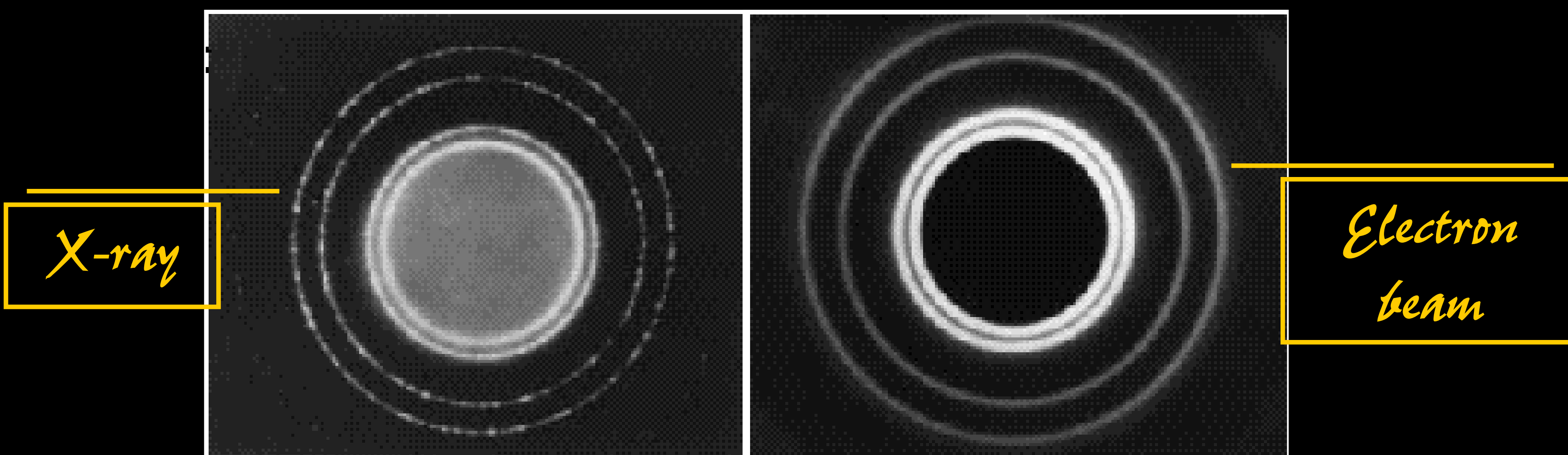
1989 double-slit experiment

7,100,3000, 20,000 and 70,000 electrons

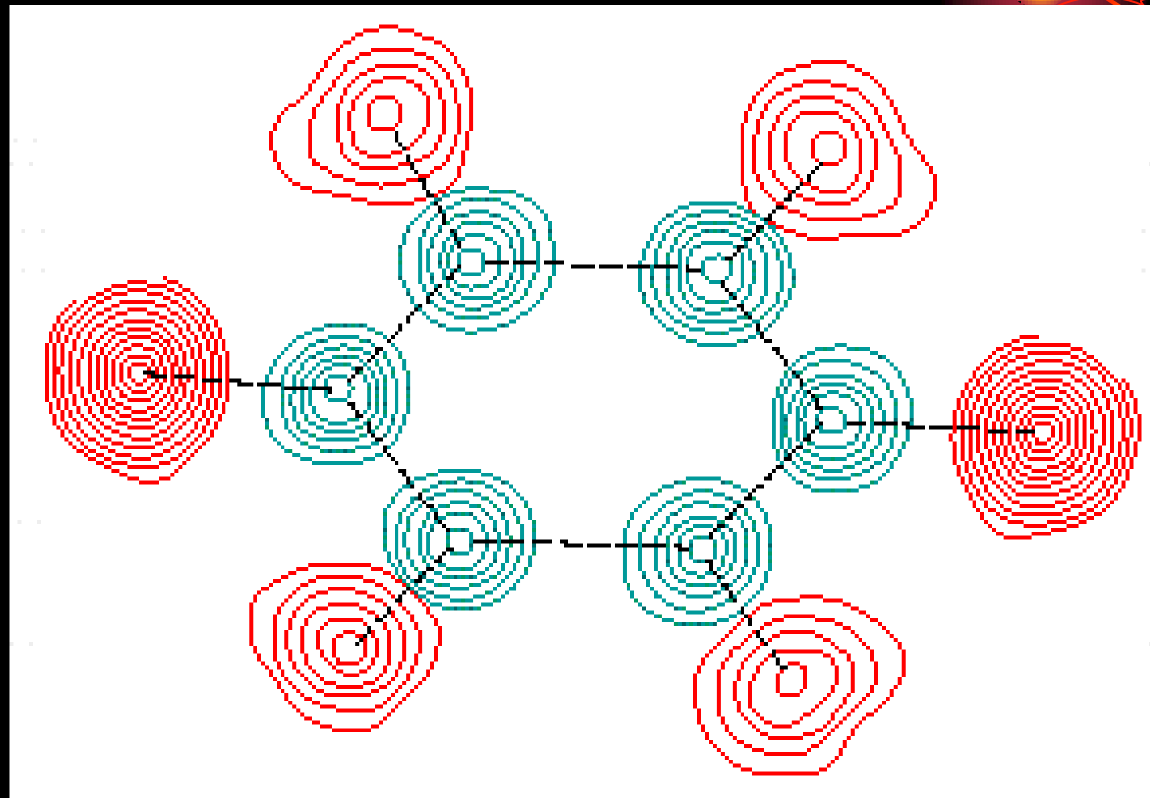


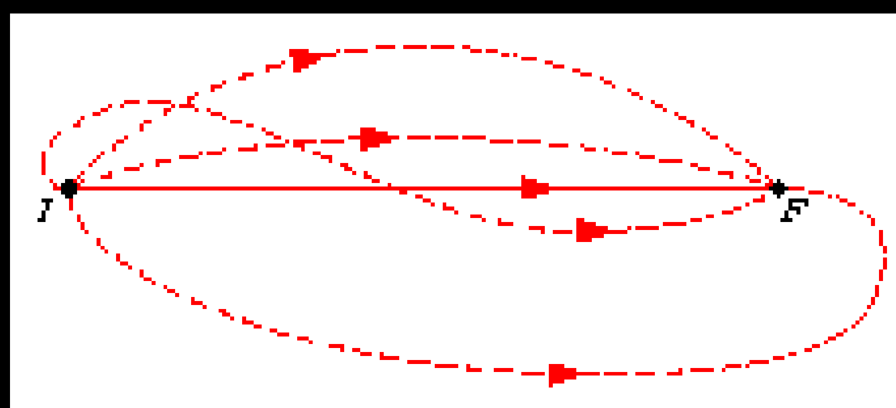
Experimental Verifications





苯環的中子繞射





12-5 Schrodinger's Equation

- Matter waves and the wave function

$$\Psi(x,y,z,t)=\psi(x,y,z)e^{-i\omega t}$$

- The probability (per unit time) is ∝

$$|\psi|^2 \quad \text{ie. } \psi^*\psi$$

Complex conjugate
共軛複數

The Schrodinger Equation from A Simple Wave Function

$$\Psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$$

$$\psi = A\sin(kx) + B\cos(kx) \quad (1D)$$

$$p = h/\lambda = \hbar k$$

$$E = p^2/2m = \hbar^2 k^2/2m$$

1D Time-independent SE

$$\psi = A\sin(kx) + B\cos(kx)$$

$$d^2\psi / dx^2 = -k^2\psi \quad k^2 = -\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

$$E = -\frac{1}{\psi}\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

3D Time-dependent SE

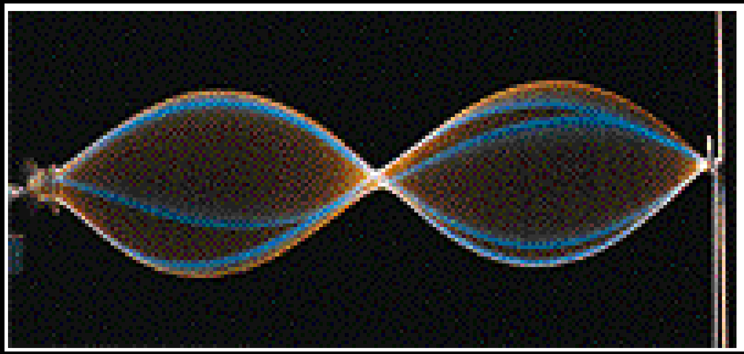
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

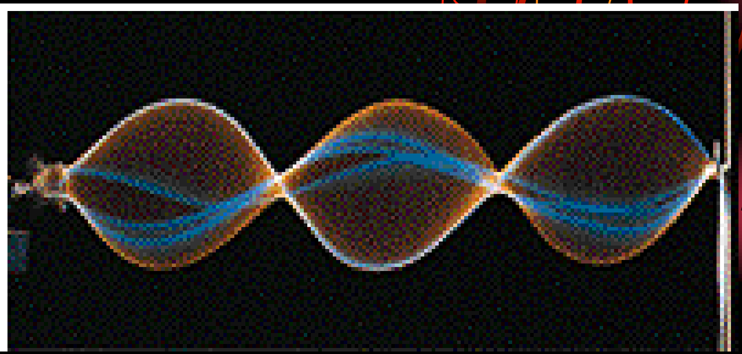
$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = E\psi$$

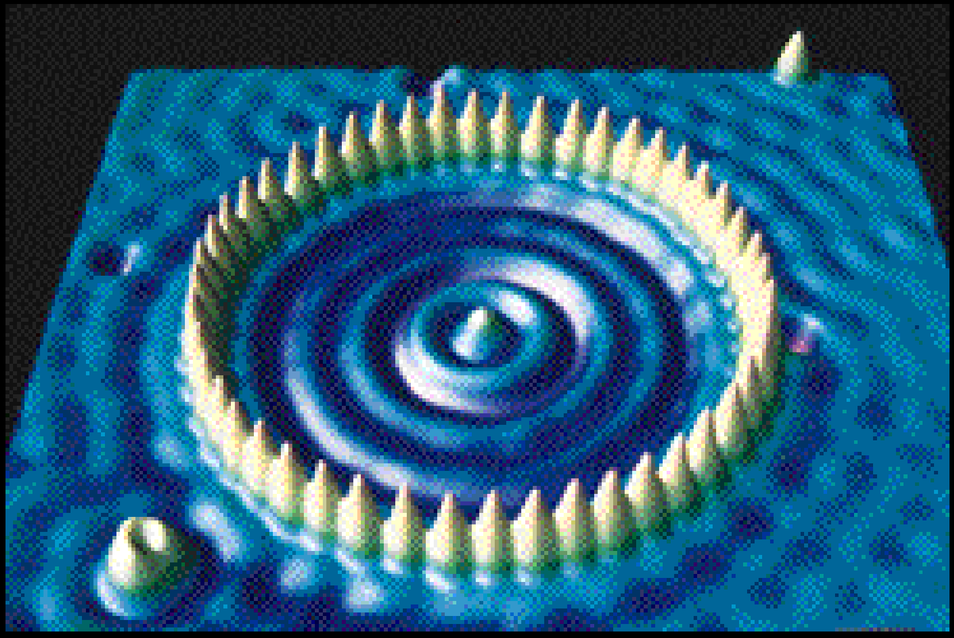
$$-\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial}{\partial t}\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial}{\partial t}\psi$$

12-6 Waves on Strings and Matter Waves







駐波與量子化 Quantization

駐波：

$$\lambda = \frac{2L}{n} \quad f = \frac{v}{\lambda} = n\frac{v}{2L} \quad n = 0,1,2,\cdots$$

Confinement of a Wave leads to Quantization – discrete states and discrete energies

12-7 Trapping an Electron

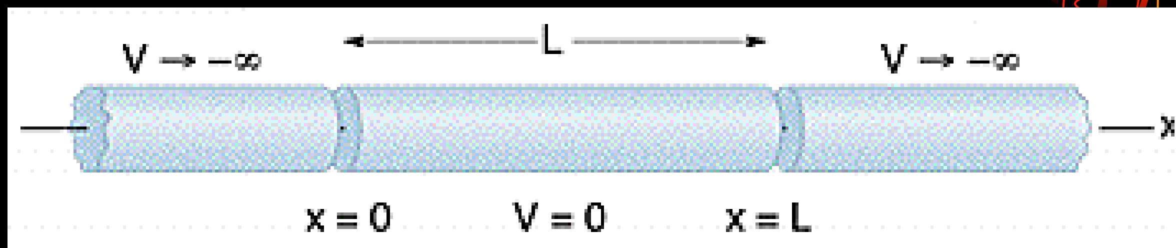
For a string：

$$L = \frac{n\lambda}{2} \quad n = 1,2,3,\cdots$$

$$y_n = A\sin(\frac{n\pi}{L})x, \quad n = 1,2,3,\cdots$$

n : quantum number

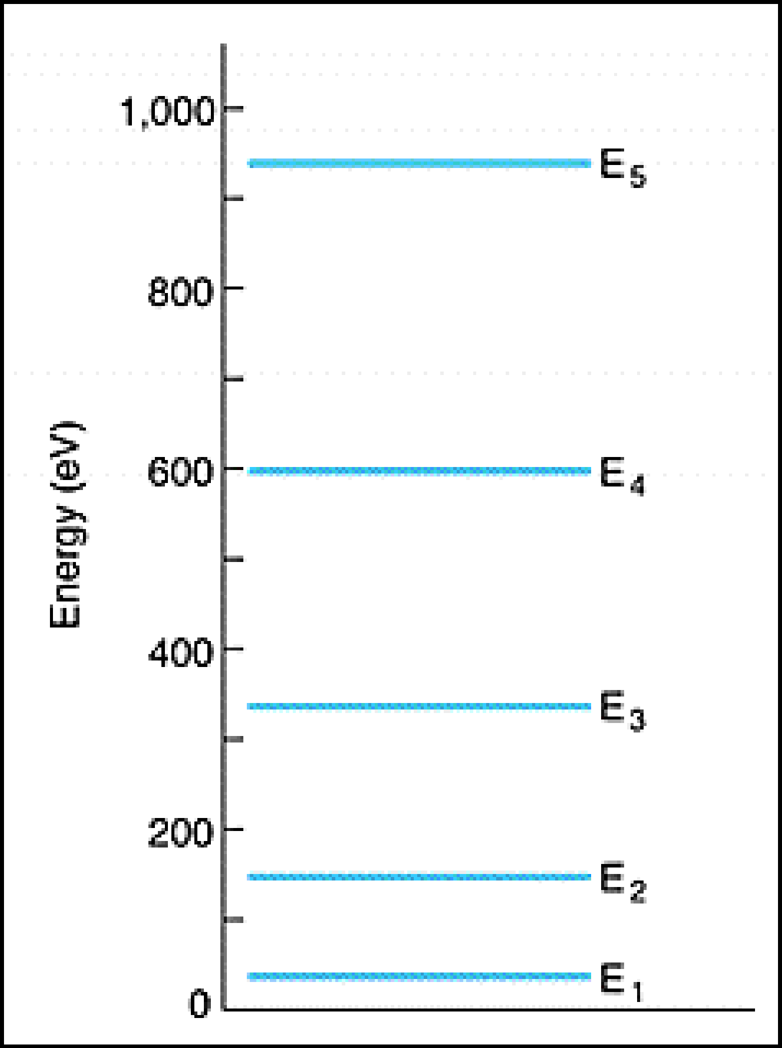
Finding the Quantized Energies of an infinitely deep potential energy well



$$\lambda = h / p = h / \sqrt{2mE},\ L = n\lambda / 2$$

$$E_n = n^2 h^2 / 8mL^2, \quad n = 1, 2, 3, \cdots$$

The Energy Levels 能階



- The ground state and excited states
- The Zero-Point Energy

n can't be 0

The Wave Function and Probability Density

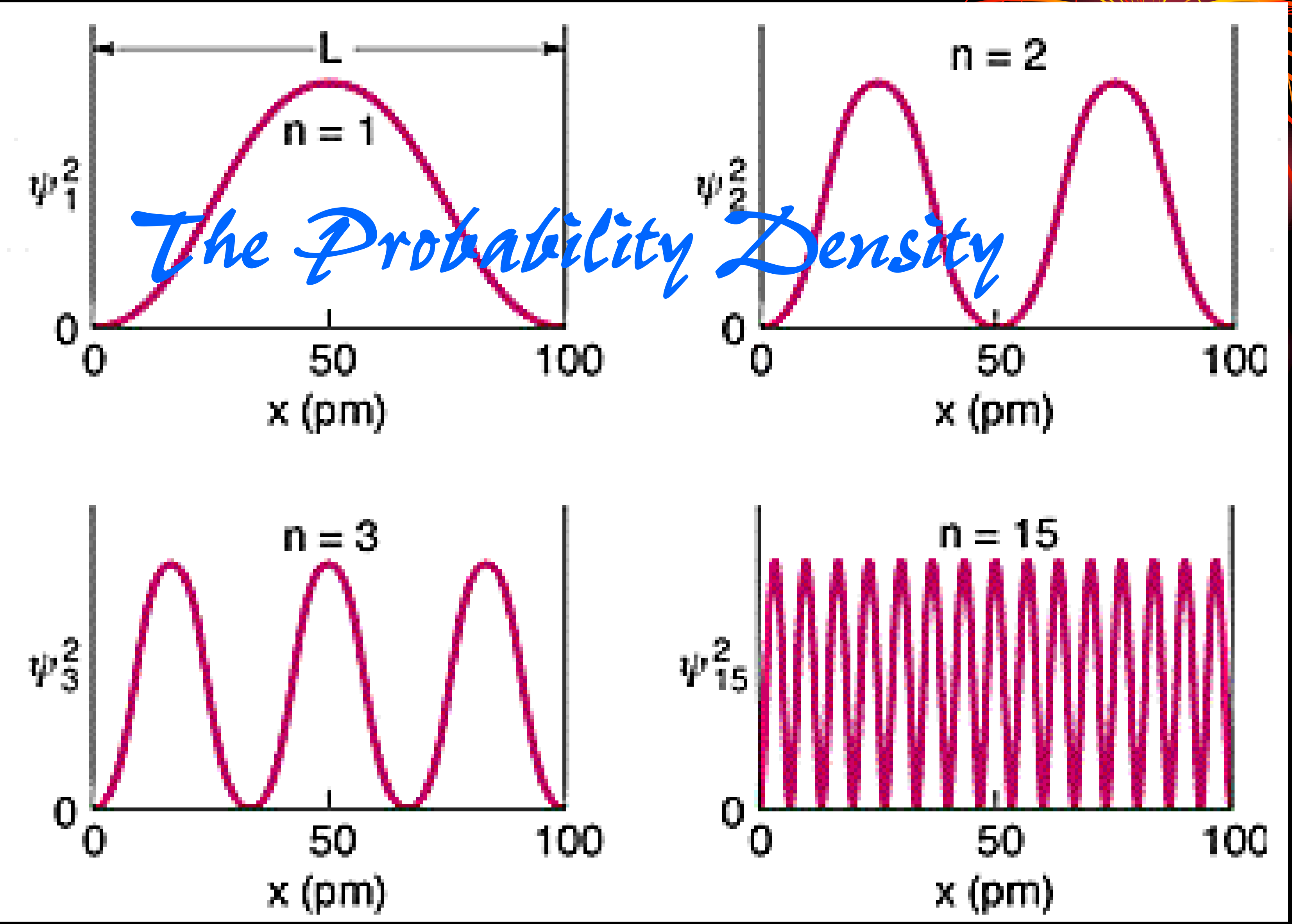
For a string

$$y_n = A\sin(\frac{n\pi}{L})x, \qquad n = 1,2,3,\cdots$$

$$\psi_n = A\sin(\frac{n\pi}{L})x, \qquad n = 1,2,3,\cdots$$

$$\psi_n^2 = A^2\sin^2(\frac{n\pi}{L})x, \quad n = 1,2,3,\cdots$$

The Probability Density



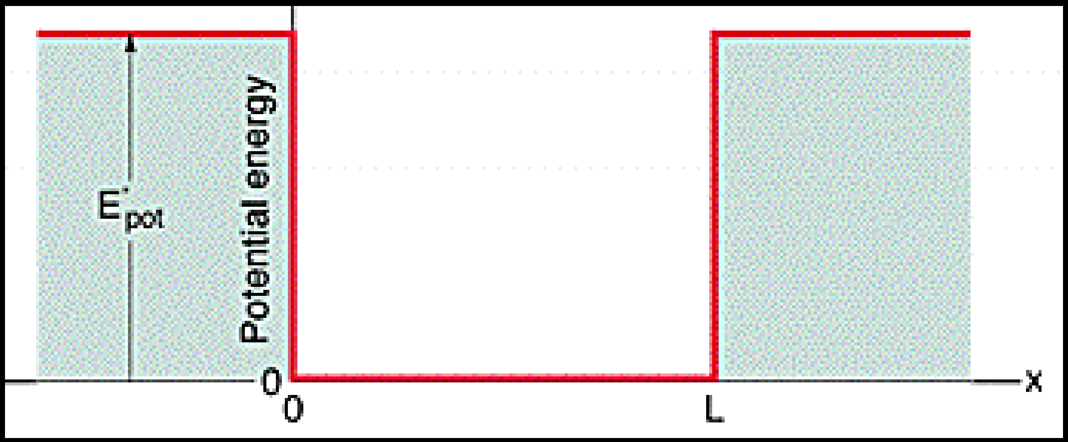
Correspondence principle (對應原理)

At large enough quantum numbers, the predictions of quantum mechanics merge smoothly with those of classical physics

- Normalization (歸一化)

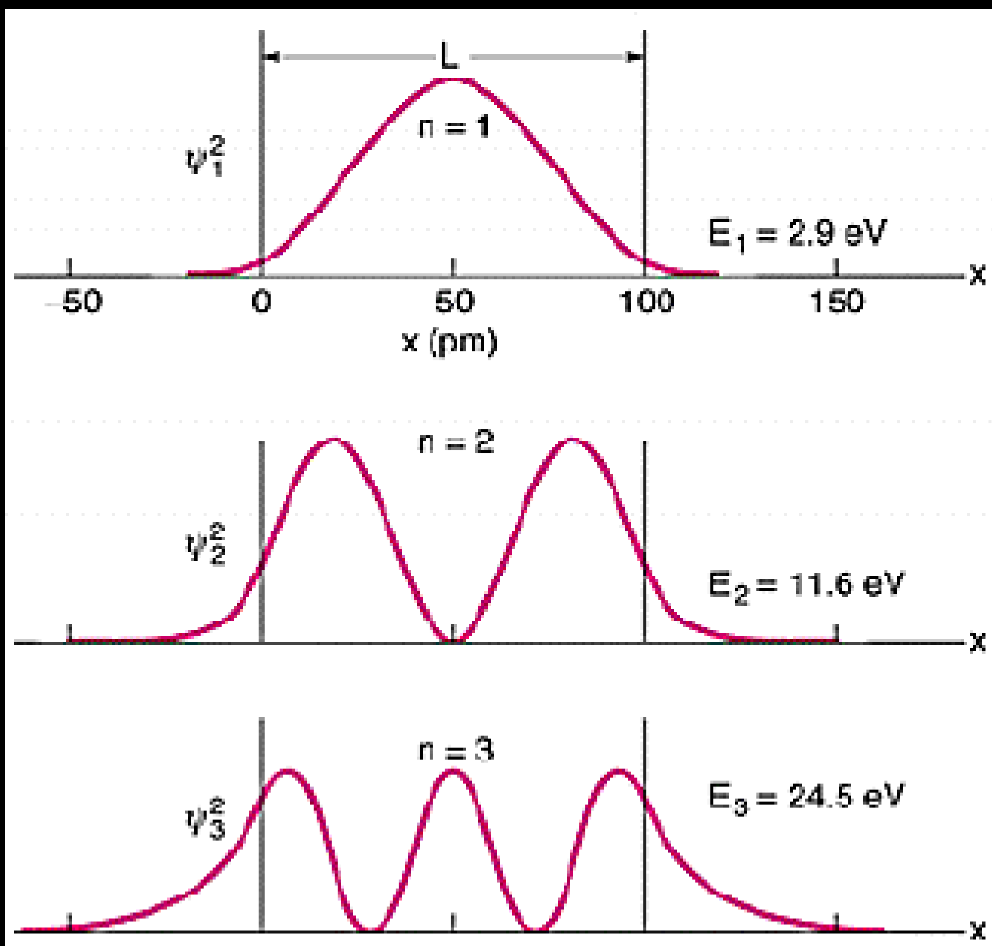
$$\int_{-\infty}^{\infty} \psi_n^2(x)\,dx = 1 \rightarrow A = \sqrt{2/L}$$

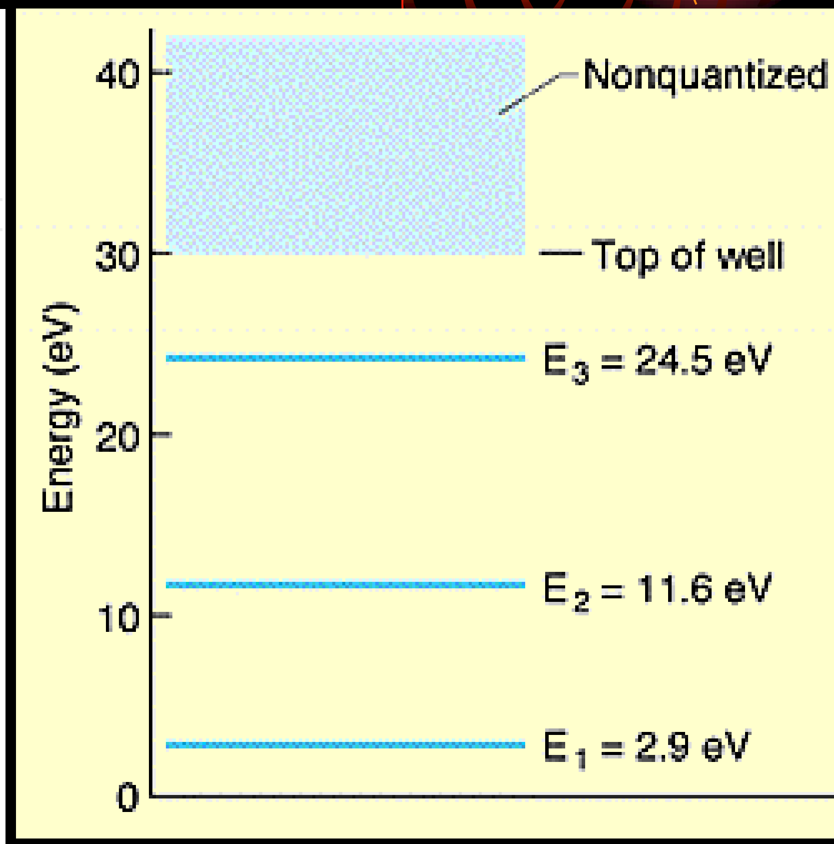
A Finite Well 有限位能井



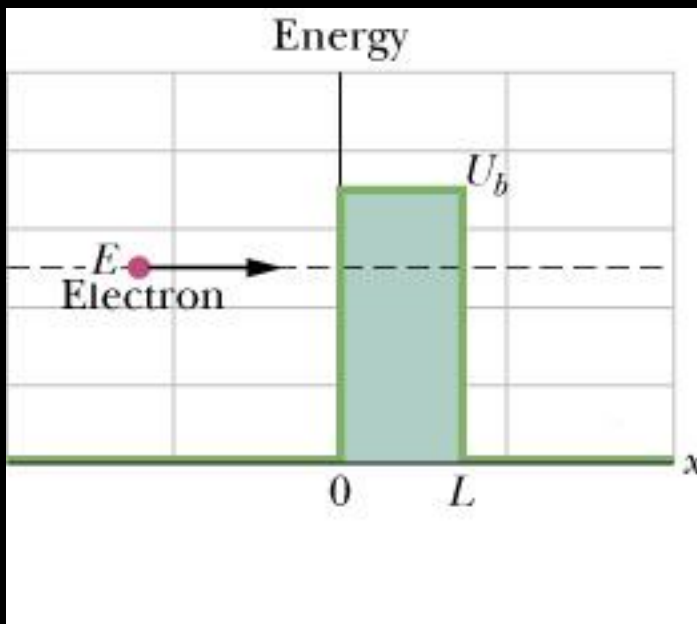
$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}[E-E_{pot}(x)]\psi=0$$

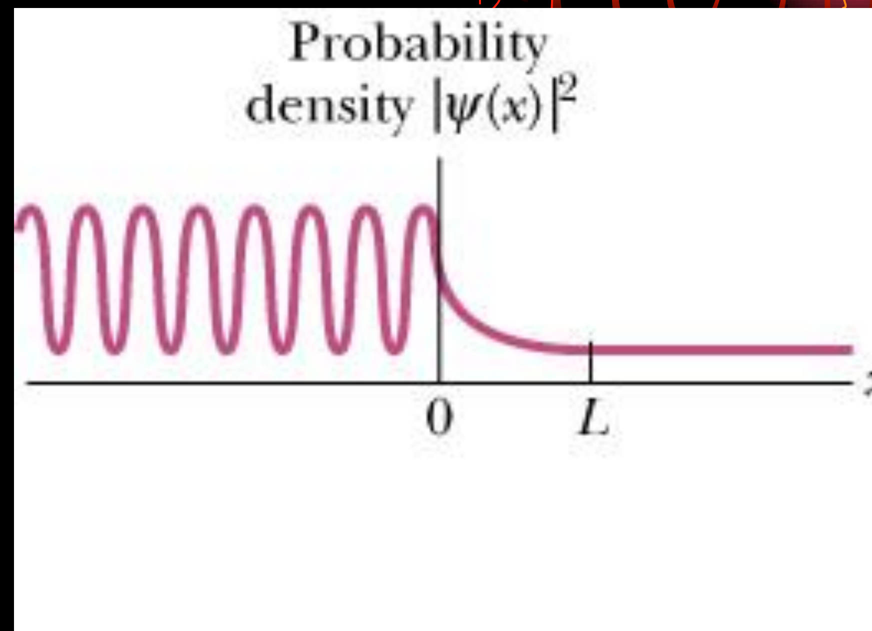
The probability densities and energy levels





Barrier Tunneling
穿隧效應

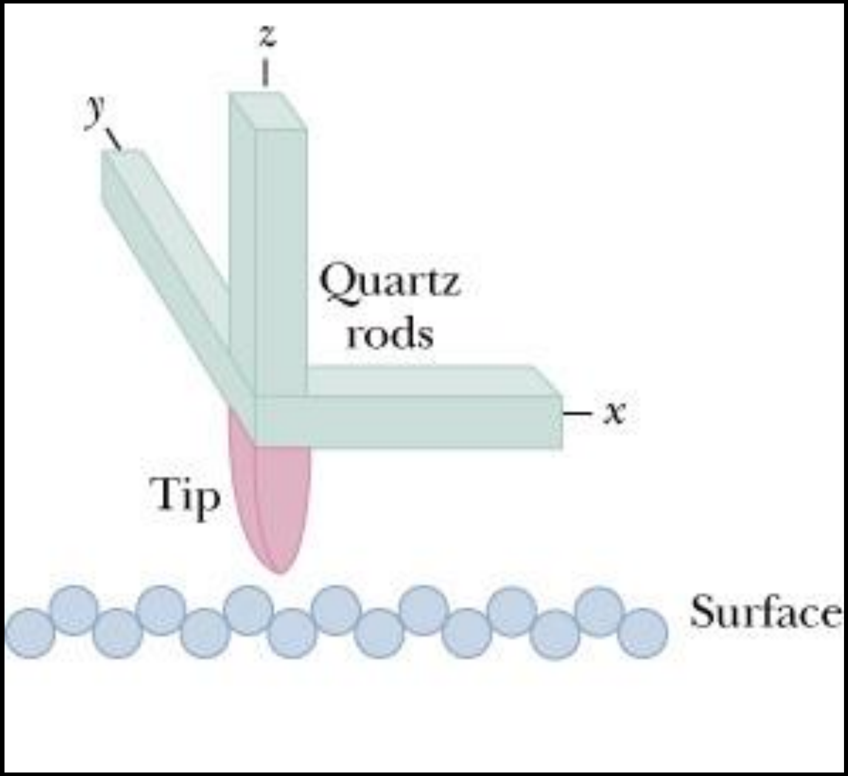


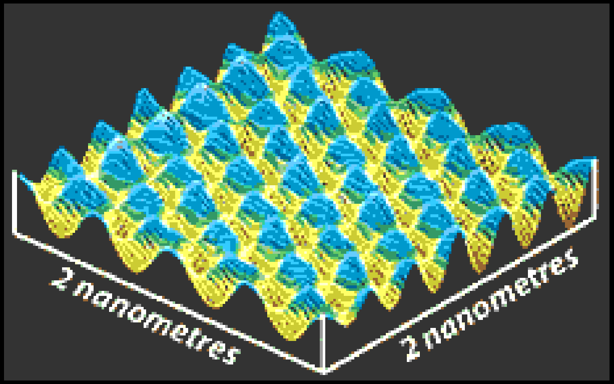


- Transmission coefficient

$$T = e^{-2bL} \qquad k = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}$$

STM 掃描式穿隧顯微鏡

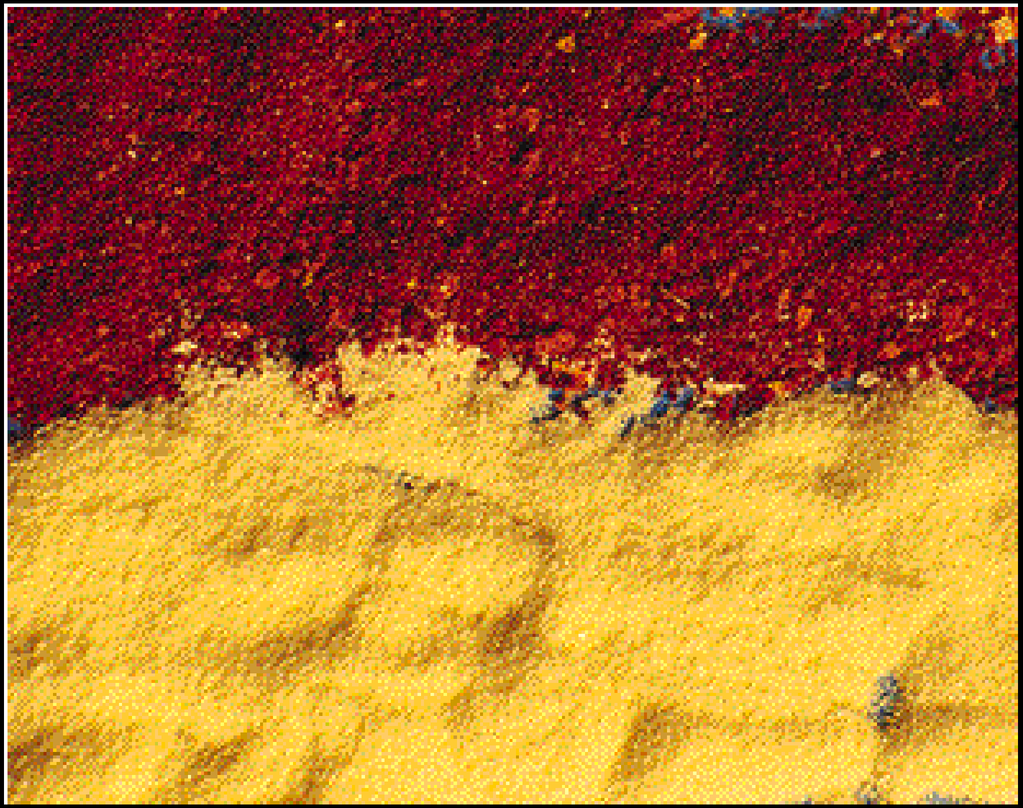




Piezoelectricity of quartz

12-8 Three Electron Traps

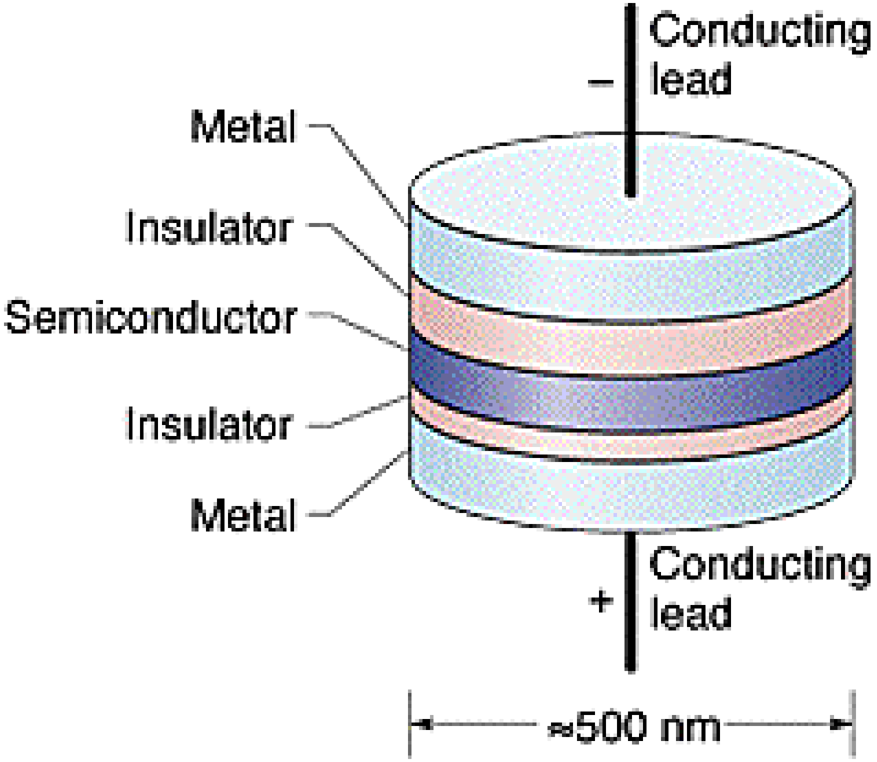
- Nanocrystallites 硒化鎘奈米晶粒 那種顏色的顆粒比較小

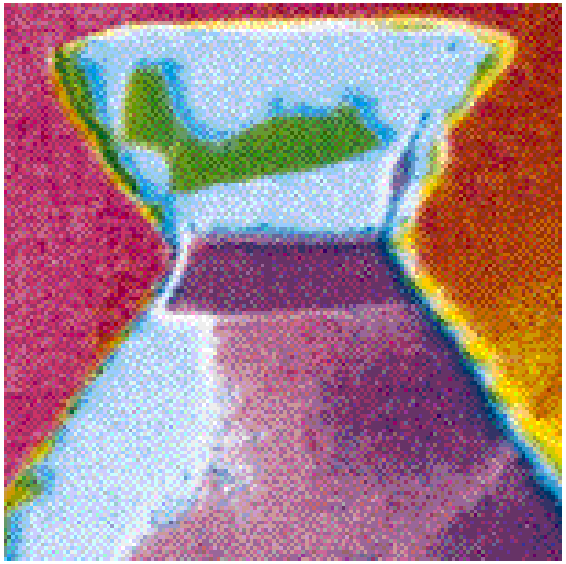


$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$\lambda_t = \frac{c}{f_t} = \frac{ch}{E_t}$$

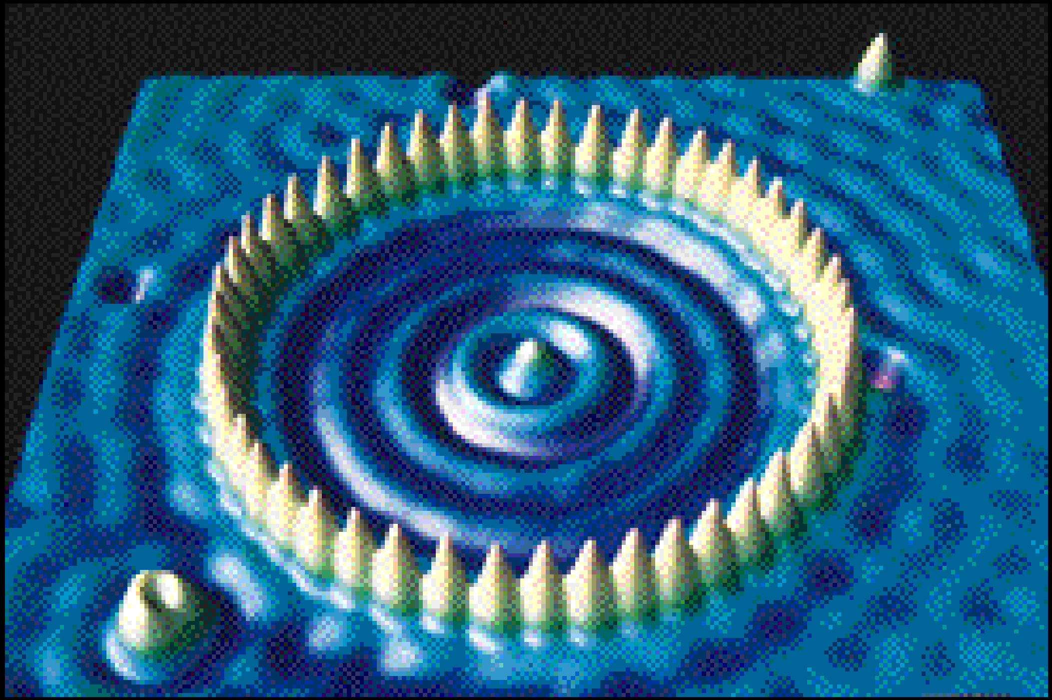
A Quantum Dot
An Artificial Atom





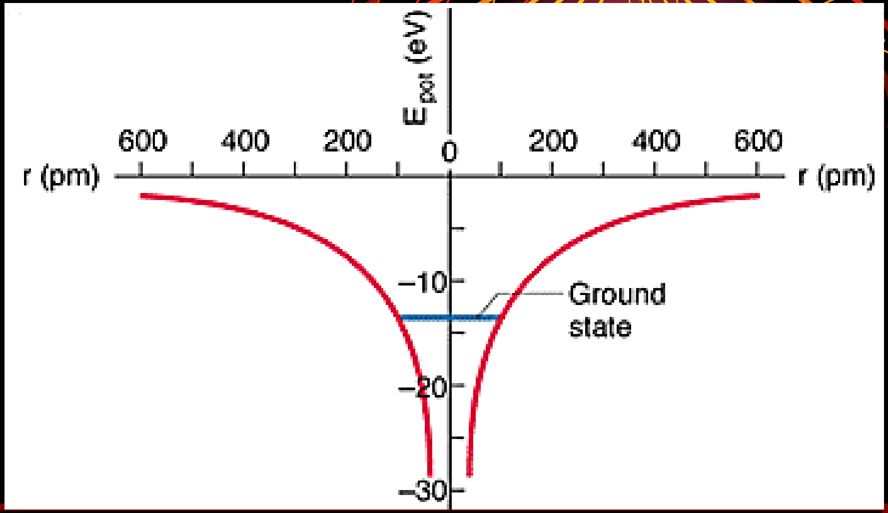
The number of electrons can be controlled

Quantum Corral
量子圍欄



12-1.9 The Hydrogen Atom

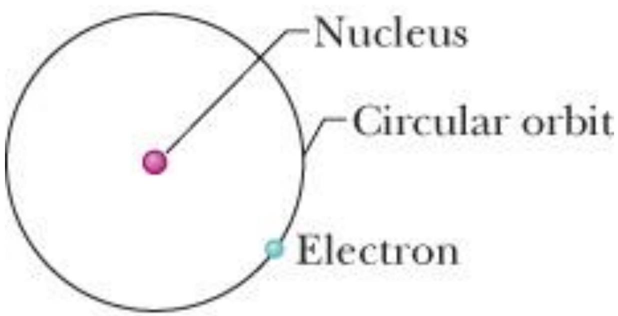
- The Energies



$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \rightarrow -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{13.6\text{ev}}{n^2}, \qquad n = 1,2,3,\cdots$$

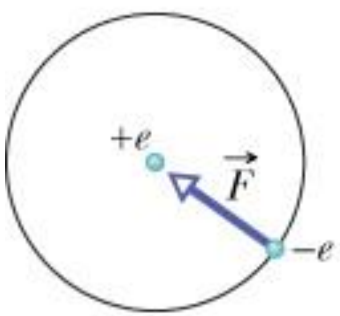
The Bohr Model of the Hydrogen Atom



(a)

Balmer's empirical (based only on observation) formula on absorption/emission of visible light for H

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad \text{for } n = 3, 4, 5, \text{ and } 6$$



(b)

Fig. 39-16

Bohr's assumptions to explain Balmer formula

1) Electron orbits nucleus

2) The magnitude of the electron's angular momentum L is quantized

$$L = n\hbar, \quad \text{for } n = 1, 2, 3, \ldots$$

Orbital Radius is Quantized in the Bohr Model

Coulomb force attracting electron toward nucleus $F = k\frac{|q_1||q_2|}{r^2}$

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = ma = m\left(-\frac{v^2}{r}\right)$$

Quantize angular momentum $\boldsymbol{\ell}$: $\ell = rmv\sin\phi = rmv = n\hbar \rightarrow v = \frac{n\hbar}{rm}$

Substitute v into force equation:

$$r = \frac{h^2\varepsilon_0}{\pi me^2}n^2, \quad \text{for } \; n = 1, 2, 3, \ldots \quad \Rightarrow \quad r = an^2, \quad \text{for } \; n = 1, 2, 3, \ldots$$

Where the smallest possible orbital radius (n=1) is called the Bohr radius a:

$$a = \frac{h^2\varepsilon_0}{\pi me^2} = 5.291772 \times 10^{-10} \text{ m} \approx 52.92 \text{ pm}$$

Orbital Energy is Quantized

The total mechanical energy of the electron in H is:

$$E = K + U = \tfrac{1}{2}mv^2 + \left(-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)$$

Solving the $F=ma$ equation for mv^2 and substituting into the energy equation above:

$$E = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$$

Substituting the quantized form for r: $$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} \text{ for } n = 1, 2, 3, \ldots$$

$$E_n = -\frac{2.180\times 10^{-18}\text{ J}}{n^2} = \frac{13.60\text{ eV}}{n^2}, \text{ for } n = 1, 2, 3, \ldots$$

Energy Changes

$$hf = \Delta E = E_{\text{high}} - E_{\text{low}}$$

Substituting $f = c/\lambda$ and using the energies E_n allowed for H:

$$\frac{1}{\lambda} = -\frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n_{\text{high}}^2} - \frac{1}{n_{\text{low}}^2}\right)$$

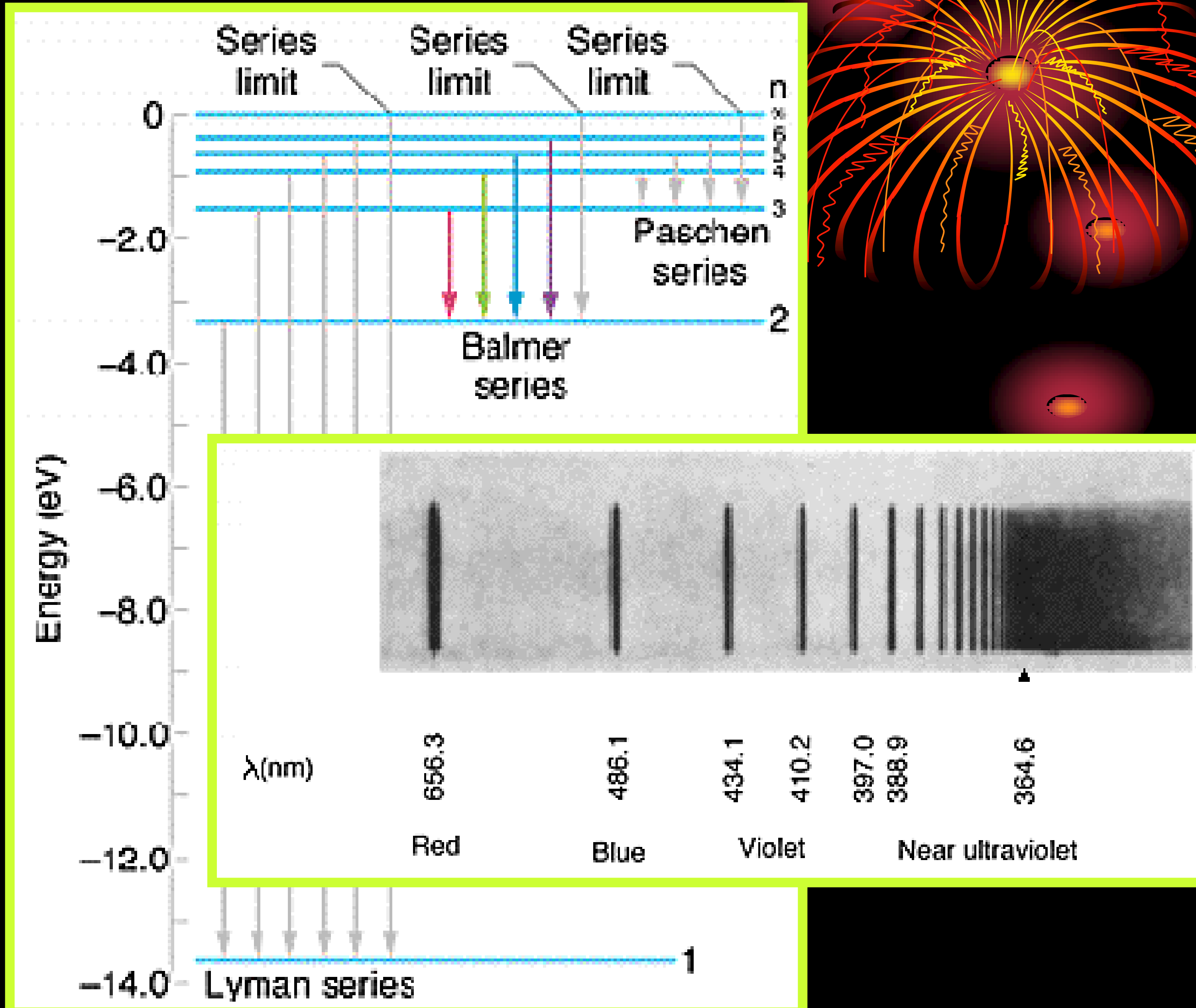
$$\frac{1}{\lambda} = R\left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right)$$

Where the Rydberg constant

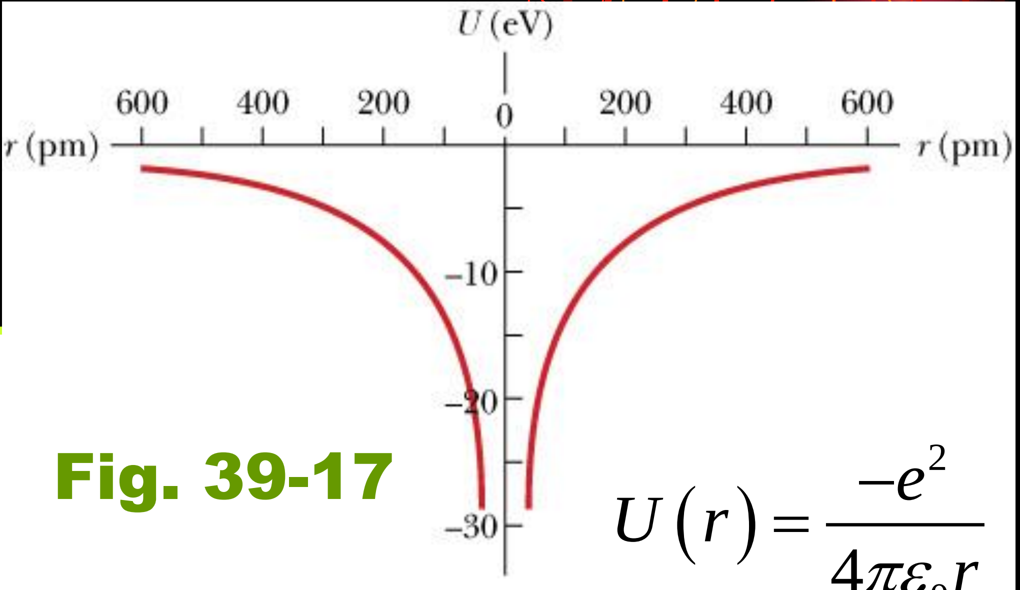
$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097373 \times 10^7 \ \text{m}^{-1}$$

This is precisely the formula Balmer used to model experimental emission and absorption measurements in hydrogen! However, the premise that the electron **orbits** the nucleus is **incorrect**! Must treat electron as matter wave.

氫原子能階與光譜線


Series limit
Series limit
Series limit
n
0
−2.0
−4.0
−6.0
−8.0
−10.0
−12.0
−14.0
Energy (eV)
3
2
1
Paschen series
Balmer series
Lyman series
λ(nm)
656.3
486.1
434.1
410.2
397.0
388.9
364.6
Red
Blue
Violet
Near ultraviolet

Schrödinger's Equation and the Hydrogen Atom



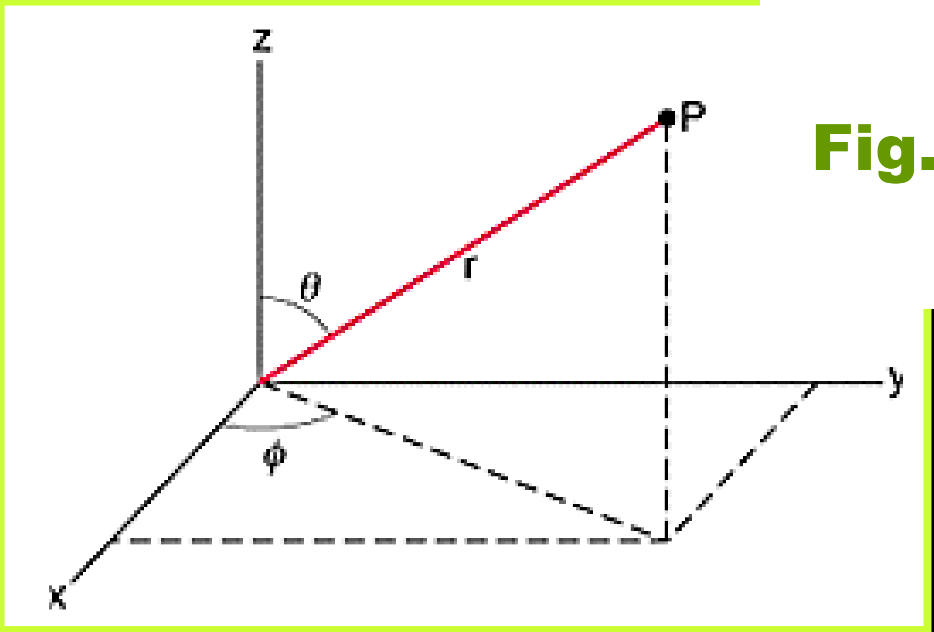


Fig. 39-17

$$U(r) = \frac{-e^2}{4\pi\varepsilon_0 r}$$

The Ground State Wave Function

$$\psi(r) = \frac{1}{\sqrt{\pi}\, a^{3/2}} e^{-r/a}$$

$$a = \frac{h^2 \varepsilon_0}{\pi m e^2} = 5.29\,\mathrm{pm}$$

(Bohr radius)

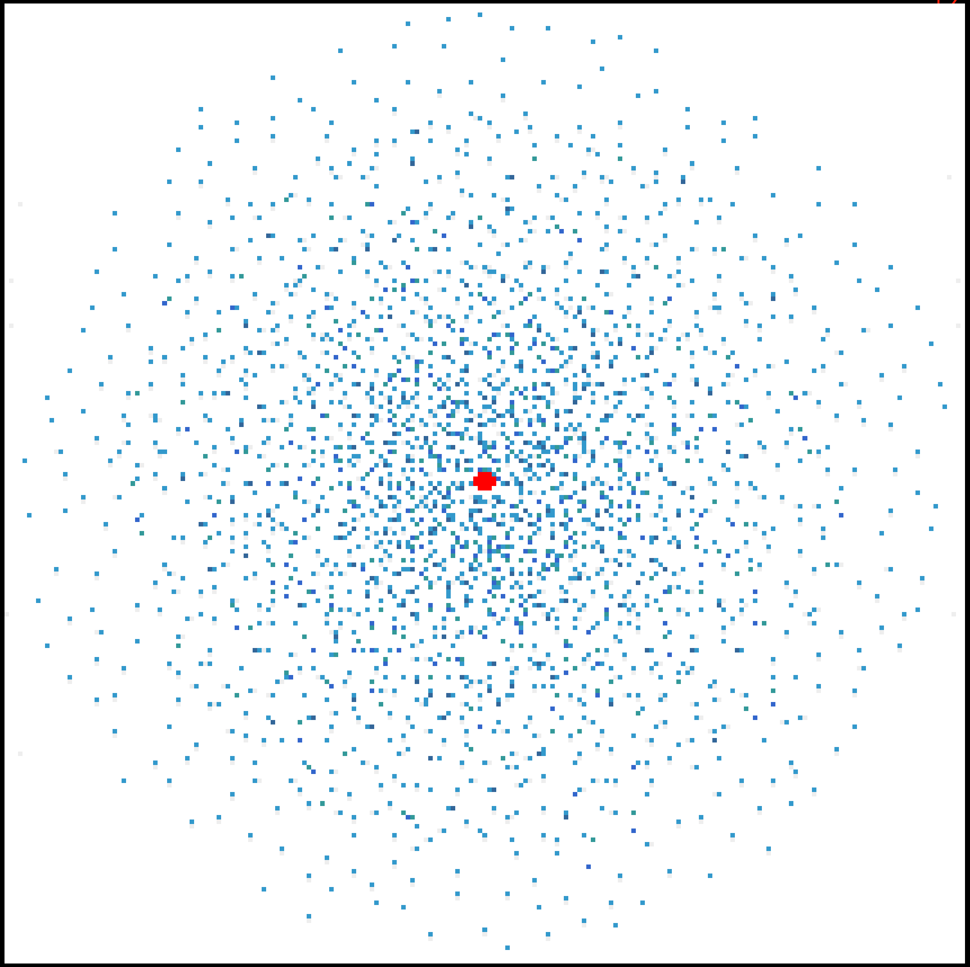
Quantum Numbers for the Hydrogen Atom

TABLE 40-1 QUANTUM NUMBERS FOR THE HYDROGEN ATOM

SYMBOL	NAME	ALLOWED VALUES
n	Principal quantum number	1, 2, 3, . . .
l	Orbital quantum number	0, 1, 2, . . ., n - 1
m_l	Orbital magnetic quantum number	- l, - (l - 1), . . ., + (l - 1), + l

For ground state, since n=1 → l=0 and m_l =0

The Ground State Dot Plot



Wave Function of the Hydrogen Atom's Ground State

Probability of finding electron within a within a small distance from a given radius

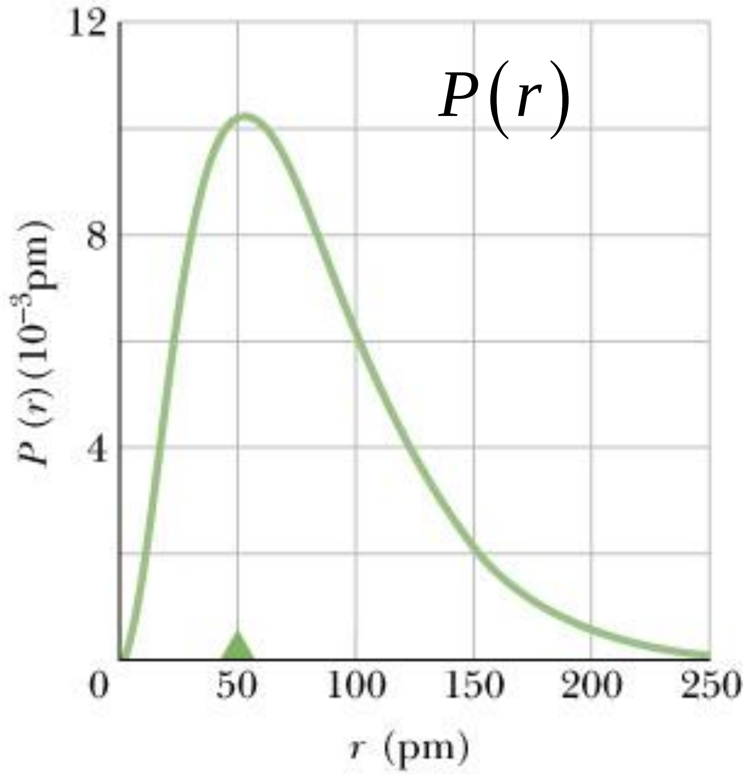


Fig. 39-20

Probability of finding electron within a small volume at a given position

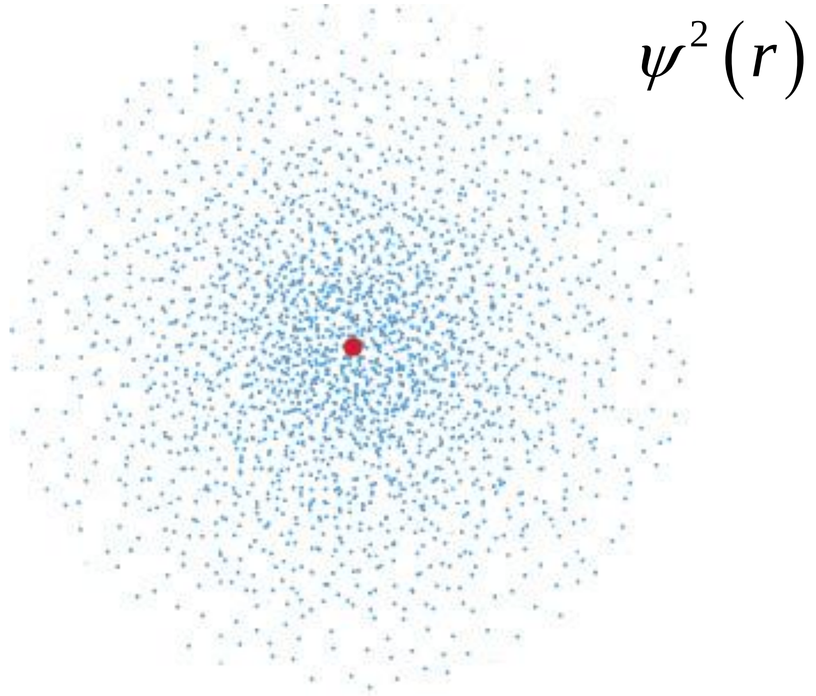


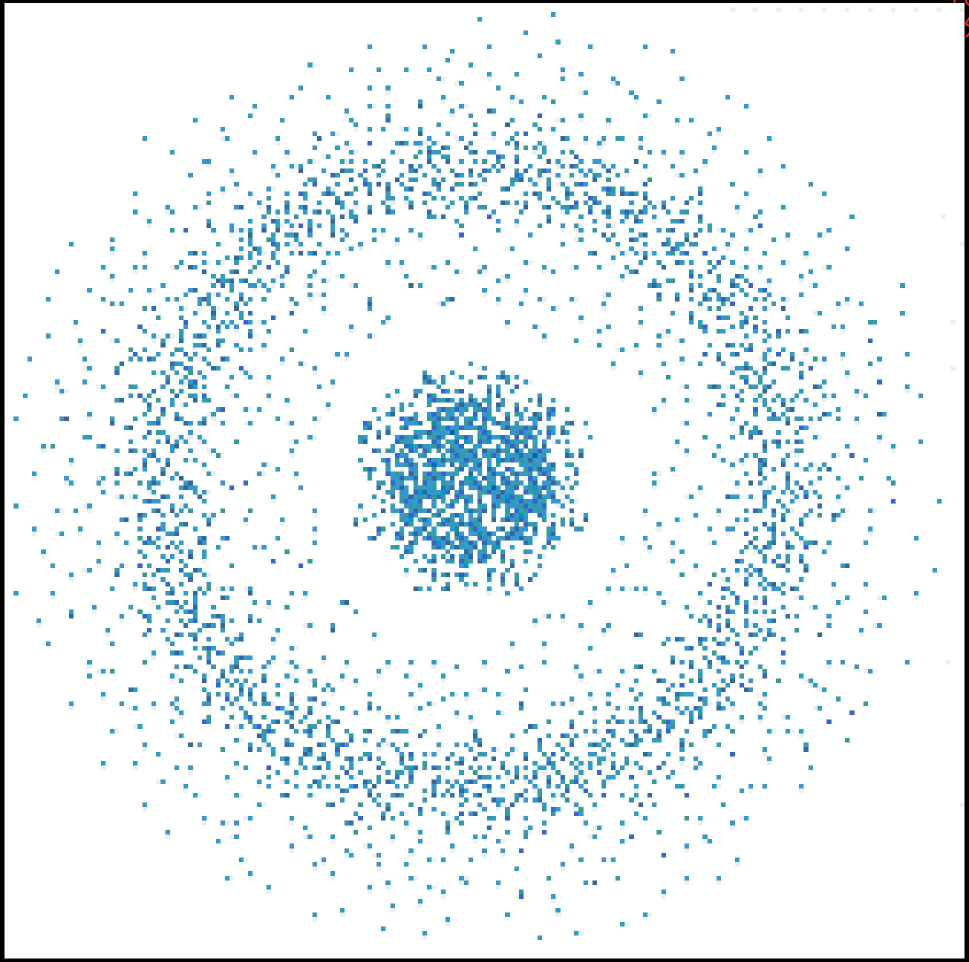
Fig. 39-21

氫原子的量子數

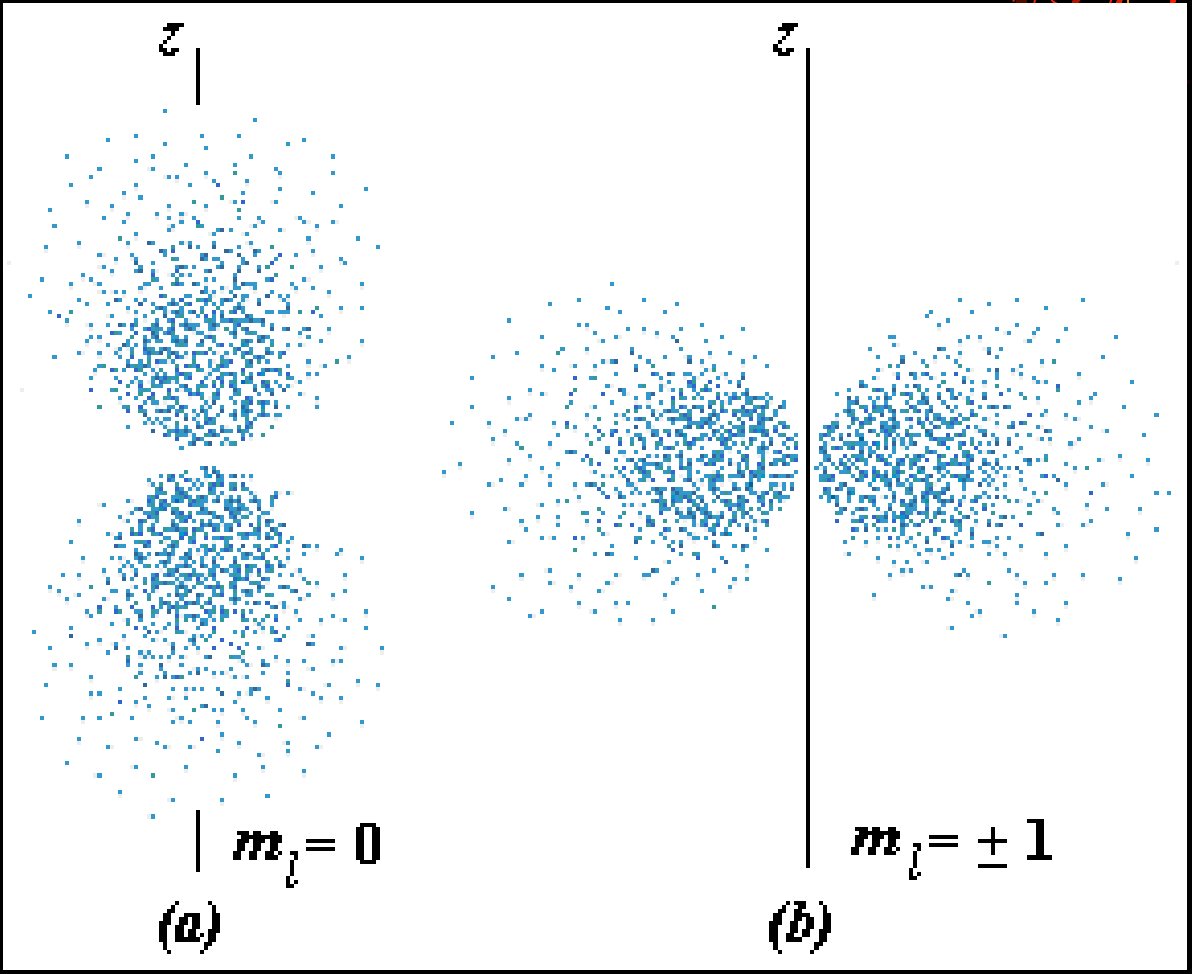
TABLE 40-2 QUANTUM NUMBERS FOR HYDROGEN ATOM STATES WITH $n = 2$

n	l	m
2	0	0
2	1	+ 1
2	1	0
2	1	- 1

$n=2, l=0, m_l=0$



$n=2, l=1$



Hydrogen Atom States with *n*>>1

As the principal quantum number increases, electronic states appear more like classical orbits.

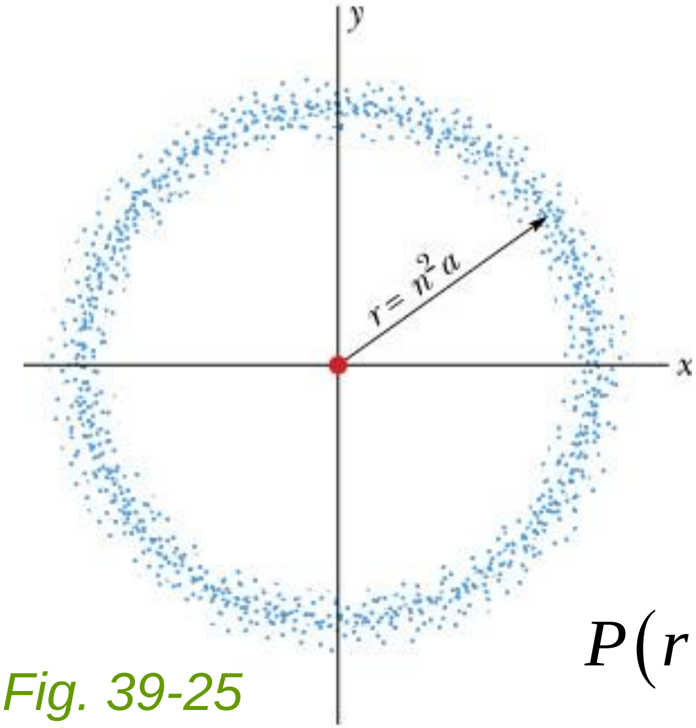


Fig. 39-25

$$P(r) \text{ for } n = 45,\ \ell = n - 1 = 44$$