

4 3 2 1
 王 張 答 請 是 否

KE 陣 matrix, matrices

$$\begin{matrix} \text{row} \uparrow \\ \downarrow \text{column} \\ \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \end{matrix}$$

$m \times n$ matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ 4 & -5 & 1 \end{pmatrix}$$

(a_{ij})

$i = 1, 2, \dots, m$

$j = 1, 2, \dots, n$

如果 $1+1 \neq 0$
 則 $\dots$

potatoes $A = (a_{ij})$ $\left| \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ -2 & -5 \end{pmatrix} \right|$

$\alpha A = (\alpha a_{ij})$

$3 \begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ 4 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 3 \\ 6 & 9 & 6 \\ 12 & 6 & 3 \end{pmatrix}$

$A+B=B+A$
 $\alpha A=A\alpha$

$\alpha \vec{v}$

$\vec{v} = (v_1, v_2, v_3)$

$\alpha \vec{v} = (\alpha v_1, \alpha v_2, \alpha v_3, \dots, \alpha v_n)$

$\begin{cases} 1+0=0+1=1 \\ 0+0=0 \\ 1+1=0 \end{cases} \quad \boxed{0, 1, 2, \dots, 7}$

矩陣 matrix, matrices

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

row

$m \times n$ matrix
column

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ 4 & -5 & 6 \end{pmatrix}$$

(a_{ij})

$i=1, 2, \dots, m$

$j=1, 2, \dots, n$

如果 $1+1 \neq 0$
則...

$$\begin{bmatrix} 1 & 0 \\ -4 & 5 \end{bmatrix}$$

potatoes $A = (a_{ij})$

$B = (b_{ij})$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ -2 & -5 \end{pmatrix}$$

$$\alpha A = (\alpha a_{ij})$$

$$3 \begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ 4 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 3 \\ 6 & 6 & 6 \\ 12 & 6 & 3 \end{pmatrix}$$

$$A+B=B+C$$

$$\alpha A = A\alpha$$

$$\alpha \vec{v}$$

$$\vec{v} = (v_1, v_2, v_3)$$

$$\alpha \vec{v} = (\alpha v_1, \alpha v_2, \alpha v_3)$$

$$\begin{cases} 1+0=0+1=1 \\ 0+0=0 \end{cases}$$

$$\boxed{0, 1, 2, \dots, 7}$$

$$C = AB = (a_{ij})(b_{kj})$$

$$(c_{ij})$$

$$c_{ij} = \sum_k a_{ik} b_{kj}$$

$$c_{i2} = \sum_k a_{ik} b_{k2}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

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KE 降 matrix, matrices

$$\begin{matrix} \text{row} \nearrow \\ \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \\ \text{column} \searrow \end{matrix}$$

$m \times n$ matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 0 \\ 4 & -5 & 6 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ -4 & 5 \end{bmatrix}$$

(a_{ij})

$i=1, 2, \dots, m$

$j=1, 2, \dots, n$

如果 $1+1 \neq 0$
則...

potatoes $A = (a_{ij})$

$B = (b_{ij})$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} -1 & 3 \\ -2 & -5 \end{pmatrix}$$

$$\alpha A = (\alpha a_{ij})$$

$$\langle \rangle 3 \begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ 4 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 & 3 \\ 6 & 6 & 6 \\ 12 & 6 & 3 \end{pmatrix}$$

$$\vec{v} = (v_1, v_2, v_3) \rightarrow \langle v |$$

$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \rightarrow |v\rangle$

$$\langle v | w \rangle = \vec{v} \cdot \vec{w}$$

bracket

$$C_{12} = \sum_{k=1}^n a_{1k} b_{k2}$$

$a_{11}b_{12} + a_{12}b_{22} + \dots + a_{1n}b_{n2}$

看看也許
新年快樂

2×2
 $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \sigma_1 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 7 & 5 \end{bmatrix}$
 $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \sigma_1 AB = -BA$
 $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \sigma_1 AA = I$
 $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \sigma_1 BB = I = CC$
 $AC = -CA$
 $BC = -CB$
 Pauli matrices

Identity matrix
 $I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$
 Square matrices
 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$C = AB = (a_{ij})(b_{kj})$
 $C_{ij} = \sum_{k=1}^r a_{ik} b_{kj}$
 $C_{12} = \sum_{k=1}^r a_{1k} b_{k2}$
 $C_{21} = \sum_{k=1}^r a_{2k} b_{k1}$
 $C_{22} = \sum_{k=1}^r a_{2k} b_{k2}$
 $C_{11} = \sum_{k=1}^r a_{1k} b_{k1}$
 $C_{12} = \sum_{k=1}^r a_{1k} b_{k2}$
 $C_{21} = \sum_{k=1}^r a_{2k} b_{k1}$
 $C_{22} = \sum_{k=1}^r a_{2k} b_{k2}$

$A = (a_{ij}) \quad A = (a_{ij}) \quad A = \begin{bmatrix} 4 & 2 & 9 \\ -3 & -1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$
 $B = (b_{ij}) \quad B = \begin{bmatrix} -2 & 2 & -4 \\ 0 & 0 & 1 \\ 3 & -4 & -1 \end{bmatrix}$
 $BA = \begin{bmatrix} -22 & -10 & -50 \\ 2 & 1 & 2 \end{bmatrix}$

$\sigma_1 \sigma_2 + \sigma_2 \sigma_1 = 0$
 $\sigma_2 \sigma_3 + \sigma_3 \sigma_2 = 0$
 $\sigma_3 \sigma_1 + \sigma_1 \sigma_3 = 0$
 $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = I$
 $\sigma_1 \sigma_2 = i \sigma_3$
 $\sigma_2 \sigma_1 = -i \sigma_3$
 $\sigma_1 \sigma_3 = i \sigma_2$
 $\sigma_3 \sigma_1 = -i \sigma_2$
 $\sigma_2 \sigma_3 = i \sigma_1$
 $\sigma_3 \sigma_2 = -i \sigma_1$

2×2
 $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \sigma_1$
 $B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \sigma_2$
 $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \sigma_3$
 $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AB = -BA$
 $AA = I$
 $BB = I$
 $CC = I$
 $AC = -CA$
 $BC = -CB$

Identity matrix
 $I = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$

Square matrices
 $IA = A$
 $AI = A$

Pauli matrices

$\delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$

$\sigma_i \sigma_j + \sigma_j \sigma_i = 0$
 $\sigma_i \sigma_j + \sigma_j \sigma_i = 2\delta_{ij} I$

$\sigma_i \sigma_j = i \sum_{k=1}^3 \epsilon_{ijk} \sigma_k$

$\sigma_1 \sigma_2 = i \sigma_3$
 $\sigma_2 \sigma_3 = i \sigma_1$
 $\sigma_3 \sigma_1 = i \sigma_2$

$\sigma_i \sigma_j = -i \sum_{k=1}^3 \epsilon_{ijk} \sigma_k$

$\sigma_1 \sigma_2 = i \sigma_3$
 $\sigma_2 \sigma_3 = i \sigma_1$
 $\sigma_3 \sigma_1 = i \sigma_2$

$S_z = \frac{\hbar}{2} \sigma_3 = \begin{bmatrix} \frac{\hbar}{2} & 0 \\ 0 & -\frac{\hbar}{2} \end{bmatrix}$
 $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

自旋
 $\uparrow$
 $\downarrow$

$\text{Spin } \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$
 $k = \frac{1}{2}$

(S_x, S_y, S_z)
 $(\sigma_1, \sigma_2, \sigma_3)$

$A = (a_{ij})$
 $A = (a_{ij})$
 $A = \begin{bmatrix} 4 & 2 & 9 \\ -3 & -1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$B = (b_{ij})$
 $B = \begin{bmatrix} -2 & 2 & -4 \\ 0 & 0 & 1 \\ 3 & -4 & -1 \end{bmatrix}$

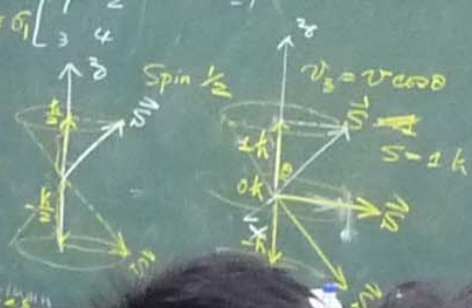
Levi-Civita symbols
 $\epsilon_{123} = 1$
 $\epsilon_{132} = -1$
 $\epsilon_{213} = -1$
 $\epsilon_{231} = 1$
 $\epsilon_{312} = 1$
 $\epsilon_{321} = -1$

$AB = \begin{bmatrix} 19 & -28 & -23 \\ 9 & -10 & -10 \\ -1 & -4 & -9 \end{bmatrix}$
 $BA = \begin{bmatrix} -22 & -10 & -58 \\ 2 & 1 & 2 \\ 22 & 9 & 20 \end{bmatrix}$

$AB \neq BA$

$[\uparrow] = |\uparrow\rangle, [\downarrow] = |\downarrow\rangle$
 自旋 $\uparrow \downarrow$
 $S_{\uparrow} = \frac{1}{2}, S_{\downarrow} = \frac{1}{2}$
 $[\uparrow] = |\uparrow\rangle, [\downarrow] = |\downarrow\rangle$

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \sigma_1$ 2×2
 $B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
 $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \sigma_3$
 $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
 Pauli matrices



$$A = (a_{ij}) \quad n=0,1,2, \dots$$

$$B = (b_{ij})$$

$$C = (c_{ij})$$

$$D = (d_{ij})$$

$$E = (e_{ij})$$

$$F = (f_{ij})$$

$$G = (g_{ij})$$

$$H = (h_{ij})$$

$$I = (i_{ij})$$

$$J = (j_{ij})$$

$$K = (k_{ij})$$

$$L = (l_{ij})$$

$$M = (m_{ij})$$

$$N = (n_{ij})$$

$$O = (o_{ij})$$

$$P = (p_{ij})$$

$$Q = (q_{ij})$$

$$R = (r_{ij})$$

$$S = (s_{ij})$$

$$T = (t_{ij})$$

$$U = (u_{ij})$$

$$V = (v_{ij})$$

$$W = (w_{ij})$$

$$X = (x_{ij})$$

$$Y = (y_{ij})$$

$$Z = (z_{ij})$$

$$AA = A^2$$

$$AB = BA$$

$$AC = CA$$

$$AD = DA$$

$$AE = EA$$

$$AF = FA$$

$$AG = GA$$

$$AH = HA$$

$$AI = IA$$

$$AJ = JA$$

$$AK = KA$$

$$AL = LA$$

$$AM = MA$$

$$AN = NA$$

$$AO = OA$$

$$AP = PA$$

$$AQ = QA$$

$$AR = RA$$

$$AS = SA$$

$$AT = TA$$

$$AU = UA$$

$$AV = VA$$

$$AW = WA$$

$$AX = XA$$

$$AY = YA$$

$$AZ = ZA$$

$$BA = AB$$

$$CB = BC$$

$$DB = BD$$

$$EB = BE$$

$$FB = BF$$

$$GB = BG$$

$$HB = BH$$

$$IB = BI$$

$$JB = BJ$$

$$KB = BK$$

$$LB = BL$$

$$MB = BM$$

$$NB = BN$$

$$OB = BO$$

$$PB = BP$$

$$QB = BQ$$

$$RB = BR$$

$$SB = BS$$

$$TB = BT$$

$$UB = BU$$

$$VB = BV$$

$$WB = BW$$

$$XB = BX$$

$$YB = BY$$

$$ZB = BZ$$

$$AA = A^2$$

$$AB = BA$$

$$AC = CA$$

$$AD = DA$$

$$AE = EA$$

$$AF = FA$$

$$AG = GA$$

$$AH = HA$$

$$AI = IA$$

$$AJ = JA$$

$$AK = KA$$

$$AL = LA$$

$$AM = MA$$

$$AN = NA$$

$$AO = OA$$

$$AP = PA$$

$$AQ = QA$$

$$AR = RA$$

$$AS = SA$$

$$AT = TA$$

$$AU = UA$$

$$AV = VA$$

$$AW = WA$$

$$AX = XA$$

$$AY = YA$$

$$AZ = ZA$$

$$BA = AB$$

$$CB = BC$$

$$DB = BD$$

$$EB = BE$$

$$FB = BF$$

$$GB = BG$$

$$HB = BH$$

$$IB = BI$$

$$JB = BJ$$

$$KB = BK$$

$$LB = BL$$

$$MB = BM$$

$$NB = BN$$

$$OB = BO$$

$$PB = BP$$

$$QB = BQ$$

$$RB = BR$$

$$SB = BS$$

$$TB = BT$$

$$UB = BU$$

$$VB = BV$$

$$WB = BW$$

$$XB = BX$$

$$YB = BY$$

$$ZB = BZ$$

