

$$f^{(n)}(x) = \frac{d^n f(x)}{dx^n} = \lim_{\Delta x \rightarrow 0} \frac{f^{(n-1)}(x+\Delta x) - f^{(n-1)}(x)}{\Delta x}$$

$$\frac{d}{dx} x^\alpha = \alpha x^{\alpha-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} f(g(x)) = \frac{d f(g(x))}{d g(x)} \frac{d g(x)}{dx} \quad \text{chain rule}$$

$$\frac{d e^{ax}}{dx} = \frac{d e^{ax}}{d(ax)} \frac{d(ax)}{dx} = a e^{ax}$$

$$\frac{d \sin ax}{dx} = a \cos ax \quad \frac{d (3x^2+6x-2)^a}{dx} = a (3x^2+6x-2)^{a-1} (6x+6)$$

微分 Differentiation

Derivative $\frac{d f(x)}{dx} = f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\frac{d^2}{dx^2} f(x) = f''(x) = \lim_{\Delta x \rightarrow 0} \frac{f'(x+\Delta x) - f'(x)}{\Delta x}$$

$$\frac{d}{dx} f'(x)$$

$$\frac{d uv}{dx} = u'v + uv'$$

$$(uv)'$$

$$(uv)'' = (u'v + uv)'$$

$$= u''v + u'v' + u'v' + uv''$$

$$= u''v + 2u'v' + uv''$$

$$(uv)'''$$

$$u''v + 3u'v' + 3u'v' + v''$$

請簽到
 2. 前3有遺失物請認領
 3. 請與凱仁同學找助教

$$y = \ln x$$

$$x = e^y$$

$$dx = e^y$$

$$f^{(n)}(x) = \frac{d^n f(x)}{dx^n} = \lim_{\Delta x \rightarrow 0} \frac{f^{(n-1)}(x+\Delta x) - f^{(n-1)}(x)}{\Delta x}$$

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{u'v - u(\frac{1}{v})'}{v^2}$$

$$y(x) = x^3$$

$$xy = \left(\frac{y}{x}\right)^3$$

$$\frac{d}{dx} f(g(x)) = \frac{d f(g(x))}{d g(x)} \frac{d g(x)}{dx} \quad \text{chain rule}$$

$$\frac{d e^{ax}}{dx} = \frac{d e^{ax}}{d(ax)} \frac{d(ax)}{dx}$$

$$= a e^{ax}$$

$$\frac{d \sin ax}{dx} = a \cos ax$$

$$\frac{d}{dx} (3x^2 + 6x - 2)^a = a(3x^2 + 6x - 2)^{a-1} (6x + 6)$$

微分 Differentiation

Derivative

$$\frac{df(x)}{dx} = f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{d^2 f(x)}{dx^2} = f''(x) = \lim_{\Delta x \rightarrow 0} \frac{f'(x+\Delta x) - f'(x)}{\Delta x}$$

$$\frac{d}{dx} f'(x)$$

$$y = e^{\ln y} = e^{x \ln a}$$

$$\frac{dy}{dx} = e^{x \ln a} \cdot \ln a$$

$$= a^x \ln a$$

$$y = x^x$$

$$\ln y = \ln(x^x) = x \ln x$$

$$\frac{d \ln y}{dy} \left(\frac{dy}{dx} \right) = (x)' \ln x + x (\ln x)'$$

$$= \ln x + 1$$

請簽到
 1. 前若有遺失物，請認領
 2. 請將訊仁同學找助教
 3. 請將訊仁同學找助教

$$y = \ln x$$

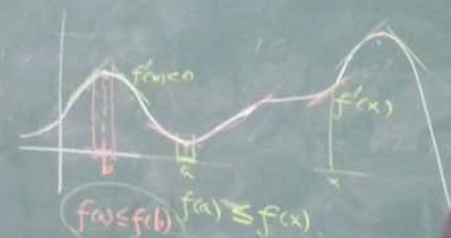
$$x = e^y$$

$$\frac{dx}{dy} = e^y$$

$$f^{(n)}(x) = \frac{d^n f(x)}{dx^n} = \lim_{\Delta x \rightarrow 0} \frac{f^{(n-1)}(x+\Delta x) - f^{(n-1)}(x)}{\Delta x}$$

$$\frac{d(u/v)}{dx} = \frac{u'v - uv'}{v^2}$$

$y(x) = 2x^3$
 $x(y) = (\frac{y}{2})^{1/3}$
 $\frac{dx}{dy} = 1/4 \cdot 1/3$



請到
前之有
請到

$$\frac{d}{dx} f(g(x)) = \frac{d f(g(x))}{d g(x)} \frac{d g(x)}{dx} \quad \text{chain rule}$$

$$\frac{d e^{ax}}{dx} = \frac{d e^{ax}}{d(ax)} \frac{d(ax)}{dx}$$

$$= a e^{ax}$$

$$\frac{d \sin ax}{dx} = a \cos ax$$

$$\frac{d}{dx} (3x^2 + 6x - 2)^a = a(3x^2 + 6x - 2)^{a-1} (6x + 6)$$

$$y = \arcsin x$$

$$= \sin^{-1} x$$

$$x = \sin y$$

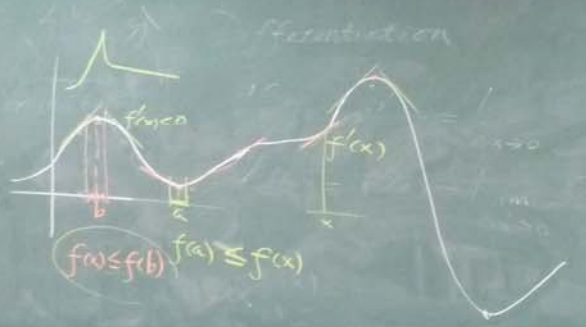
$$\frac{dx}{dy} = \cos y = \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{1}{(\frac{dx}{dy})} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{d\theta} \sin \theta = \frac{d \cos \theta}{d\theta} = -\frac{\sin \theta}{d\theta} = -\cos \theta$$

$$= -\cos \theta = -\sin \theta$$

請發到
 之前有遺失物請認領
 計算機 手機 充電器 記事本
 請蘇凱仁同學協助



I. First derivative test.

I.1 $f'(x)$ 從 $+$ 變 $-$ $\Rightarrow f(a)$ 極小

$\begin{cases} f'(x) > 0 & \text{if } x < a \\ f'(x) < 0 & \text{if } x > a \end{cases}$

I.2 $\begin{cases} f'(x) < 0, & x < a \\ f'(x) > 0, & x > a \end{cases} \Rightarrow f(a)$ 是極大

I.3 $f'(x) \neq 0$
 $f'(x)$ 在 $(a-\delta, a+\delta)$
 ≥ 0
 不改變符號
 $\Rightarrow f(a)$ 不是極大/小

$$\frac{d}{dx} f(g(x)) = \frac{df(g(x))}{dg(x)} \frac{dg(x)}{dx} \quad \text{chain rule}$$

$$\frac{d e^{ax}}{dx} = \frac{d e^{ax}}{d(ax)} \frac{d(ax)}{dx} = a e^{ax}$$

$$\frac{d \sin(ax)}{dx} = a \cos(ax) \quad \frac{d (3x^2 + 6x - 2)^a}{dx} = a (3x^2 + 6x - 2)^{a-1} (6x + 6)$$

$y = x^2$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d \tan \theta}{d\theta} = \frac{d \frac{\sin \theta}{\cos \theta}}{d\theta} = \frac{\cos \theta}{\cos \theta} + \sin \theta \left(-\frac{1}{\cos^2 \theta}\right) = 1 + \tan^2 \theta = \sec^2 \theta$$

I. First derivative test.

I.1 $f(x)$ 從 $+$ 變 $-$ $\Rightarrow f(a)$ 極小.

$\begin{cases} f'(x) > 0 & \text{if } x < a \\ f'(x) < 0 & \text{if } x > a \end{cases}$

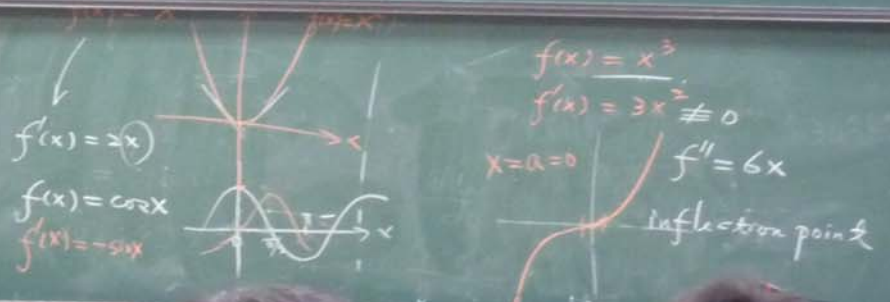
I.2 $f(x) \neq 0$
 $f(x)$ 在 $(a-\delta, a+\delta)$
 $\equiv 0$
 不改變符號

$$\frac{d}{dx} f(g(x)) = \frac{df(g(x))}{dg(x)} \frac{dg(x)}{dx} \quad \text{chain rule}$$

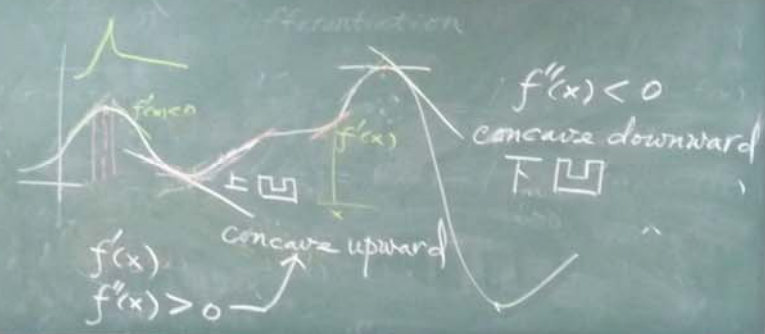
$$\frac{d}{dx} e^{ax} = \frac{de^{ax}}{de^{ax}} \frac{d(ax)}{dx} = a e^{ax}$$

$$\frac{d}{dx} \sin ax = a \cos ax \quad \frac{d}{dx} (3x^2 + 6x - 2)^a = a(3x^2 + 6x - 2)^{a-1} (6x + 6)$$

請簽到
 之前有遺失物請認領
 計算機 手機充電器 記事本
 請郵訊仁同學代助教



請簽到
 之前有遺失物請認領
 計算機、手機充電器、記事本
 請郵訊仁同學找助教

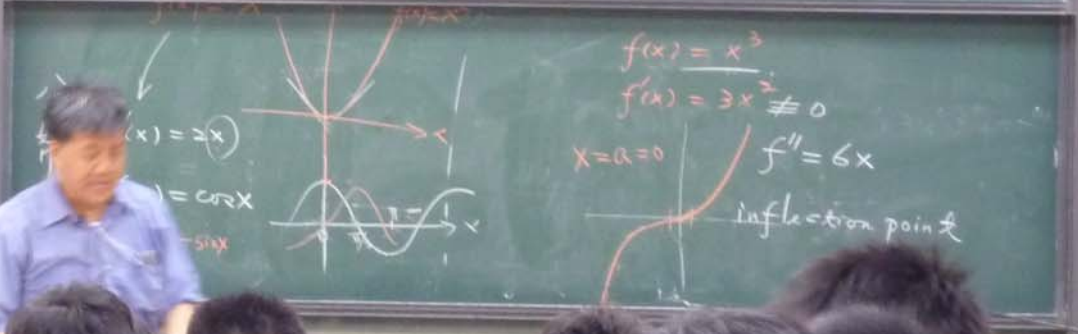


$$\frac{d}{dx} f(g(x)) = \frac{df(g(x))}{dg(x)} \frac{dg(x)}{dx} \quad \text{chain rule}$$

$$\frac{d}{dx} e^{ax} = \frac{de^{ax}}{da(x)} \frac{da(x)}{dx} = ae^{ax}$$

$$\frac{d}{dx} \sin ax = a \cos ax \quad \frac{d}{dx} (3x^2 + 6x - 2)^a = a(3x^2 + 6x - 2)^{a-1} (6x + 6)$$

$f'(a) = 0$, and $f''(a) > 0 \Rightarrow f(a)$ 是極小值
 $f'(a) = 0$ and $f''(a) < 0 \Rightarrow f(a)$ is a maximum
 $f'(x) < 0, x < a \rightarrow f(a)$ 是極大
 $f'(x) > 0, x > a$
 minimum
 $I \rightarrow f'(x) \neq 0$
 $f(x)$ 在 $(a-\delta, a+\delta)$
 ≥ 0
 不改變符號
 $\Rightarrow f(a)$ 不是極小值



$f(x) = 2x$
 $f'(x) = 2$
 $f''(x) = 0$
 $f(x) = \cos x$
 $f'(x) = -\sin x$

請簽到
 之前有遺失物請認領
 計算機 手機 充電器 記事本



$$\frac{d}{dx} f(g(x)) = \frac{df(g(x))}{dg(x)} \frac{dg(x)}{dx} \quad \text{chain rule}$$

$$\frac{d}{dx} e^{ax} = \frac{de^{ax}}{d(ax)} \frac{d(ax)}{dx} = ae^{ax}$$

$$\frac{d}{dx} \sin ax = a \cos ax \quad \frac{d}{dx} (3x^2 + 6x - 2)^a = a(3x^2 + 6x - 2)^{a-1} (6x + 6)$$

Minimum
 $f(a)$ 是極小值
 $I \Rightarrow f(x) \neq 0$
 $f(x)$ 在 $(a-\delta, a+\delta)$
 ≥ 0

$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$
 $f(x) = c_1 e^{ix} + c_2 e^{2ix} + c_3 e^{3ix} + \dots$
 $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$
 $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$
 $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$e^{ix} = \cos x + i \sin x$
 $e^{i\theta} = \cos \theta + i \sin \theta$
 $\Rightarrow f(x)$ 不改變符號

$f(x) = x^2 \quad f'(x) = 2x$
 $f(x) = f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!} (x-a)^n + \frac{(x-a)^{n+1}}{(n+1)!}$

$f(x) = x^3$
 $x \leq \frac{2}{3} \leq a$
 $\frac{2}{3} \leq x \leq a$

