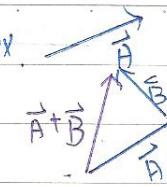


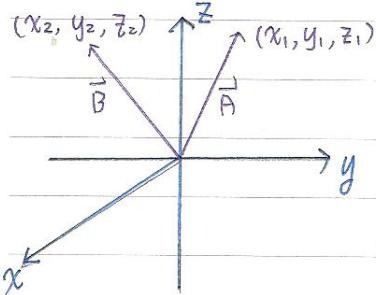
向量：(包括長度、方向)

ex.



$$\textcircled{1} \quad \vec{A} + \vec{B} \Rightarrow \vec{A} + \vec{B}$$

$$\textcircled{2} \quad \vec{A} - \vec{B} \Rightarrow \vec{A} + \vec{-B} \Rightarrow \vec{A} - \vec{B}$$



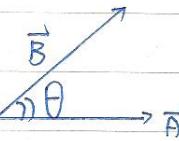
$$\vec{A} = x_1 \hat{x} + y_1 \hat{y} + z_1 \hat{z}$$

$$\vec{B} = x_2 \hat{x} + y_2 \hat{y} + z_2 \hat{z}$$

$$\Rightarrow \vec{A} + \vec{B} = (x_1 + x_2) \hat{x} + (y_1 + y_2) \hat{y} + (z_1 + z_2) \hat{z}$$

$$\Rightarrow \vec{A} - \vec{B} = (x_1 - x_2) \hat{x} + (y_1 - y_2) \hat{y} + (z_1 - z_2) \hat{z}$$

內積： $\vec{A} \cdot \vec{B} \equiv |\vec{A}| \cdot |\vec{B}| \cdot \cos \theta$



$$(x_A \hat{x} + y_A \hat{y} + z_A \hat{z}) \cdot (x_B \hat{x} + y_B \hat{y} + z_B \hat{z})$$

$$= x_A x_B + y_A y_B + z_A z_B$$

外積： $\vec{A} \times \vec{B} = \vec{C}$ $|\vec{C}| = |\vec{A}| |\vec{B}| \sin \theta$ (ex. 力矩)

$$(x_A \hat{x} + y_A \hat{y} + z_A \hat{z}) \times (x_B \hat{x} + y_B \hat{y} + z_B \hat{z})$$

$$= x_A y_B \hat{z} + x_A y_B (-\hat{y}) + y_A x_B (-\hat{z}) + y_A z_B \hat{x} + z_A x_B \hat{y} + z_A y_B (-\hat{x})$$

$$= (y_A z_B - z_A y_B) \hat{x} + (z_A x_B - x_A z_B) \hat{y} + (x_A y_B - y_A x_B) \hat{z}$$

$$\begin{vmatrix} x & y & z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = ?$$

[力矩]

$$\vec{r}(t) = x(t) \hat{x} + y(t) \hat{y} + z(t) \hat{z}$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dx(t)}{dt} \hat{x} + \frac{dy(t)}{dt} \hat{y} + \frac{dz(t)}{dt} \hat{z}$$

微分

$$f(x) = ax^2 + bx + c \Rightarrow \text{多项式函数}$$

函数 $\left\{ \begin{array}{l} \text{(1) 多项式} \\ \text{(2) 三角函数} \end{array} \right.$

$$\text{(2) 三角函数, } e^{ax} \ln x \text{ ex. } \frac{e^{ax} + e^{-ax}}{2}$$

f对x的1階倒數

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{d}{dx} f(x) = (f')$$

很小, 趨近於0

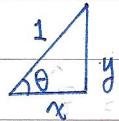
Ex. 多項式 $f(x) = ax^n$

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{a[(x+\Delta x)^n - x^n]}{\Delta x} = \frac{a(nx^{n-1})}{\Delta x}$$

$$(x+\Delta x)^n = x^n + nx^{n-1}\Delta x + \dots + (\Delta x)^n \approx x^n$$

(趨近於0, 忽略)

三角函數的微分



$$\cos \theta = \frac{x}{1}$$

$$\sin \theta = y$$

$$\frac{d \cos \theta}{d \theta} = \lim_{\Delta \theta \rightarrow 0} \frac{\cos(\theta + \Delta \theta) - \cos \theta}{\Delta \theta} = -\sin \theta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\theta + \Delta \theta) = \cos \theta \cos \Delta \theta - \sin \theta \sin \Delta \theta = \cos \theta - \sin \theta \cdot \Delta \theta$$

★ $\begin{cases} \frac{d \cos \theta}{d \theta} = -\sin \theta \\ \frac{d \sin \theta}{d \theta} = \cos \theta \end{cases}$ $\frac{d \tan \theta}{d \theta} = \frac{d(\frac{\sin \theta}{\cos \theta})}{d \theta} = \cos \theta \cdot \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = 1 + \tan^2 \theta = \sec^2 \theta$

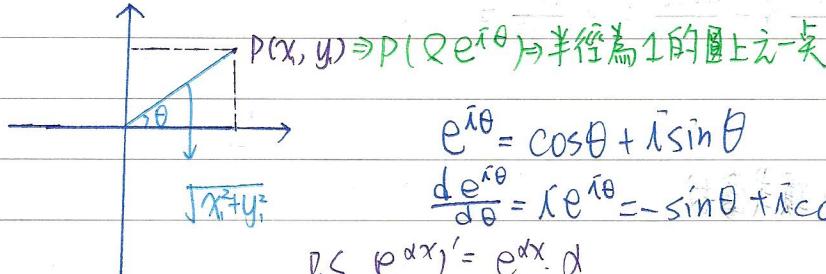
★ $\frac{d(\frac{1}{\cos \theta})}{d \theta} = \frac{d(\frac{1}{\cos \theta})}{d(\cos \theta)} \cdot \frac{d(\cos \theta)}{d \theta} = -\frac{1}{\cos^2 \theta} (-\sin \theta)$

$$f(x) = 6x^6 = 3x^3 \cdot 2x^3$$

$$\frac{df}{dx} = 36x^5$$

$$\hookrightarrow 3x^3 \cdot 6x^2 + 9x^2 \cdot 2x^3 = 18x^5 + 18x^5 = 36x^5$$

複數平面 $\frac{de^{ix}}{dx} = \frac{de^{ix}}{d(ix)} \frac{d(ix)}{dx} = e^{ix} \cdot i$



10/31

牛克定律 $F = -kx \Rightarrow ma = -kx \Rightarrow m \frac{d^2x(t)}{dt^2} + kx(t) = 0$
或 \Rightarrow 位置对時間的二次微分

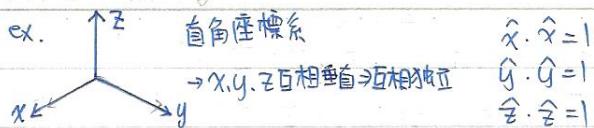
$$\star \frac{d \ln x}{dx} = \frac{1}{x}$$

$$\star \text{ex. } \frac{d x^6}{dx} = 6x^5 \quad \frac{d(x^6 + b)}{dx} = 6x^5$$

$$\int 6x^5 dx = x^6 + C$$

課程進度：力學 → 流體力學 → 熱力學

三度空間基底 bases



$$\hat{x} \cdot \hat{x} = 1$$

$$\hat{y} \cdot \hat{y} = 1$$

$$\hat{z} \cdot \hat{z} = 1$$

三度空間位置隨時間的變化情形： $\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y} + z(t)\hat{z}$

位置 $3 \equiv 3x$

要看位置隨時間的變化，ex. 5 min $\Rightarrow \frac{\vec{r}(t=5) - \vec{r}(t=0)}{5} = \langle \vec{v} \rangle$ 平均速度

$\vec{v}(t)$ 三 $\lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$ 瞬時速度

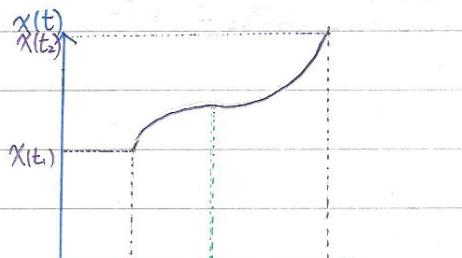
瞬間速度

$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left(\frac{d\vec{r}}{dt} \right) = \frac{d^2\vec{r}}{dt^2}$ 瞬時加速度

*古典力學：沒有受外力 F ，靜者恒靜，動者恒作等速度運動

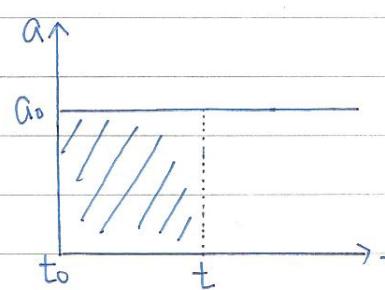
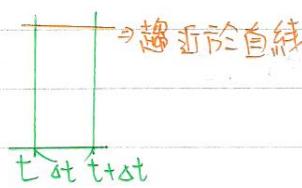
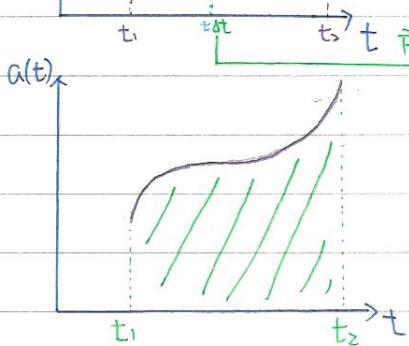
$$a(t) = \frac{dv}{dt} \quad dv = a(t)dt \quad \Delta v = a(t)\Delta t \Rightarrow \sum_{i=1}^n \Delta v_i = a(t_i)\Delta t_i$$

$$\int_{v(t_1)}^{v(t_2)} dv = \int_{t_1}^{t_2} a(t) dt$$



$$v(t) = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

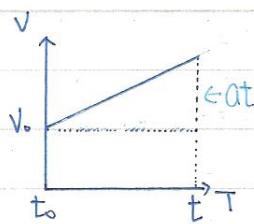
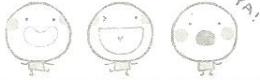
$v(t)$ 即為 速率



$$v(t) - v_0 = at$$

$$\Rightarrow v(t) = v_0 + at$$

$$\Rightarrow \int_{v_0}^{v(t)} dv = \int_0^t a dt' = a \int_0^t dt'$$



$$\begin{aligned} \text{当作原来} \\ X(t) - X_0 &= V_0 t + \frac{1}{2} a t^2 \\ \Rightarrow X_t &= V_0 t + \frac{1}{2} a t^2 \end{aligned}$$

$$\begin{aligned} \frac{V(t) - V_0}{t} &= a \\ X(t) &= V_0 t + \frac{1}{2} \frac{V(t) - V_0}{t} \cdot t^2 \\ &= V_0 t + \frac{1}{2} t (V(t) - V_0) \\ &= \frac{1}{2} V_0 t + \frac{1}{2} V(t) - \frac{1}{2} V_0 t = \frac{V_0 t + Vt}{2} \end{aligned}$$

$$\begin{aligned} \frac{V(t) - V_0}{a} &= t \\ S \frac{X(t) - X_0}{a} &= V_0 \frac{V(t) - V_0}{a} + \frac{1}{2} a \left(\frac{V(t) - V_0}{a} \right)^2 = \frac{V_0(V_0 - V_0)}{a} + \frac{(V(t) - V_0)^2}{2a} = \frac{(V(t) - V_0)(2V_0 + V(t) - V_0)}{2a} \\ S &= \frac{(V(t) - V_0)(V_0 + V_0)}{2a} \quad V^2(t) - V_0^2 = 2aS \end{aligned}$$

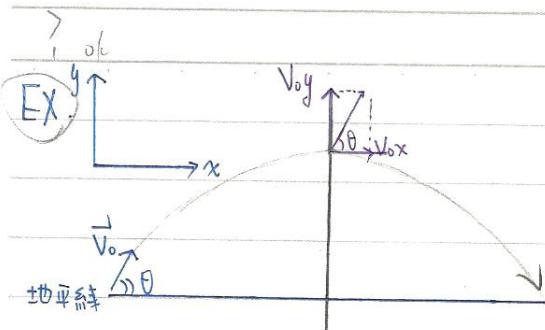
Ex. <自由落体>有一物体從40m處掉下，求掉到地面時的瞬時速度及要花多久？($g=10$)

$$\begin{aligned} V_0 &= 0 & V^2(t) - V_0^2 &= 2gS & V_0 &= 0 & t &= 2\sqrt{2}, \\ 40m & \downarrow V(t) & V^2(t) &= 2 \times 10 \times 40 = 800 & V(t) &= gt & \\ & & V(t) &= 20\sqrt{2} \approx 28m/s & X(t) &= \frac{1}{2} g t^2 & \\ & & & & 40 &= \frac{1}{2} \times 10 \times t^2 & \end{aligned}$$

$$\begin{aligned} \text{Ex. } & V_0 = 10m/s & (50\text{m}) \xrightarrow[40\text{m}]{} & \text{最高處 } V(t) = V_0 + gt & D = 10 - 10t \Rightarrow t = 1 \\ & \downarrow 40m & V_0 = 10m/s & & S = V_0 t + \frac{1}{2} g t^2 \\ & & & & = 10 \times 1 - \frac{1}{2} \times 10 \times 1 = 5 \\ \text{求掉到地面時的瞬時速率? } & 30m/s & & & V^2(t) - V_0^2 = 2gS \\ & & & & V^2(t) = 900 \quad V(t) = 30 \quad A = 30m/s \end{aligned}$$

Ex. 以 $10 m/s$ 的初速往上拋，求最高處距地面多少 m? 5m

$$\begin{aligned} & \downarrow 5m \\ & V = 10m/s \uparrow & 0 = 10 - 10t & t = 1 & S = 10 \times 1 - \frac{1}{2} \times 10 \times 1 = 5 & A = 5m \end{aligned}$$



$$x(t) = V_0 x t = V_0 \cos \theta t$$

$$y(t) = V_0 y t - \frac{1}{2} g t^2$$

$$V_y(t) = V_0 y - g t$$

$$0 = V_0 \sin \theta - g t \quad \text{max} = 1$$

$$S = V_0 \cos \theta \frac{2 V_0 \sin \theta}{g} = \frac{2 V_0^2 \sin \theta \cos \theta}{g} = \frac{V_0^2 \sin 2\theta}{g}$$

$$2t = \frac{2V_0 \sin \theta}{g}$$

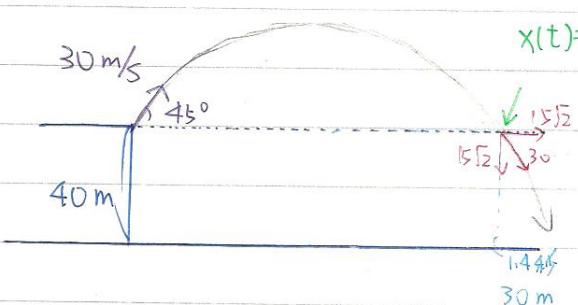
$$V_y(t) = V_0 y - g t$$

$$t = \frac{V_0 \sin \theta}{g} = \frac{V_0 y}{g} \quad \leftarrow t = \frac{V_0 y}{g}$$

$\Rightarrow \theta = 90^\circ$ 时 $\sin 2\theta$ 最大

\Rightarrow 即 $\theta = 45^\circ$ 时，可去最远

Ex.



$$x(t) = \frac{V_0^2}{g} = \frac{900}{10} = 90$$

$$40 = \frac{t}{2} (15\sqrt{2} + 15\sqrt{2} + gt)$$

$$80 = t (30\sqrt{2} + 10t) = 30\sqrt{2}t + 10t^2$$

$$t^2 + 3\sqrt{2}t - 8 = 0$$

$$t \approx 1.4$$

$$1.4 \times 15\sqrt{2} \approx 30$$

Date 2011.11.26. (四)

$$\vec{a} \rightarrow \vec{v} \rightarrow \vec{s}$$

$$\vec{F} = m \times \vec{a}$$

PS. 重力、電磁力、弱作用力、強作用力

重力

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}$$

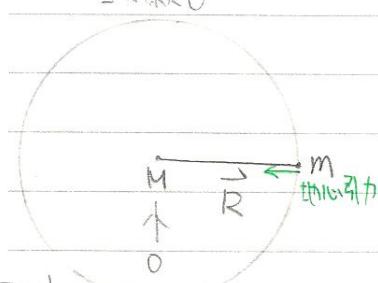
MKS制 $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \text{ kg}^{-2}$

当成原点O

$$\vec{F}_g = -G \frac{Mm}{R^2} \hat{r}$$

$= m \left(-\frac{GM}{R^2} \right) \hat{r}$

把一有质量的东西放在一空间，会使
空间产生变化，产生重力场



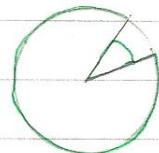
$$g = \frac{GM}{R^2}$$

Earth

球面 $\oint \vec{F}_g \cdot d\vec{a} = -\frac{GM}{R^2} 4\pi R^2$

$$= -4\pi GM$$

高斯定理



面積跟重力場內積會成定值

(只要是有封閉的面)

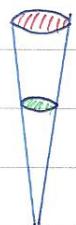
面積積分出來一樣



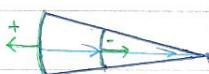
AB面積與R成正比

重力與R成反比

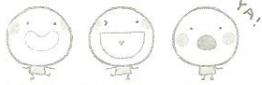
抵消掉



側面

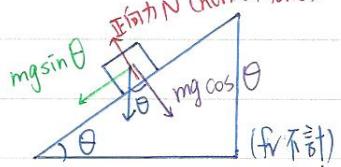


面積會抵消掉



任何在地球表面上的東西，皆有受到重力作用。若沒有支撑就会掉到地球表面。

正向力 N (normal force)



$$a = g \sin \theta$$

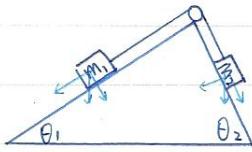
$$2as = v^2 \Rightarrow 2g \sin \theta s \Rightarrow v^2 \Rightarrow v = \sqrt{2g \sin \theta s}$$

ex. $\theta = 30^\circ$, $r_k = 0.2$

$$mgsin\theta$$

$$f_r = 0.2mg \cos \theta$$

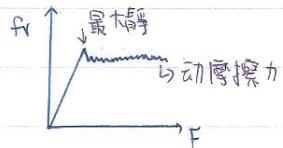
$$0.2mg \cos \theta$$



$$m_1 g \sin \theta_1 - m_2 g \sin \theta_2 = (m_1 + m_2) a$$

$$m_1 g - m_2 g = (m_1 + m_2) a$$

摩擦力
跟正向力摩擦系数有關
摩擦係數
動摩擦係數 μ_k
靜摩擦係數 μ_s



$$f_r = \mu_s N$$

② 向心力
離心力 $m \frac{v^2}{r}$ (圓周運動)

③ 阻力 ①有些跟 V^2 有關 ②有些跟 V 有關 (ex. 風速 v)

④ 跳力

$$g \downarrow$$

$$V = gt$$

終端速度跟 A (截面積) P 有关

$$= \frac{1}{2} C_P A V^2$$

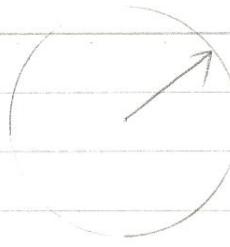
終端速度

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r_{12}^2} \hat{r}$$

① 重力: $\vec{F}_g = G \frac{Mm}{R^2} \hat{R}$ 地球質量

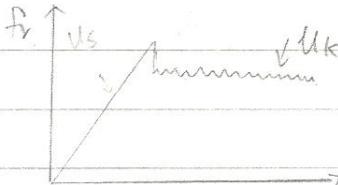
$$\text{萬用} = m (-G \frac{M}{R^2} \hat{R})$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / (\text{kg})^2$$



② 向心力 圓周運動 $F = m \frac{v^2}{r} = mr\omega^2$

③ 摩擦力 (觀察) 跟材質、正向力、時間，跟運動方向相反



$$\vec{F} = -\mu k_f s \times N$$

④ 空氣阻力 滑板 面積 終端速度

$$\text{Drag Force } F_D = \frac{1}{2} C_D A \rho V^2 \rightarrow V_t^2 = \frac{2mg}{\rho C_D A} \Rightarrow V = \sqrt{\frac{2mg}{\rho C_D A}}$$

(碰撞時，與相對速率、截面積、密度有關)

$$\text{球面體 } m = \frac{4}{3} \pi r^3 \quad A = \pi r^2$$

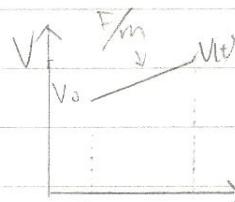
$$V = \sqrt{\frac{2g}{\rho C_D} \frac{4}{3} \pi r^3 / \pi r^2} = \sqrt{\frac{8g}{3\rho C_D}} r$$

運動 (II) 鏡

$$F = ma$$

$$F_d = mad \uparrow a$$

$$F_d = V_0 t + \frac{1}{2} \frac{(F)}{m} t^2$$



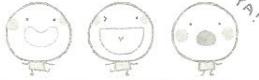
$$V(t) = V_0 + at \Rightarrow V(t) - V_0 = at \Rightarrow t = \frac{V(t) - V_0}{a}$$

$$\hookrightarrow d = V_0 \cdot \frac{V(t) - V_0}{a} + \frac{1}{2} a \left(\frac{V(t) - V_0}{a} \right)^2 = \frac{V_0(V_t) - V_0^2}{a} + \frac{V(t)^2 - 2V_0V_t + V_0^2}{2a} = \frac{V(t)^2 - V_0^2}{2a}$$

$$2ad = V^2(t) - V_0^2$$

$$F_d = mad = \frac{1}{2} m (V(t)^2 - V_0^2) = \frac{1}{2} m V^2(t) - \frac{1}{2} m V_0^2 \quad (\text{功能變化} = \text{所做功})$$

$\hookrightarrow \text{功}$



$$V = \sqrt{2gh}$$

$$mgh \downarrow h \quad mgh = \frac{1}{2}mv^2 = \frac{1}{2} \cdot \frac{1}{2}$$

位能

(重力)

$$\Delta W = \vec{F} \times \vec{S}$$

→ 在很遙遠的地方 $dW = \vec{F} \cdot d\vec{s}$

$$W = \int dW = \int \vec{F} \cdot d\vec{s}$$

$$\vec{F} = -m(G \frac{M}{r^2}) \hat{r}$$

$$W = \int \vec{F} \cdot d\vec{r}$$

$$= \int mG \frac{M}{r^2} dr$$

$$= +mG \frac{M}{r} \Big|_{r_2}^{r_1} = mGu \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = mGM \frac{1}{r_1} - mGM \frac{1}{r_2}$$

< 在無窮遠，把 r_1 當成基準点 $\Rightarrow mGu \frac{1}{r_1}$ >

$$mGM \left(\frac{1}{r_2} - \frac{1}{R+h} \right) = mGM \frac{h}{R(R+h)} = mGM \frac{h}{R^2} = mgh$$

$$\text{彈簧 } F = -kx$$

$$W = \int \vec{F} \cdot d\vec{x} = \frac{1}{2} kx^2$$

$$mgh = \frac{1}{2}mv^2 = \frac{1}{2}m(v^2 - v_0^2) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

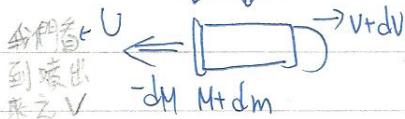
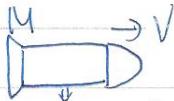
$$mgh + \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + D_{mg0}$$

能量守恒（保守力）

$$\frac{d\vec{p}}{dt} = \vec{F}_{ext}$$

問：氣體 Rate: $\frac{dM}{dt}$

$V_{rel} \rightarrow$ 氣體噴出來之 v



不計重力

$$\text{動量守恒: } Mv = (-dm)U + (M+dm)(v+dv)$$

$$U + V_{rel} = v + dv$$

$$= (-dm)(v+dv - V_{rel}) + (M+dm)(v+dv)$$

$$= (dm)V_{rel} + Mv + Mdv$$

$$\Rightarrow -(dm)V_{rel} = Mdv$$

$$-\frac{dm}{dt}V_{rel} = M \frac{dv}{dt} = Ma$$

單位時間氣體噴出來的量

$$-dm V_{rel} = M dv \Rightarrow -\frac{dm}{M} = V_{rel} dv \Rightarrow \int_{M_i}^{M_f} \frac{dm}{M} = \int \frac{1}{V_{rel}} dv \Rightarrow \ln \frac{M_i}{M_f} = \frac{V_f}{V_{rel}}$$

常數

$$\Rightarrow V_f = V_{rel} \ln \frac{M_i}{M_f}$$

* 這學期上了什麼

$$\vec{F} = m\vec{a}$$

r(曲率)很小

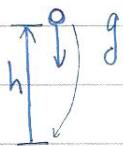
$m\frac{v^2}{r} = ma$ (圓周運動) ex. 行星間的運動、遊樂設施、電子繞原子核

$$m\frac{v^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GM}{r}$$

牛頓萬有引力定律

$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

$$\Rightarrow T^2 = \frac{4\pi^2}{GM} r^3$$



$$h = \frac{1}{2}gt^2 \quad v = gt \Rightarrow t = \frac{v}{g}$$

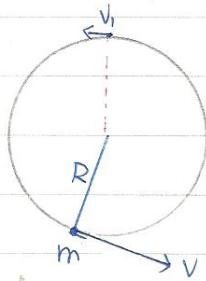
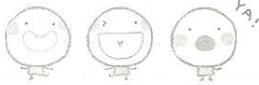
$$= \frac{1}{2}g \cdot \frac{v^2}{g^2}$$

$$= \frac{v^2}{2g}$$

$$2gh = v^2 \Rightarrow m2gh = mv^2 \Rightarrow \frac{1}{2}mv^2 = mgh$$

(重力對物体所做的功等於其動能變化)

$$U + K = E \quad (\text{位能} + \text{動能} = \text{總能量})$$



$$mg(2R) + \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2$$

$$g(2R) = \frac{1}{2}m(v^2 - v_1^2)$$

$$\frac{v_1^2}{12} = g$$

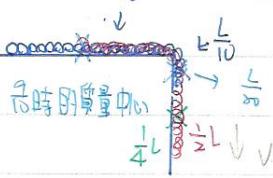
T \downarrow m 做圓周運動繩子須有張力，何時不用張力？

$$2gR = \frac{1}{2}v^2 - \frac{1}{2}gR$$

$$5gR = v^2 \Rightarrow v = \sqrt{5gR}$$

繩長 L

$$\frac{9}{10}L$$



$$\frac{9}{10}mgh - \frac{1}{10}mg\left(\frac{1}{20}L\right)$$

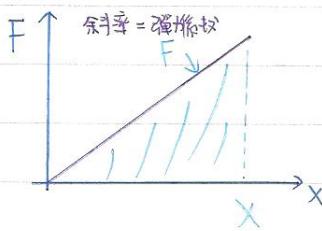
- 開始位置能 \rightarrow 在桌面上的部分

$$0 - \frac{1}{2}mg\left(\frac{1}{4}L\right) + \frac{1}{2}mv^2 = - \frac{1}{10}mg\left(\frac{1}{20}L\right)$$

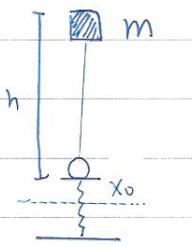
\therefore 是整條繩子在移動

[虎克定律]

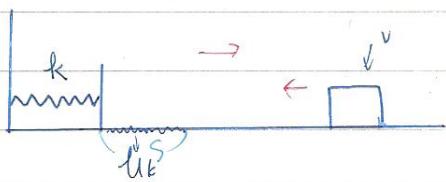
$$F = -kx$$



$$\frac{1}{2}kx^2 \quad (\text{位能})$$



$$mg(h+x_0) = \frac{1}{2}kx^2$$



$$\frac{1}{2}mv^2 = \frac{1}{2}mv_f^2 + U_k mg S$$

$$\frac{1}{2}mv^2 = U_k mg S + \frac{1}{2}kx_0^2$$

⇒ 弹簧在缩最大量

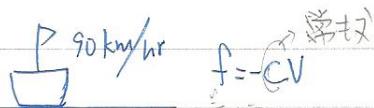
弹簧过去再回来的 time.

m 和 2m 的物体碰撞

$$m \rightarrow v \quad 2m \quad mv = (2m+m)v_f$$

$$v_f = \frac{1}{3}v$$

$$\begin{aligned} &\text{初 } \frac{1}{2}mv^2 \\ &\text{末 } \frac{1}{2}3m\left(\frac{1}{3}v\right)^2 \end{aligned} \quad \text{减少 } \frac{1}{2}mv^2\left(\frac{1}{3}\right)$$



$$ma = -CV$$

$$m \frac{dv}{dt} = -CV$$

初速 v_i

$$\frac{dv}{v} = -C dt$$

$$\int_{v_i}^{v_f} \frac{dv}{v} = -C \int_0^{t_f} dt$$

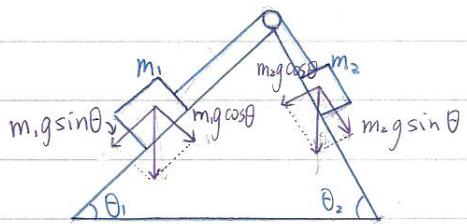
$$\Rightarrow \ln \frac{v_f}{v_i} = -C t_f \quad \ln 3 = C t_f$$

$$\ln \frac{1}{3} = -C t_f \quad t_f = \frac{m}{C} \ln 3$$

ps. reduce mass
(reduce)

PS. 第三定律

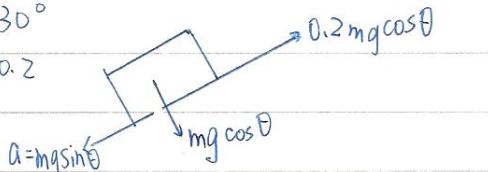
动量守恒



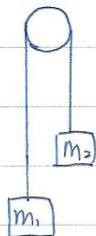
$$\begin{aligned} & m_1 g \sin \theta - m_2 g \sin \theta \\ & = (m_1 - m_2) a \end{aligned}$$

$$\theta = 30^\circ$$

$$\mu_k = 0.2$$



$$\begin{aligned} F &= m g \sin \theta - 0.2 m g \cos \theta \\ &= m g (\sin \theta - 0.2 \cos \theta) \\ &= m g \times 0.3268 \end{aligned}$$



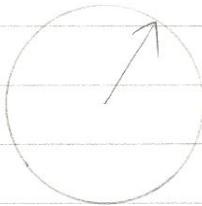
$$m_1 g - m_2 g = (m_1 + m_2) a$$

$$\vec{F}_{12} = -G \frac{m_1 m_2}{(r_{12})^2} \hat{r} \rightarrow \text{地球質量}$$

$$1. \text{重力: } \vec{F} = -G \frac{Mm}{R^2} \hat{R}$$

$$\xrightarrow{\text{表面吸引}} = m (-G \frac{M}{R^2} \hat{R})$$

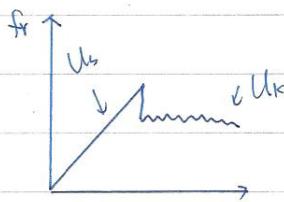
$$\hookrightarrow G = 6.67 \times 10^{-11} \text{ (N} \cdot \text{m}^2/\text{kg}^2)$$



$$2. \text{向心力: 圓周運動 } F = m \frac{v^2}{r} = m r \omega^2$$

3. 摩擦力<靠觀察> 跟材質、正向力...有關

跟運動方向相反 $\vec{F} = -\mu_k(s) \times N$



4. 空氣阻力 (Drag Force) \rightarrow 終端速度

$$F_D = \frac{1}{2} C_P A V^2 \Rightarrow V^2 = \frac{2mg}{\rho C A} \Rightarrow V = \sqrt{\frac{2mg}{\rho C A}}$$

(碰撞時與相對速率、截面積、密度有關)

ex. 雨滴 $m = \frac{4}{3} \pi r^3 \rho A \pi r^2$

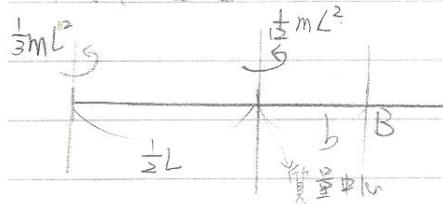
$$V = \sqrt{\frac{2g}{PC} \cdot \frac{4}{3} \pi r^3 / \pi r^2} = \sqrt{\frac{8g}{3PC}} r$$

$$m \leftarrow I = \sum_{i=1}^n m_i r_i^2$$

$\int \lambda d\lambda$

提一通 $d\lambda$ 体积 ρdV
面密度
体积密度

平行軸定理



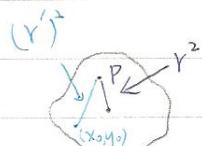
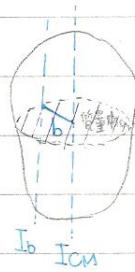
$$\frac{1}{3}mL^2 = \frac{1}{12}mL^2 + m\left(\frac{1}{2}L\right)^2$$

$$I_b = I_{cm} + Mb^2$$

质量中心の转动惯量

$$\text{质量中心定義 } X_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{M}$$

$$\sum m_i x_i = M X_{cm}$$



$$I_b = \sum_{i=1}^n m_i r_i^2$$

$$= \sum_{i=1}^n m_i [(x_i - x_0)^2 + (y_i - y_0)^2]$$

$$= \sum_{i=1}^n m_i [x_i^2 - 2x_i x_0 + x_0^2 + y_i^2 - 2y_i y_0 + y_0^2]$$

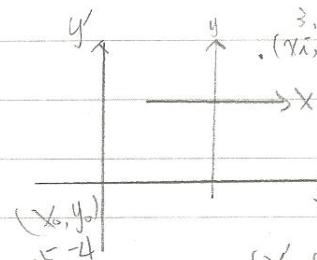
↓
 I_{cm}

$$= \sum_{i=1}^n m_i x_i^2 + \sum_{i=1}^n m_i y_i^2 - 2 \sum_{i=1}^n m_i x_i x_0 - 2 \sum_{i=1}^n m_i y_i y_0$$

$$\Rightarrow x_0 \sum_{i=1}^n m_i x_i + y_0 \sum_{i=1}^n m_i y_i = 0$$

設厚さ為0.

$$\Rightarrow x_0 \cdot M X_{cm} + y_0 \cdot M Y_{cm} = 0$$

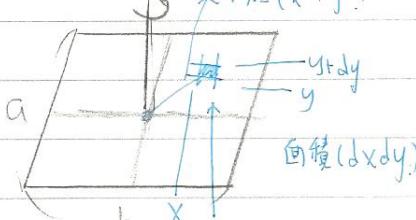


$$(x, y) \rightarrow (x', y')$$

$$x' = x - y \text{ 軸 } x_0, y_0$$

$$y' = y$$

$$r_i = (x_i^2 + y_i^2)$$



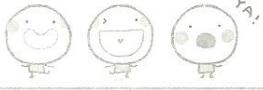
$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} (x^2 + y^2) \rho dx dy$$

$$= \partial b \int_{-\frac{h}{2}}^{\frac{h}{2}} x^2 dx + \partial a \int_{-\frac{b}{2}}^{\frac{b}{2}} y^2 dy$$

$$= \partial b \frac{a^3}{12} + \partial a \frac{b^3}{12} = \partial ab \left(\frac{a^2}{12} + \frac{b^2}{12} \right) = M \left(\frac{a^2}{12} + \frac{b^2}{12} \right)$$

$$\frac{dx^n}{dx} = n x^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

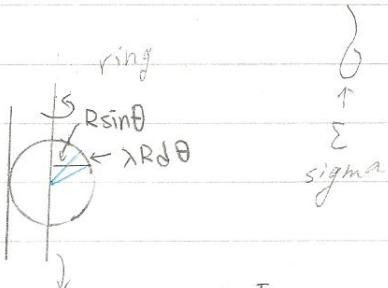


面積 $\lambda R d\theta$ 面積 $= \lambda R d\theta$

$$\int_{\theta=0}^{2\pi} R^2 \lambda R d\theta = \lambda R^3 2\pi = MR^2$$

$I_{cm} = MR^2$

$$I_b = MR^2 + MR^2 = 2MR^2$$



disk

$$\int_{r=0}^R \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \rho (rsin\theta) r d\theta dr = \sum \text{disk}$$

$$\int_{r=0}^R \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} r^3 dr = \pi R^4 = \frac{1}{4} MR^2$$

軸動 Rotational motion

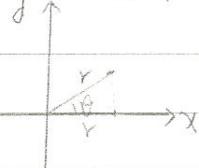
Rigid body 刚体

× 圓周運動一定距向心力

$$\text{向心力} = m \frac{v^2}{r}$$

$$\text{动能} = \frac{1}{2} m v^2$$

切線速率



(x, y, z) θ

$$\vec{v} = \frac{dx}{dt} \hat{x} + \frac{dy}{dt} \hat{y} + \frac{dz}{dt} \hat{z}$$

$$\omega = \frac{d\theta}{dt} \hat{z}$$

角度單位時間內的變化量

$$\vec{F} = \frac{d\vec{p}}{dt}$$

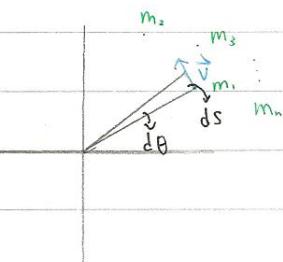
$\vec{p} = m\vec{v}$ 角量

$$E_k = \frac{1}{2} m (\vec{v})^2 = \frac{\vec{p}^2}{2m}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

$$(\alpha = r \times \omega)$$



$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t} = \frac{ds}{dt} = \frac{r d\theta}{dt} = r\omega$$

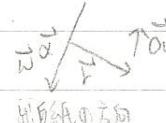
$$\Rightarrow \frac{1}{2} m V^2 = \frac{1}{2} m r^2 \omega^2 = \frac{1}{2} (m r^2) \underline{\underline{\omega^2}}$$

$$\Rightarrow \sum_{i=1}^n \frac{1}{2} m_i v_i^2 \Rightarrow \sum_{i=1}^n \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2} \left(\sum_{i=1}^n m_i r_i^2 \right) \underline{\underline{\omega^2}}$$

↑ 重運動量

$$\vec{T} = \vec{r} \times \vec{F} \quad \text{力矩} = \vec{r} \times m\vec{a} = m\vec{r} \times \vec{a} = (mr^2)\vec{\omega} = I\vec{\omega}$$

$$\vec{L} = \vec{r} \times \vec{p} \quad \text{角動量} = \vec{r} \times m\vec{v} = m\vec{r} \cdot \vec{v} = I\vec{\omega}$$



順時針方向

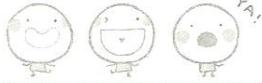
$$\vec{L} = \sum_{i=1}^n \vec{L}_i = \sum_{i=1}^n \vec{r}_i \times \vec{p}_i$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt} \sum_{i=1}^n \vec{r}_i \times \vec{p}_i = \sum_{i=1}^n \frac{d}{dt} (\vec{r}_i \times \vec{p}_i) = \sum_{i=1}^n \left(\frac{d\vec{r}_i}{dt} \right) \times \vec{p}_i + \sum_{i=1}^n \vec{r}_i \times \frac{d\vec{p}_i}{dt}$$

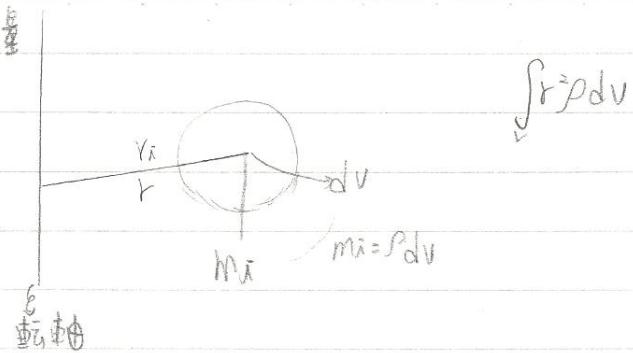
$$\Rightarrow \frac{d\vec{L}}{dt} = \sum_{i=1}^n \vec{v}_i \times \vec{p}_i + \sum_{i=1}^n \vec{r}_i \times \vec{F}_i = \vec{T}$$

$$\therefore \vec{v}_i \times \vec{p}_i = \vec{E}_i$$

$$K.E. = \sum_{i=1}^n \frac{1}{2} m_i V_i^2 = \sum_{i=1}^n \frac{1}{2} m_i r_i^2 \omega^2 = \frac{\omega^2}{2} \sum_{i=1}^n m_i r_i^2 = \frac{I}{2} \omega^2$$



算转动惯量



I - dim rad with constant linear density

l.m

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \lambda dx$$

線性密度

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \lambda dx = \lambda \frac{x^3}{3} \Big|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{\lambda}{3} \left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3 \right]$$

$$= \frac{\lambda}{3} \frac{L^3}{4} = \frac{1}{12} mL^2$$

$\lambda \times L$ = 質量

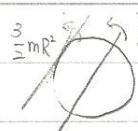
$$I = \int_0^L x^2 \lambda dx = \lambda \frac{x^3}{3} \Big|_0^L = \frac{\lambda}{3} L^3$$

$I = \frac{1}{12} mL^2$ $I' = \frac{1}{3} mL^2$

$$\phi - \Phi = \odot$$

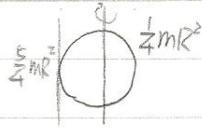
$$H - \Phi = \odot$$

Date 101. 3. 17



$$\frac{1}{2}mR^2$$

半徑為 R

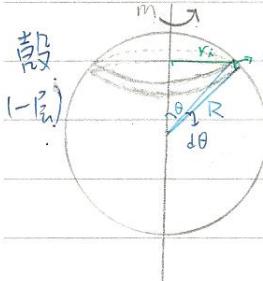


$$\frac{1}{4}mR^2$$

平行軸定理

疊加原理

$$I = \sum m_i r_i^2$$



$$\int_{\theta=0}^{\pi} (R \sin \theta)^2 \rho 2\pi R \sin \theta R d\theta = 2\pi R^4 \rho \int_{\theta=0}^{\pi} \sin^3 \theta d\theta$$

$$= 2\pi R^4 \rho \int_{\theta=0}^{\pi} (\cos^2 \theta - 1) d(\cos \theta)$$

$$= \frac{4}{3} \cdot 2\pi R^4 \rho \left[\frac{1}{3} \cos^3 \theta \right]_{\theta=0}^{\pi} = \frac{4}{3} \cdot 2\pi R^4 \rho \cdot \frac{2}{3} R^2 = \frac{8}{9} \pi R^6 \rho$$

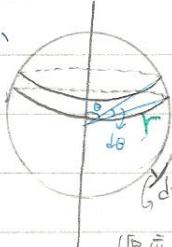
球體質量

$$= \frac{8}{9} m R^6 \text{ 転動慣量}$$

$$d\cos \theta = -\sin \theta d\theta$$

$$= \frac{8}{9} m R^6 \text{ 転動慣量}$$

半径

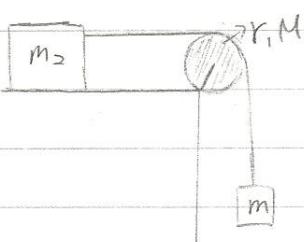


$$\int_{r=0}^R \int_{\theta=0}^{\pi} (rsin\theta)^2 \rho 2\pi rsin\theta r dr d\theta$$

$$= 2\pi \rho \frac{4}{3} \int_{r=0}^R r^4 dr = 2\pi \rho \frac{4}{3} \cdot \frac{1}{5} R^5 = \frac{8}{15} \pi R^5 \rho$$

球體體積

$$\Rightarrow I = \frac{2}{5} m R^2$$



Cylindrical l. R. m $\Omega = \frac{1}{2}$

Shell R. m. $\Omega = \frac{2}{3}$

Solid sphere R. m. $\Omega = \frac{2}{5}$

$\square \cdot m$

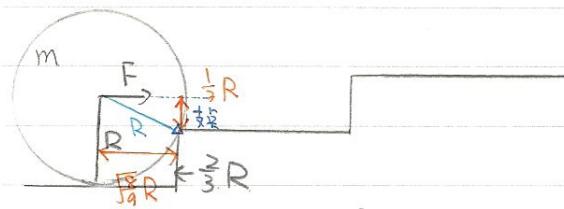
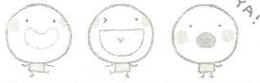
$$\square: mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

$$\square: mgh = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

質量中心線性速度 角速度

$$= \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{V_{cm}}{R}\right)^2$$

$$= \frac{1}{2}(1 + \square) mV_{cm}^2 \Rightarrow V_{cm} = \sqrt{\frac{2}{1+\square} gh}$$



$$\frac{1}{3}RF \geq \frac{\sqrt{8}}{3}Rmg$$

$$F \geq \sqrt{8}mg$$

力 矩

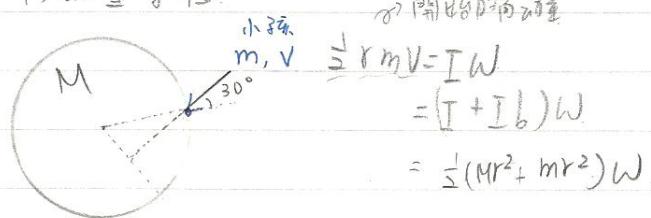
$$\vec{T} = \vec{I} \times \vec{\omega}$$

$$\vec{T} = \vec{r} \times \vec{F}$$

$$R^2 - \frac{1}{3}R^2 = \frac{8}{9}R^2$$

$$\sqrt{5} = \frac{\sqrt{8}}{3}R$$

角動量守恒

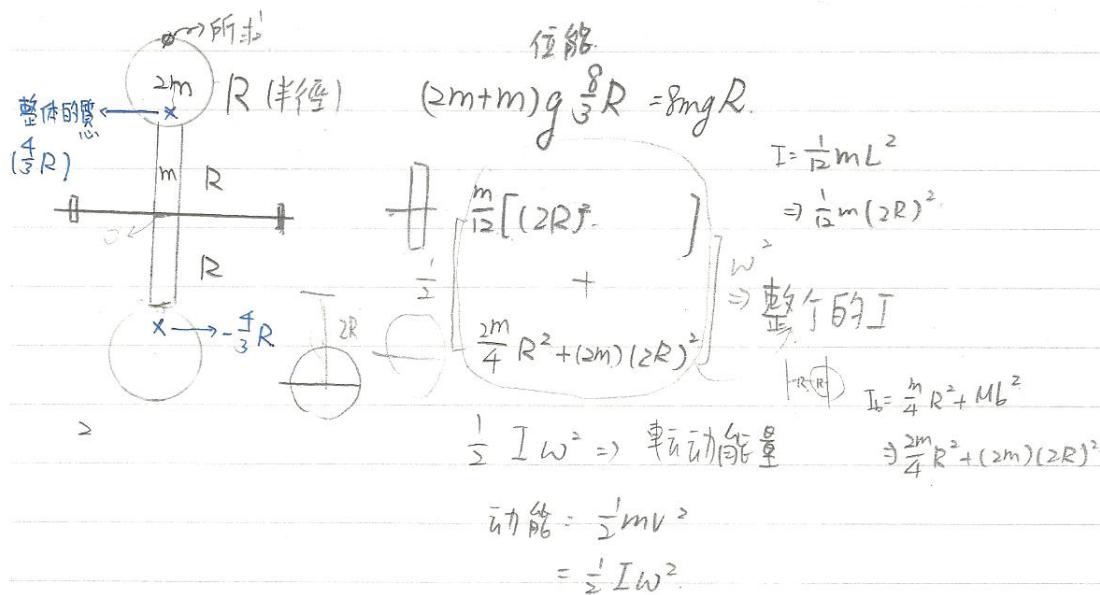


→ 開始的角動量

$$m, v \quad \frac{1}{2}rmv = Iw$$

$$= (I + Ib)w$$

$$= \frac{1}{2}(MR^2 + mr^2)w$$



位能

$$(2m+m)g \frac{8}{3}R = 8mgR.$$

$$I = \frac{1}{2}mL^2$$

$$\Rightarrow \frac{1}{2}m(2R)^2$$

⇒ 整个的 I

$$I_b = \frac{1}{4}R^2 + Mb^2$$

$$\frac{1}{2}Iw^2 \Rightarrow \text{轉動能量}$$

$$\Rightarrow \frac{2m}{4}R^2 + (2m)(2R)^2$$

$$\text{动能} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}Iw^2$$

位能 = 轉動能量

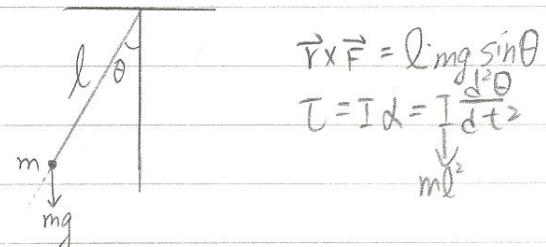
簡諧運動

Simple harmonic oscillation

$$F = -kx$$

$$m \frac{dx}{dt^2}$$

$$m \frac{d^2x}{dt^2} + kx = 0$$

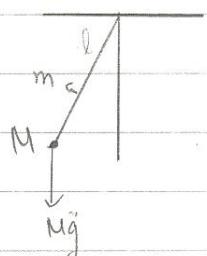


$$\vec{T} \times \vec{F} = I mg \sin\theta$$

$$T = I \alpha = I \frac{d^2\theta}{dt^2}$$

不計繩重

$$ml^2 \frac{d^2\theta}{dt^2} + lmg \sin\theta = 0 \Rightarrow l \frac{d^2\theta}{dt^2} + g\theta = 0$$



$$\text{質心系統}$$

$$l' = \left(\frac{1}{2} I_m + I_M \right) / m + M$$



不規則物体擺動
先找出質心，視為
單擺

赤道 g 值較南北小

$$x(t) = A \cos(\omega t)$$

$$\theta(t) = \Theta_0 \cos(\omega t)$$

$$m(-\omega^2) \cos(\omega t) + k \cos(\omega t) = 0$$

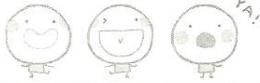
$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \left(\frac{k}{m} \right)^{\frac{1}{2}}$$

$$\omega^2 = \frac{g}{l} \Rightarrow \omega = \left(\frac{g}{l} \right)^{\frac{1}{2}}$$

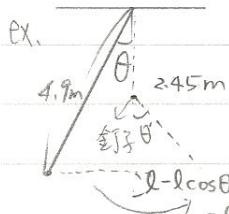
$$\omega T = 2\pi \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

角週期
頻率

$$T = 2\pi \sqrt{\frac{g}{l}}$$



高度差



$$l(1-\cos\theta) = \frac{l}{2}(1-\cos\theta')$$

$$\frac{l}{2} - l\cos\theta = \frac{1}{2}l - \frac{1}{2}l\cos\theta'$$

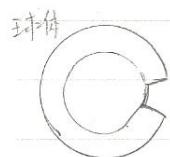
$$2\cos\theta - 1 = \cos\theta'$$

$$= l(1-\cos\theta) T = \pi\sqrt{\frac{l}{g}} + \pi\sqrt{\frac{l}{2g}} \quad (T = 2\pi\sqrt{\frac{l}{g}})$$



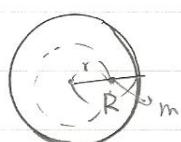
$$-G\frac{M(m)}{r^2}$$

$$-G\frac{M(m)}{r^2} \times 4\pi r^2 = -4\pi GM(m) \Rightarrow \text{重力場總量}$$



$$-4\pi G \sum_{i=1}^n m_i \quad (\text{高斯定理})$$

只要是封閉球體，重力場總量不變。



$$\frac{m(-4\pi G(\frac{4}{3}\pi r^3\rho))}{4\pi r^2} \frac{M}{\frac{4}{3}\pi R^3}$$

$$= -m\frac{GM}{R^3}r \Rightarrow m \text{ 所受的力}$$

$$m\frac{d^2r}{dt^2} + m\frac{GM}{R^3}r = 0$$

$$F = -kx - bx$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\frac{de^{i\theta}}{d\theta} = ie^{i\theta}$$

$$x(t) = Ae^{at}$$

$$\Rightarrow mA(a^2)e^{at} + bA(a)e^{at} + kAe^{at} = 0$$

$$(m\omega^2 + b\alpha + k)Ae^{at} = 0$$

↑ 頭↑
" 積分 ↑
0 0 0

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

$$= -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$

$$x(t) = Ae^{-\frac{b}{2m}t} \cos(\omega t)$$

$$\frac{b^2}{4m^2} \ll \left| -\frac{k}{m} \right| \Rightarrow \omega^2 \ll 4km$$

$$-\left(\frac{k}{m} - \left(\frac{b}{2m}\right)^2\right)$$

$$\omega = \sqrt{\left(\frac{k}{m} - \left(\frac{b}{2m}\right)^2\right)}$$

$$\boxed{\frac{d^2y}{dx^2} + xy = 0}$$

Date 4月 4 14

$$F = -kx - bx \Rightarrow m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$F = -kx - bx + F \cos \omega_0 t \Rightarrow m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F \cos \omega_0 t$$

$$\begin{aligned} & -\text{微分 } F \omega_0 (-\sin \theta) \omega_0 t \\ & = \text{微分 } F (\omega_0)^2 \cos \theta t \end{aligned}$$

电阻.

$$L \frac{d^2\theta}{dt^2} + R \frac{d\theta}{dt} + \frac{Q}{C} = 0$$

↓
电量
↓
电容

$$L \frac{d^2\theta}{dt^2} + R \frac{d\theta}{dt} + \frac{Q}{C} = A \cos \omega t.$$

Date 2012. 5. 12 物

熱力学

玻茲曼常數

統計 $S = k \ln S_L$ $\Delta S > 0$
 Boltzman
 Bose-Einstein
 Fermi-Dirac

熱力学第0定律 ① (A) (B) 指觸後平衡溫度為 T_{AB}

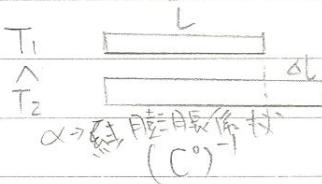
$$T_A > T_B$$

$T_A > T_{AB} > T_B \Rightarrow$ 热由高溫到低溫

$$\text{② } T_A = T_B \Rightarrow T_A = T_C \quad \text{(A) (B)}$$

$$T_B = T_C \quad \text{(C)}$$

熱脹冷縮



$$\Delta L = \alpha L (T_2 - T_1)$$

coefficient of linear expansion

$$Al \quad 23 \times 10^{-6}$$

$$Brass \quad 19 \times 10^{-6}$$

$$\text{Concrete} \quad 12$$

$$\text{Copper} \quad 17$$

$$Pb \quad 29$$

$$\text{Glass (common)} \quad 8.5$$

$$Ni \quad 13$$

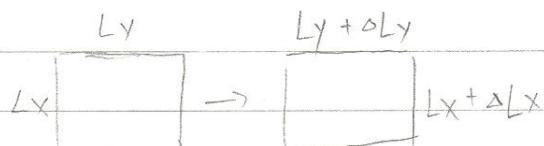
$$\text{Glass (Pyrex)} \quad 3.3$$

$$\text{Quartz (fused)} \quad 0.5^\circ$$

$$Au \quad 14$$

$$Ag \quad 19$$

$$Fe \quad 12$$



$$\Delta S = (L_x + \Delta L_x)(L_y + \Delta L_y) - L_x L_y$$

$$= \Delta L_x L_y + \Delta L_y L_x + \underline{\Delta L_x \Delta L_y}$$

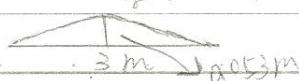
$$= \Delta L_x L_y + \Delta L_y L_x \quad (T_2 - T_1)$$

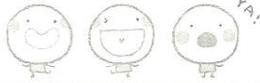
$$L_x L_y = S$$

$$= 120 S (T_2 - T_1)$$

B 面積膨脹係數

concrete膨脹
~3.00047





Bronze \rightarrow Steel

γ 体積膨脹係數 ($\times 10^{-4}$)

$\Delta t = 20^\circ$

汽油 = 950

$$Q = c m \Delta T$$

熱量 比熱

比熱 (單位: J/kg)

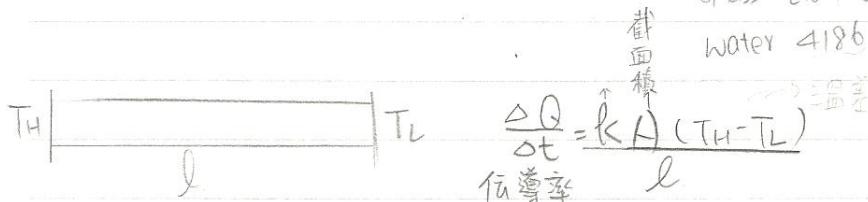
Al 9.00×10^3

Cu 387

Al $137^\circ C$ 3500

Glass 2.05×10^3

Water 4186



k (J/s.m.°C)

Al 240 Air 0.0256

Brass 110 H₂ 0.180

Copper 390 N₂ 0.0258

Glass 0.80 O₂ 0.0265

ice (0°C) 2.2 Diamond 2450

Body fat 0.20 Wool 0.040

water 0.60

$$\frac{k_A A (T_H - T_x)}{l_A} = \frac{k_B A (T_x - T_L)}{l_B}$$

$$T_x (k_A l_B + k_B l_A) = k_A l_B T_H + k_B l_A T_L$$

$$T_x = \frac{k_A l_B T_H + k_B l_A T_L}{k_A l_B + k_B l_A}$$

$$\frac{\Delta Q}{\Delta t} = \frac{k_A (T_H - T_x)}{l_A} = \frac{k_A T_H - k_A \frac{k_B l_B T_H + k_B l_A T_L}{k_A l_B + k_B l_A}}{l_A}$$