

$$f'(x) \equiv \frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x) = x^2 \quad x = 1$$

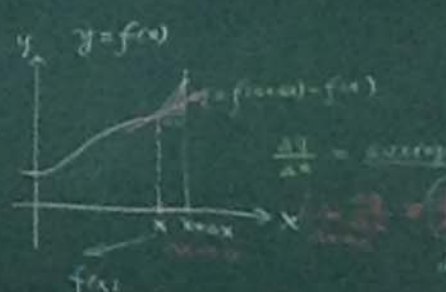
$$\lim_{\Delta x \rightarrow 0} \left\{ \frac{(1+\Delta x)^2 - 1^2}{\Delta x} \right\} = \lim_{\Delta x \rightarrow 0} \frac{2 + 2\Delta x}{\Delta x} = 2$$

$$f'(1) = 2$$

$$\frac{d^2 f(x)}{dx^2} \equiv f''(x)$$


$$\lim_{\Delta x \rightarrow 0} \frac{f''(x+\Delta x) - f''(x)}{\Delta x}$$

$$f''(x), f'''(x)$$



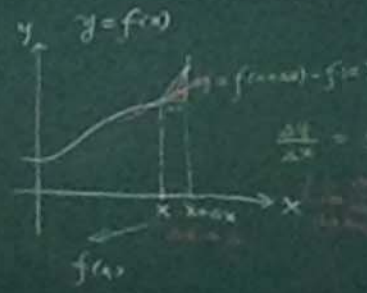


$f(x) = \sqrt{x}$



$\frac{df(x)}{dx} = f'(x)$   
"  
 $\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$   
 $f'(x), f''(x)$

$y = f(x)$



$\Delta y = f(x+\Delta x) - f(x)$

$\frac{\Delta y}{\Delta x} = \text{average rate of change}$

$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \text{instantaneous rate of change}$

derivative  $\frac{df(x)}{dx}$



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$$f'(x) = \frac{df(x)}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{df(x)}{dx} = f'(x)$$

$$f(x) = x^2, \quad x=1$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{(1+\Delta x)^2 - 1^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{2\Delta x + \Delta x^2}{\Delta x} = 2$$

$$f'(x), f''(x)$$

$$f'(1) = 2$$

$$f(x) = x^2$$

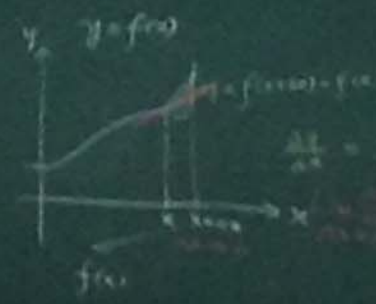
$$\lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$f'(x) = \frac{df(x)}{dx}$$

$$f''(x) = \frac{d}{dx} \left( \frac{df(x)}{dx} \right)$$

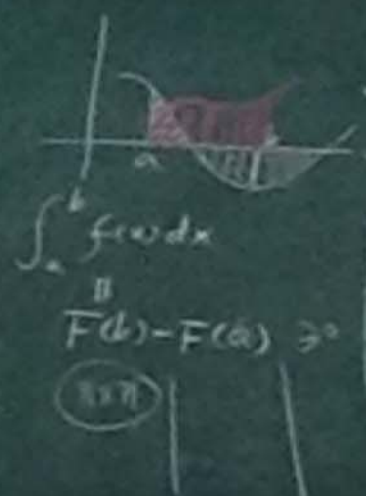
$$= \frac{d^2 f(x)}{dx^2}$$

$$= \frac{d^2 f(x)}{dx^2}$$





$e = (1 + \frac{1}{n})^n$   
 $(1 + \frac{1}{n})^n$   
 $n=1 \quad 2$   
 $n=2 \quad 2.25$   
 $n=3 \quad 2.37$   
 $n=10 \quad 2.59$   
 $n=100 \quad 2.718$   
 $n=1000 \quad 2.718$   
 $n \rightarrow \infty \quad 2.718281828459$



$\int kx dx = \frac{1}{2} kx^2 + C$   
 $\int k dx = kx + C$   
 $\int f(x) dx = F(x)$   
 $\frac{dF(x)}{dx} = f(x)$



$$y(x) = \cos x$$

$$y'(x) = -\sin x$$

$$y''(x) = -\cos x = -y(x)$$

$$y'' + y = 0$$

$$y(0) = 1, y'(0) = 0$$

$$f(x) = e^x$$

$$f'(x) = f(x)$$

$$f(0) = 1$$

$$\downarrow$$

$$f(x) = e^x$$

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$\frac{d^2 f(x)}{dx^2} = f''(x)$$

$$y(x) = \sin x$$

$$y'(x) = \cos x$$

$$y''(x) = -\sin x = -y(x)$$

$$y'' + y = 0$$

$$y(0) = 0, y'(0) = 1$$

$$\frac{df}{dx} = \frac{d}{dx} f(g(x)) = \lim_{h \rightarrow 0} \left\{ \frac{f(g(x+h)) - f(g(x))}{h} \right\}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \frac{g(x+h) - g(x)}{h} \right\}$$

$$= \left( \frac{df}{dy} \right) \frac{dy}{dx} \rightarrow \text{chain rule}$$

$$\frac{d}{dx} \sin(x)$$

||

$$\frac{d(\sin(x))}{d(\sin(x))} \frac{d(\sin(x))}{dx}$$

||

$$1 \cdot \cos(x)$$

$$f(x) = (x^2 + 3)^{10}$$

$$= x^2 + 6x + 9$$

$$f'(x) = 10x^2 + 12x$$

$$f''(x) = \frac{df}{dx} \frac{dx}{dx}$$

$$= 20x + 12$$

$$y(x) = \cos x$$

$$y'(x) = -\sin x$$

$$y''(x) = -\cos x = -y(x)$$

$$y'' + y = 0$$

$$y(0) = 1, y'(0) = 0$$



$$f(x) = e^x$$

$$f'(x) = f(x)$$

$$f(0) = 1$$

$$f(x) = e^x$$

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$\frac{d^2 f(x)}{dx^2} = f'(x)$$

$$y(x) = \sin x$$

$$y'(x) = \cos x$$

$$y''(x) = -\sin x = -y(x)$$

$$\begin{cases} y'' + y = 0 \\ y(0) = 0, y'(0) = 1 \end{cases}$$

$$\left(\frac{u}{v}\right)' = \frac{u'}{v} + u \left(\frac{1}{v}\right)'$$

$$= \frac{u'}{v} + u \left(-\frac{1}{v^2}\right) v'$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\left(\frac{1}{2^{\cos x}}\right)' = -\frac{1}{2^{\cos x}} \ln 2 \cdot (-\sin x)$$

$$f(x) = (x^2 + 3)(x^3 + x)$$

$$f'(x) = \frac{d}{dx} \left[ (x^2 + 3)(x^3 + x) \right]$$

$$= (x^2 + 3)'(x^3 + x) + (x^2 + 3)(x^3 + x)'$$

$$= 2x(x^3 + x) + (x^2 + 3)(3x^2 + 1)$$

$$f(x) = e^{2x} \sin x$$

$$f'(x) = 2e^{2x} \sin x + e^{2x} \cos x$$



$$\frac{dx^n}{dx} = nx^{n-1}$$

$$\frac{d \ln x}{dx} = \frac{1}{x}$$



$$y' = \frac{-1 - y \cos(xy)}{1 + x \cos(xy)}$$

$$(x+y + \sin(xy)) = 0 \Rightarrow y(x)$$

$$\rightarrow 1 + y' + \cos(xy)(y + xy') = 0$$

$$x^2 - xy - y^2 = 1 \rightarrow y(x)$$

$$y = \frac{1}{2}(-x \pm \sqrt{5x^2 - 4})$$

$$\frac{dy}{dx} = -\frac{1}{2} \pm \frac{1}{2} \frac{10x}{\sqrt{5x^2 - 4}}$$

$$= \frac{1}{2}(-1 \pm \frac{5x}{\sqrt{5x^2 - 4}})$$

$\Rightarrow$

$$2x - y - xy' - 2yy' = 0$$

$$y' = \frac{2x - y}{x + 2y} = \frac{\pm \sqrt{5x^2 - 4}}{x + 2y}$$

$$\frac{dy^2}{dx} = \frac{dy^2}{dy} \frac{dy}{dx} = 2y y'$$

$$2x - y = 2x + \frac{x}{2} + \frac{1}{2} \sqrt{5x^2 - 4}$$

$$= -\frac{1}{2} \pm \frac{x}{2} \sqrt{5x^2 - 4}$$



$$y(x) = \cos x$$

$$y'(x) = -\sin x$$

$$y''(x) = -\cos x = -y(x)$$

$$f(x) = e^x$$

$$f'(x) = f(x)$$

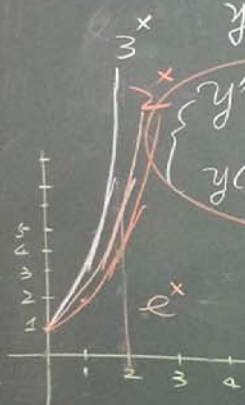
$$f(0) = 1$$

$$\frac{d^2 f(x)}{dx^2} \equiv f''(x)$$

$$y(x) = \sin x$$

$$y'(x) = \cos x$$

$$y''(x) = -\sin x = -y(x)$$



$$y'' + y = 0$$

$$y(0) = 1, y'(0) = 0$$

$$f(x) = e^x$$

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$e = 2.718281828$$

$$y'' + y = 0$$

$$y(0) = 0, y'(0) = 1$$

$$\frac{d \ln x}{dx} = \frac{1}{x}$$

$$\frac{d e^x}{dx} = e^x$$

反函数

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

$$y = \sin^{-1} x = \arcsin x$$

$$\neq \frac{1}{\sin x}$$

$$y = y(x) = \ln x$$

$$x = e^y$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{e^y} = \frac{1}{x}$$

$$y = \sin^{-1} x$$

$$x = \sin y$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dx}{dy} = \cos y = \sqrt{1 - x^2}$$

$$\frac{d \sin^{-1} x}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

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