Boost invariant formulation of the chiral kinetic theory

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In cooperation with Kenji Fukushima,

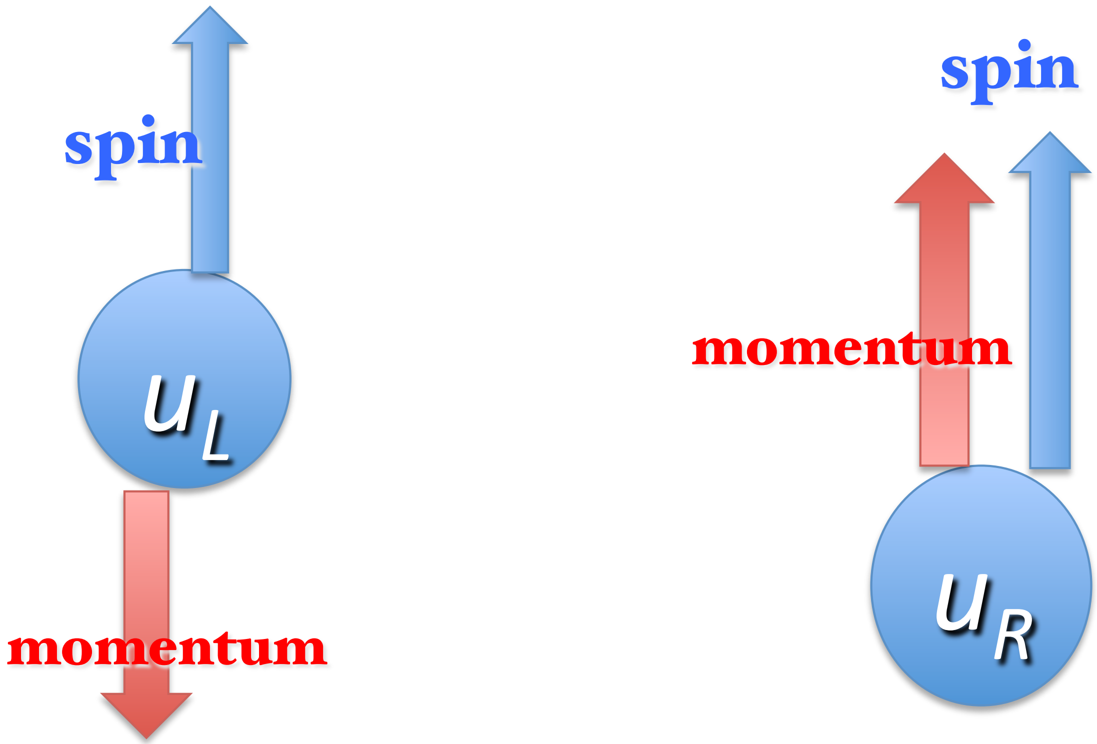
Shu Ebihara, Phys.Rev. D96 (2017), 016016

Outline

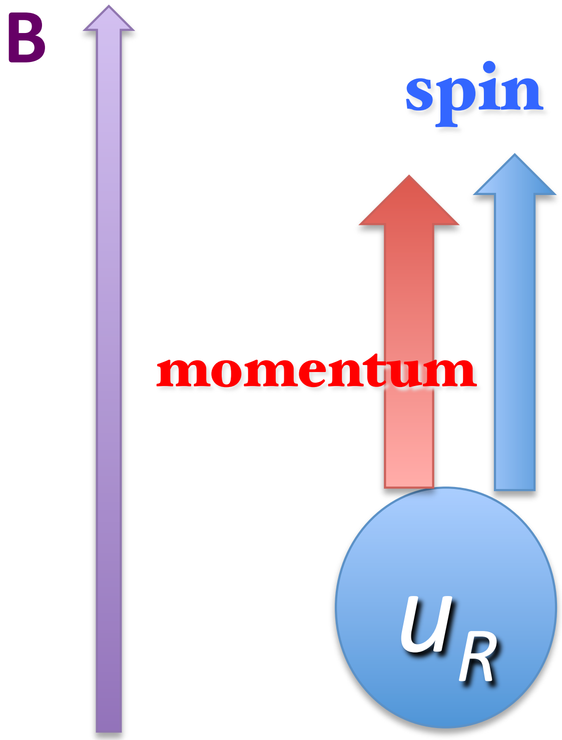
- Introduction to Chiral kinetic theory (CKT)
- CKT with Bjorken expansion
- Summary

Introduction to Chiral kinetic theory

Chirality and massless fermions



Chiral Magnetic Effect (I)



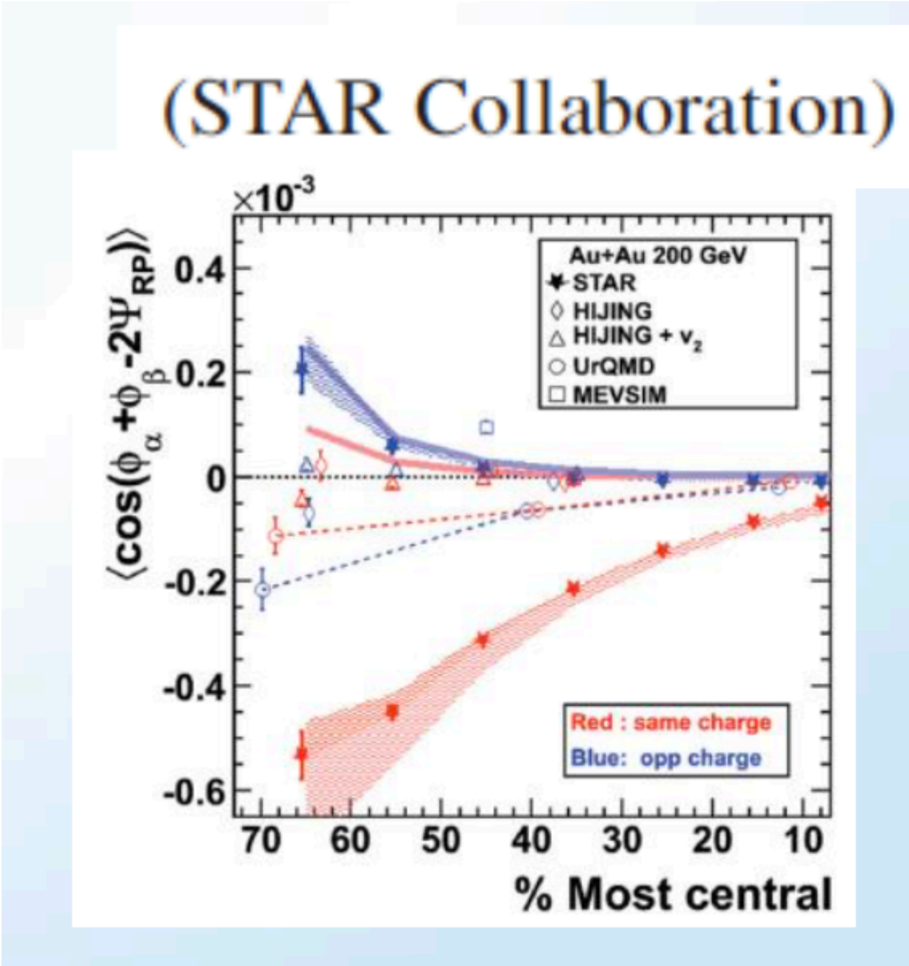
- Magnetic fields
- Nonzero axial chemical potential
  - Number of Left handed fermions ≠ Number of Right handed fermions
- Charge current

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B},$$

Ref: Kharzeev, Fukushima, Warrigna, (08,09), etc. ... Cf. Dmitri's Talk

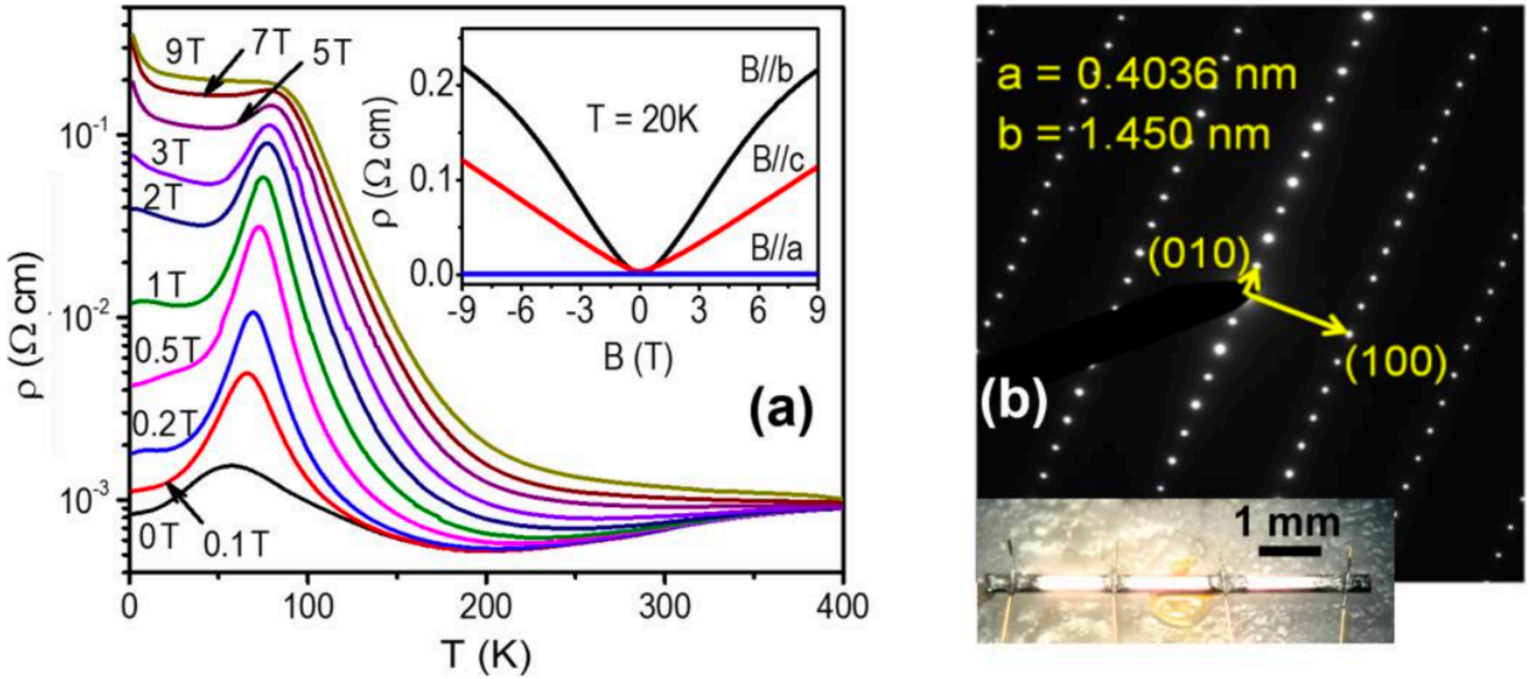
Chiral Magnetic Effect (II)

- Heavy ion collisions: charge separation



Chiral Magnetic Effect (II)

- Heavy ion collisions: charge separation
- Weyl Semi-metal: new transport effects



$ZrTe_5$: arXiv:1412.6543, **Nature Physics (2016) doi:10.1038/nphys3648**

Chiral Magnetic Effect (II)

- Heavy ion collisions: charge separation
- Weyl Semi-metal: new transport effects
- To the real world: New type of "superconductor"?

"Dissipationless transmitting and processing information and energy", Q. Li (BNL), Chiral Matter 2016, RIKEN

Kinetic theory

- “distribution function” f(x,p,t)

- Ordinary kinetic theory: Boltzmann equation

- Spin effects, massless particles
  SP, J.H. Gao, Q. Wang, PRD 83 (2011) 094017

Chiral kinetic equation (I)

- Hamiltonian formulism, effective theory

  Son, Yamamoto, PRL, (2012); PRD (2013)
- Path integration

  Stephanov, Yin, PRL (2012);

  Chen, Son, Stephanov, Yee, Yin, PRL, (2014);

  J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD (2014)
- Wigner function
  - hydrodynamics, equilibrium

    J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);
  - out-of-equilibrium, quantum field theory

    Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017) **Cf. Di-Lun's Talk**

Ordinary Boltzmann equation

$$\partial_t f + \dot{\mathbf{x}} \cdot \nabla_x f + \dot{\mathbf{p}} \cdot \nabla_p f = C[f],$$

- Particle's velocity:

$$\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}}, \qquad \varepsilon : \text{Particle's energy}$$

- Lorentz force:

$$\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B},$$

- Collision effects:

C[f]

Chiral kinetic equation

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\mathbf{x}}\cdot\nabla_x f + \sqrt{G}\dot{\mathbf{p}}\cdot\nabla_p f = C[f].$$

- Particle's velocity:

$$\sqrt{G}\dot{\mathbf{x}} = \frac{\partial\varepsilon}{\partial\mathbf{p}} + \hbar\left(\frac{\partial\varepsilon}{\partial\mathbf{p}}\cdot\mathbf{\Omega}\right)\mathbf{B} + \hbar\mathbf{E}\times\mathbf{\Omega},$$

- External force:

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial\varepsilon}{\partial\mathbf{p}}\times\mathbf{B} + \hbar(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega},$$

- Berry curvature

$$\sqrt{G} = 1 + \hbar\mathbf{B}\cdot\mathbf{\Omega}, \qquad \mathbf{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3},$$

Lorentz symmetry and side-jump (I)

- 3D action with Berry phase

  Chen, Son, Stephanov, Yee, Yin, PRL, (2014)

  – **Infinitesimal Lorentz Transform**

$$\mathbf{x}' = \mathbf{x} + \boldsymbol{\beta} t + \delta \mathbf{x}, \qquad \delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$

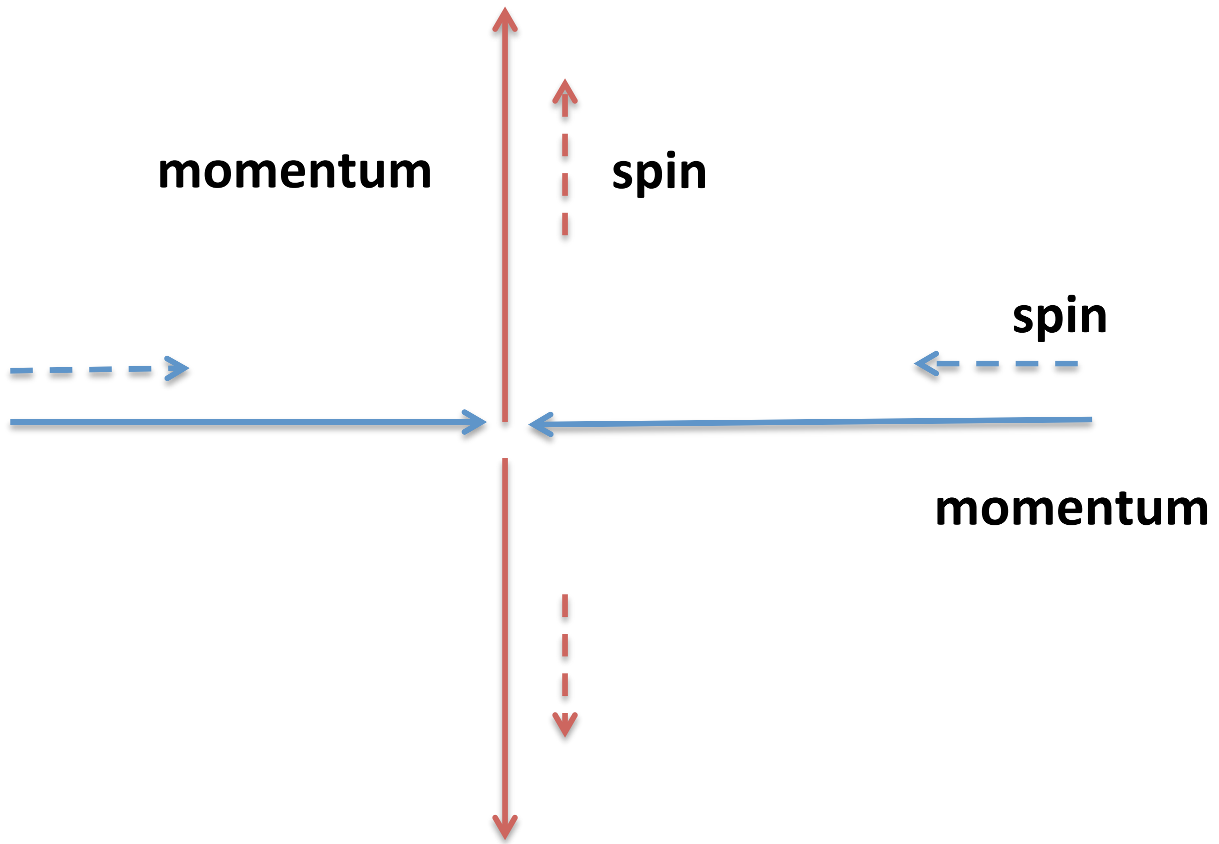
$$\mathbf{p}' = \mathbf{p} + \boldsymbol{\beta} \varepsilon + \delta \mathbf{p}, \qquad \delta \mathbf{p} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

- Global angular momentum conservation

  Chen, Son, Stephanov, PRL, (2015)

Side-jump (I)

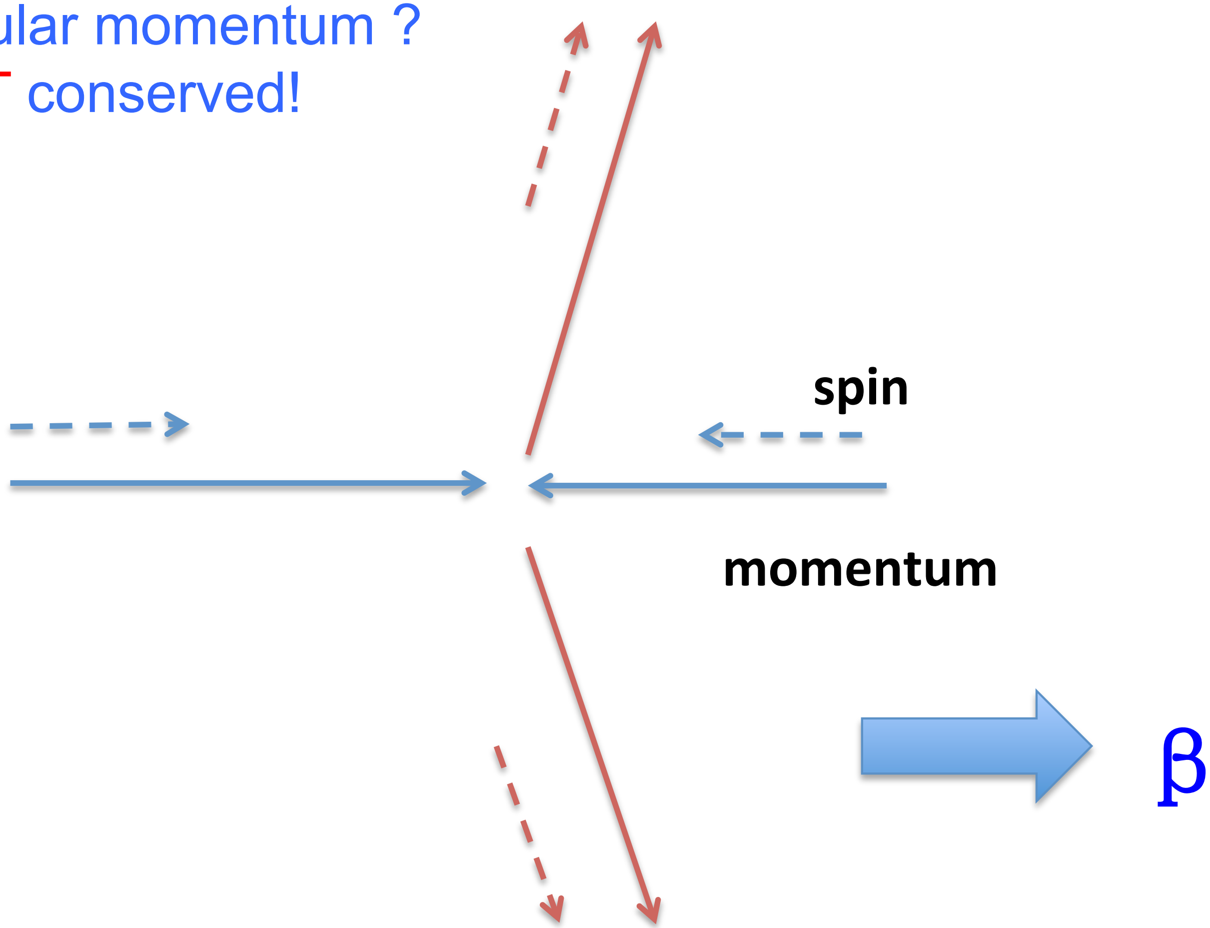
Orbital angular momentum and spin are conserved separately.



Chen, Son, Stephanov, Yee, Yin, PRL, (2014)

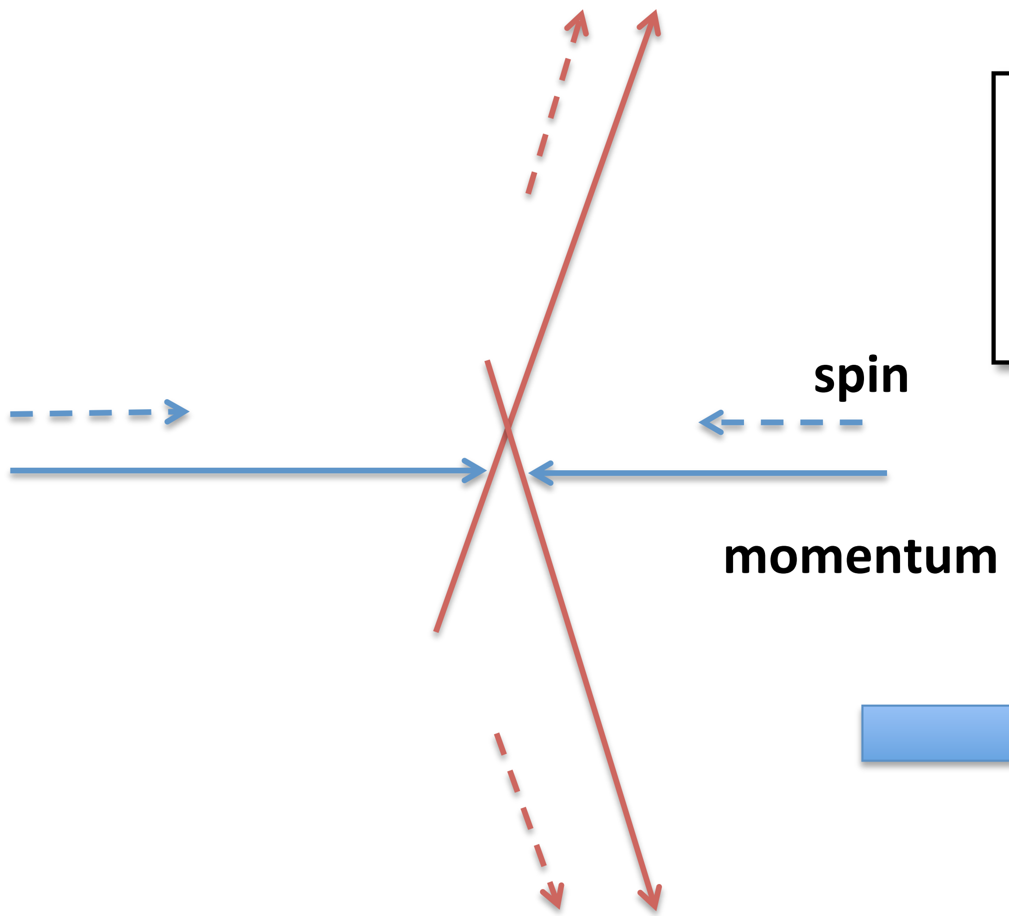
Side-jump (II)

Orbital angular momentum ?
Spin is NOT conserved!



Chen, Son, Stephanov, Yee, Yin, PRL, (2014)

Side-jump (III)



x has a shift!!!

$$
\begin{aligned}
\mathbf{x}' &= \mathbf{x} + \boldsymbol{\beta} t + \delta \mathbf{x}, \\
\mathbf{p}' &= \mathbf{p} + \boldsymbol{\beta} \varepsilon + \delta \mathbf{p},
\end{aligned}
$$

$$
\begin{aligned}
\delta \mathbf{x} &= \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|}, \\
\delta \mathbf{p} &= \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}
\end{aligned}
$$

→ β

Chen, Son, Stephanov, Yee, Yin, PRL, (2014)

Lorentz symmetry and side-jump (II)

- Field theory

$$j^{\mu} = \overline{\psi}\sigma^{\mu}\psi \to \Lambda^{\mu}_{\nu} j^{\nu}$$

- Lorentz transformation,

$$x^{\mu\prime} = \Lambda^{\mu}_{\nu} x^{\nu}, \; p^{\mu\prime} = \Lambda^{\mu}_{\nu} p^{\nu},$$

$$\boxed{f'(x', p', t') = f(x', p', t') + \hbar N^{\mu}(\partial^{x}_{\mu} + F_{\nu\mu}\partial^{\nu}_{p})f,}$$

Infinitesimal Lorentz Transform

$$\delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$

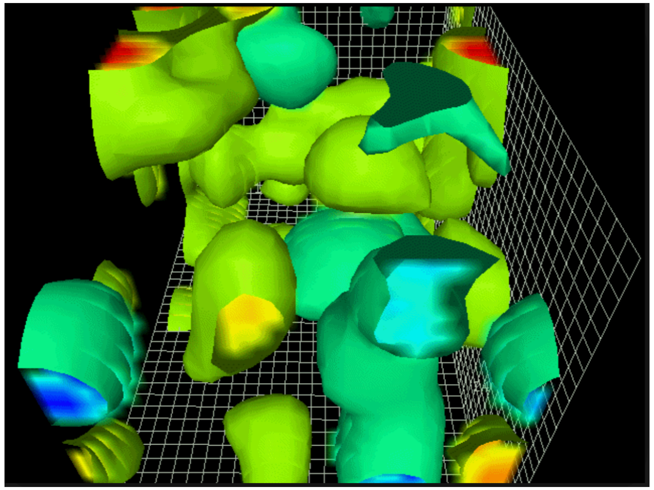
$$\delta \mathbf{p} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

Chen, Son, Stephanov, PRL, (2015); Y. Hidaka, SP, D.L. Yang, (2016)
Cf. Di-Lun's talk

Chiral kinetic theory + Bjorken expansion

In cooperation with Kenji Fukushima,
Shu Ebihara, Phys.Rev. D96 (2017), 016016

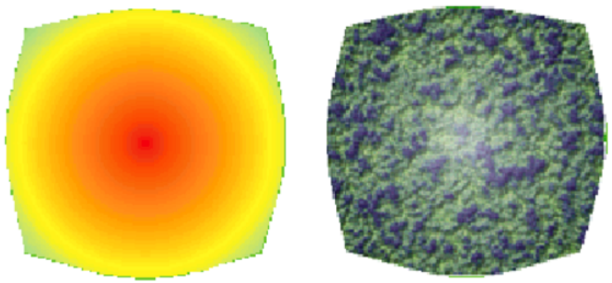
CME at early stage



Topological charges
classical statistical simulation

?

Chiral kinetic equation

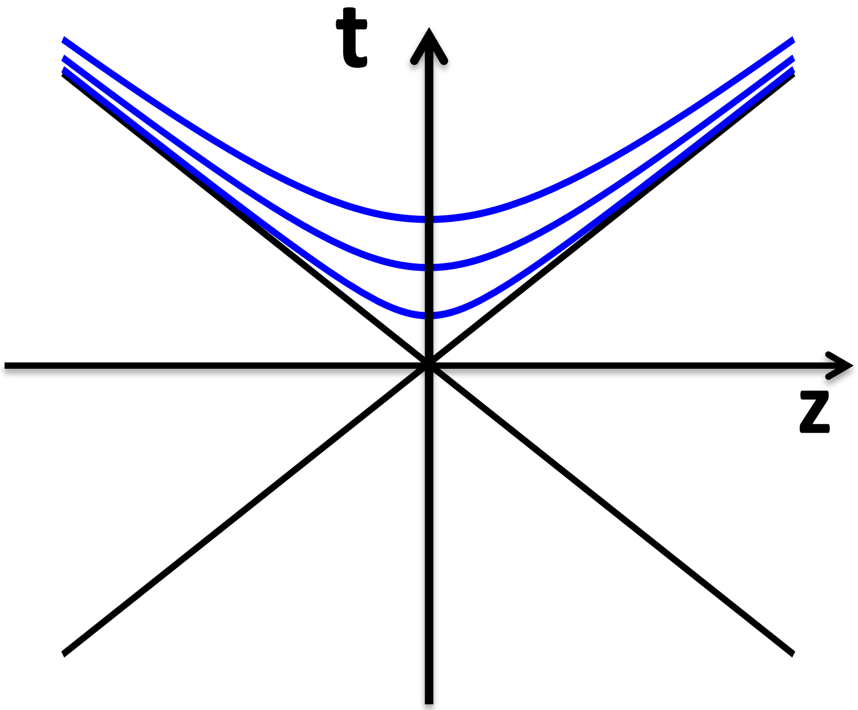


Chiral Magneto-hydrodynamics

CKT Simulations

- A. Huang, Y. Jiang, S. Shi, J. Liao, and P. Zhuang, arXiv:1703.08856
- Y. Sun, C. M. Ko, and F. Li, Phys. Rev. C94, 045204 (2016); Y. Sun and C.M. Ko Phys. Rev. C95, 034909 (2017)

Bjorken boost invariance (I)



- Central rapidity regime
- Longitudinal boost Invariance
- Proper time

$$\tau = \sqrt{t^2 - z^2}.$$

- Bjorken velocity

$$\beta_z = \frac{z}{t}$$

Bjorken boost invariance (II)

- Dirac equation

$$i\left(\gamma^0\partial_\tau + \frac{\gamma^z}{\tau}D_\eta + \gamma^i D_i\right)\hat{\psi} = 0$$

  – τ (proper time) and η (space rapidity) coordinates
  – Gauge:

$$A_\tau = 0$$

  – Spinor basis

$$\tilde{\psi} \to \tau^{1/2} e^{-\frac{\eta}{2}\gamma^0\gamma^z}\psi$$

Bjorken boost invariance (II)

- Boost invariance
  - **Assuming no η (space rapidity) dependence in gauge field**
  - Fourier transform

$$\tilde{\psi} \rightarrow e^{\mp i p_\perp \tau \cosh(y-\eta)} \chi_\mp$$

  - y: momentum rapidity, η: space rapidity

$$\left(\partial_\tau \mp i p_\perp e^{-\sigma^3 (y-\eta)} + i \frac{\sigma^3}{\tau} g A_\eta + \sigma^i D_i\right) \chi^R_\mp(\tau, \boldsymbol{x}_T; y-\eta) = 0$$

- spinor depends on y-η

F. Gelis, K. Kajantie, and T. Lappi, Phys. Rev. C71, 024904 (2005)

F. Gelis and N. Tanji, JHEP 02, 126 (2016)

S. Ebihara, K. Fukushima, and SP, Phys.Rev. D96 (2017), 016016

Ordinary boost invariant kinetic theory

- Central rapidity

$$z \simeq 0,$$

- Longitudinal boost invariance (1+1 D expansion)

$$\boxed{f(z, p_z, t) = f(z = 0, p_z - \beta_z |\mathbf{p}|, \tau)}$$

$$\tau = \sqrt{t^2 - z^2}, \ \beta_z = z/t.$$

G. Baym, PLB (1984)

Ordinary boost invariant kinetic theory

- Kinetic theory:

$$\left(\frac{\partial}{\partial t}+\frac{p_z}{|\mathbf{p}|}\partial_z\right)f(z,p_z,t)=C[f]$$

$$\Rightarrow \left(\frac{\partial}{\partial t}-\frac{p_z}{t}\frac{\partial}{\partial p_z}\right)f(z=0,p_z-\beta_z|\mathbf{p}|,\tau)=C[f]$$

- Particle number conservation:

$$\boxed{\frac{\partial}{\partial t}n+\frac{\partial}{\partial z}\left(\frac{z}{t}n\right)=0}$$

Charge density

$$n=\int d^3p f$$

System is expanded.

Boost invariance chiral kinetic theory (I)

- Central rapidity

$$z \simeq 0,$$

- Longitudinal expansion, homogenous in transverse direction $\mathbf{x}_\perp = (x, y),\ \mathbf{p}_\perp = (p_x, p_y)$

$$f(z, \mathbf{x}_\perp, p_z, \mathbf{p}_\perp, t) = f(z = 0, \mathbf{x}_\perp, p_z - \beta_z|\mathbf{p}|, \mathbf{p}_\perp, \tau) + \hbar\delta\mathbf{x} \cdot \frac{\partial}{\partial \mathbf{x}} f + \hbar\delta\mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} f$$

$$\delta\mathbf{x} = \hbar\frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$

$$\delta\mathbf{p} = \hbar\frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

$$\boldsymbol{\beta} = \frac{z}{t}\mathbf{e}_z$$

Boost invariance chiral kinetic theory(II)

We consider the Right-handed fermions only.

$$
\begin{aligned}
&(1+\boldsymbol{B}\cdot\boldsymbol{\Omega})\frac{\partial f}{\partial t}+\Bigg[(1+2\boldsymbol{B}\cdot\boldsymbol{\Omega})\hat{\boldsymbol{p}}+\boldsymbol{E}\times\boldsymbol{\Omega}\\
&\quad-\frac{p_z}{t}\boldsymbol{e}_z\times\boldsymbol{\Omega}\Bigg]_\perp\cdot\frac{\partial f}{\partial \boldsymbol{x}_\perp}+\Bigg[\boldsymbol{E}+(1+2\boldsymbol{B}\cdot\boldsymbol{\Omega})\hat{\boldsymbol{p}}\times\boldsymbol{B}\\
&\quad+(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}-\frac{B_z p_z}{t}\boldsymbol{\Omega}\\
&\quad-\frac{p}{t}\left[\hat{p}_z+(\boldsymbol{E}\times\boldsymbol{\Omega})_z\right]\boldsymbol{e}_z\Bigg]\cdot\frac{\partial f}{\partial \boldsymbol{p}}=0\,.
\end{aligned}
$$

Without external fields

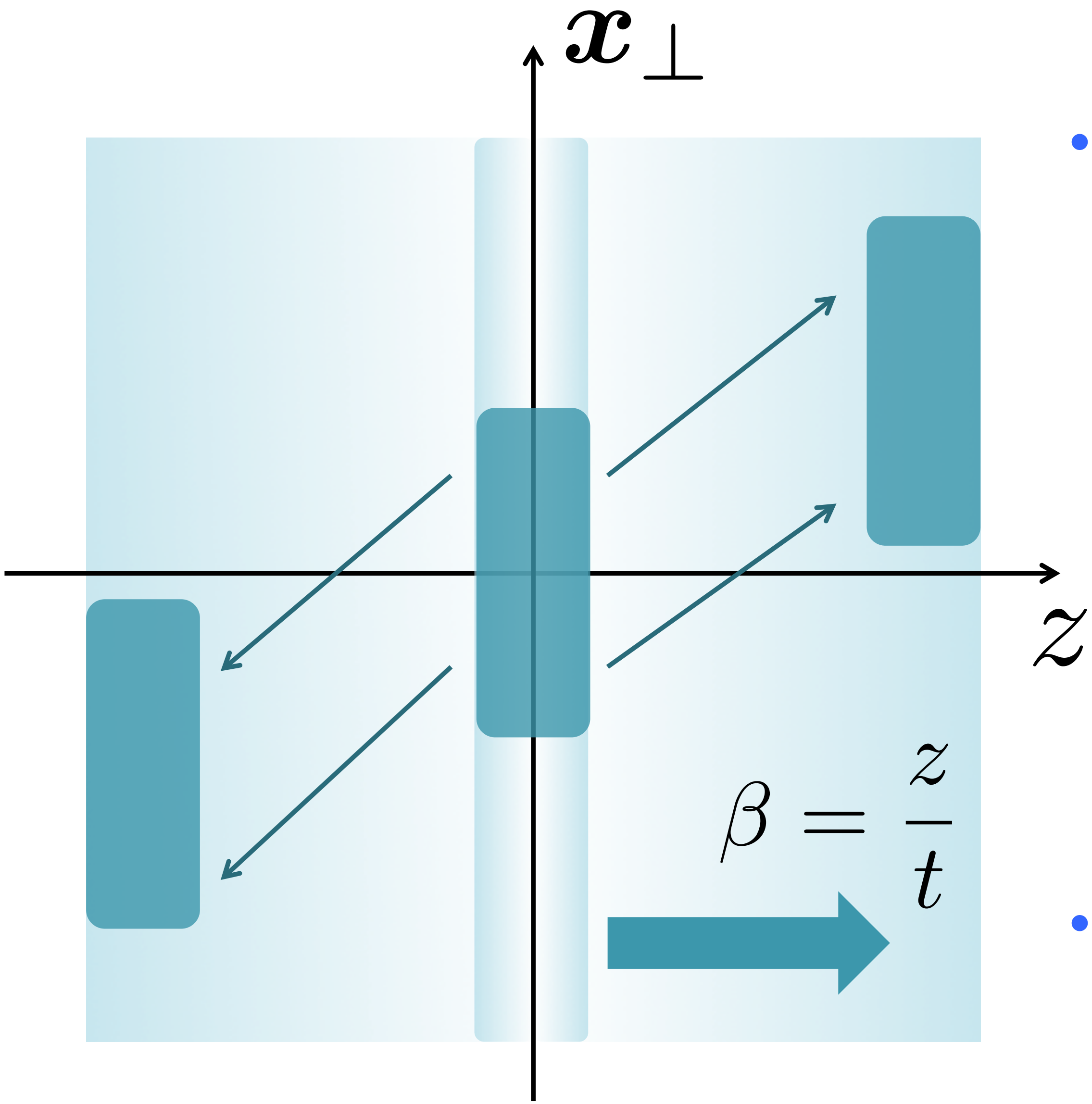
$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial z}\left(\frac{z}{t}n\right)+\boldsymbol{\nabla}_{\perp}\cdot\tilde{\boldsymbol{j}}_{\perp}=0\,,$$

- Particle number

$$n\equiv\int_{\boldsymbol{p}}f \qquad\qquad \perp=(x,y)$$

- $$\tilde{\boldsymbol{j}}_{\perp}\equiv\int_{\boldsymbol{p}}\left\{\hat{\boldsymbol{p}}+\hbar\left[-\frac{p_z}{t}\boldsymbol{e}_z\times\boldsymbol{\Omega}\right]\right\}_{\perp}f\,.$$

Measurement with side-jump



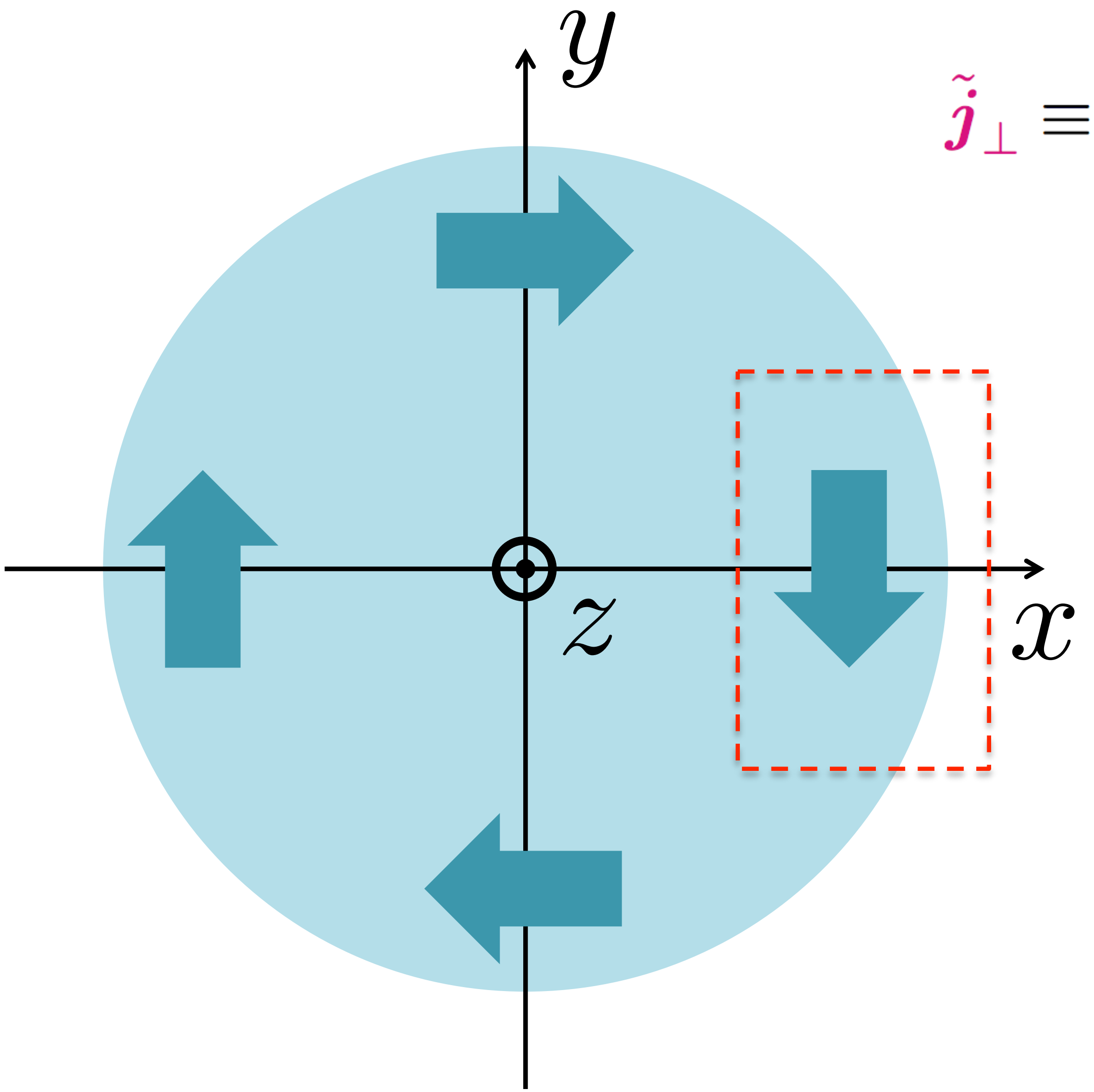
- When we boost f(z) back to z=0, because of side-jump,

$$\delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$

The media in transverse plane is required.

- It is new!

Chiral circular displacement



$$\tilde{\boldsymbol{j}}_\perp \equiv \int_{\boldsymbol{p}} \left\{ \hat{\boldsymbol{p}} + \hbar \left[-\frac{p_z}{t} \boldsymbol{e}_z \times \boldsymbol{\Omega} \right] \right\}_\perp f \,.$$

- Assuming:

$$z > 0,\ x \gg y,$$

$$\Longrightarrow p_y \sim 0,\ p_x, p_z > 0,$$

$$\boxed{-\frac{p_z}{t}(\mathbf{e}_z \times \frac{p_x}{2|\mathbf{p}|^3})_y < 0}$$

Analogy to Thomas precession (I)

- Thomas precession:

  For an accelerated object, besides the standard Lorentz boost, it is rotating in lab frame.

$$\boldsymbol{\omega}_T = -\frac{\gamma^2}{\gamma+1}\mathbf{v}\times\frac{d\mathbf{v}}{dt}$$

- e.g. Thomas half

  correction to spin-orbital interactions between electron and nucleus in hydrogenic atoms.

Analogy to Thomas precession (II)

- 4-velocity form: vorticity (!!!)

$$\boldsymbol{\omega}_T = -\frac{\gamma^2}{\gamma+1}\mathbf{v} \times \frac{d\mathbf{v}}{dt}.$$

$$\Rightarrow \boxed{\frac{1}{\gamma+1}\omega^{\mu} = \frac{1}{\gamma+1}\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_{\nu}\partial_{\alpha}u_{\beta}}$$

- Is Chiral circular displacement (or side-jump) related to the Thomas precession (or vorticity)?

Particle number conservation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}\left(\frac{z}{t}\tilde{n}\right) + \boldsymbol{\nabla}_{\perp} \cdot \tilde{\boldsymbol{j}}_{\perp} = \frac{\boldsymbol{E} \cdot \boldsymbol{B}}{(2\pi\hbar)^2} f(\boldsymbol{p} = 0)$$

- number density

$$n \equiv \int_{\boldsymbol{p}} (1 + \hbar \boldsymbol{B} \cdot \boldsymbol{\Omega}) f$$

- $$\tilde{n} \equiv n - \hbar \int_{\boldsymbol{p}} [2\hat{p}_z(\boldsymbol{E} \times \boldsymbol{\Omega})_z + \boldsymbol{B}_{\perp} \cdot \boldsymbol{\Omega}_{\perp}] f \ ,$$

CME, anomalous Hall current
+ Expansion in z direction

- $$\tilde{\boldsymbol{j}}_{\perp} \equiv \int_{\boldsymbol{p}} \left\{ \hat{\boldsymbol{p}} + \hbar \left[(2\boldsymbol{B} \cdot \boldsymbol{\Omega})\hat{\boldsymbol{p}} + \boldsymbol{E} \times \boldsymbol{\Omega} - \frac{p_z}{t} \boldsymbol{e}_z \times \boldsymbol{\Omega} \right] \right\}_{\perp} f \ .$$

CME — $(2\boldsymbol{B} \cdot \boldsymbol{\Omega})\hat{\boldsymbol{p}}$

Anomalous Hall current — $\boldsymbol{E} \times \boldsymbol{\Omega}$

Side-jump — $\frac{p_z}{t} \boldsymbol{e}_z \times \boldsymbol{\Omega}$

Summary

Summary

- Boost invariant formulation of the chiral kinetic theory
- Chiral circular displacement
- Analogy to Thomas precession (vorticity)

Thank you!