Spontaneous symmetry breaking in open systems

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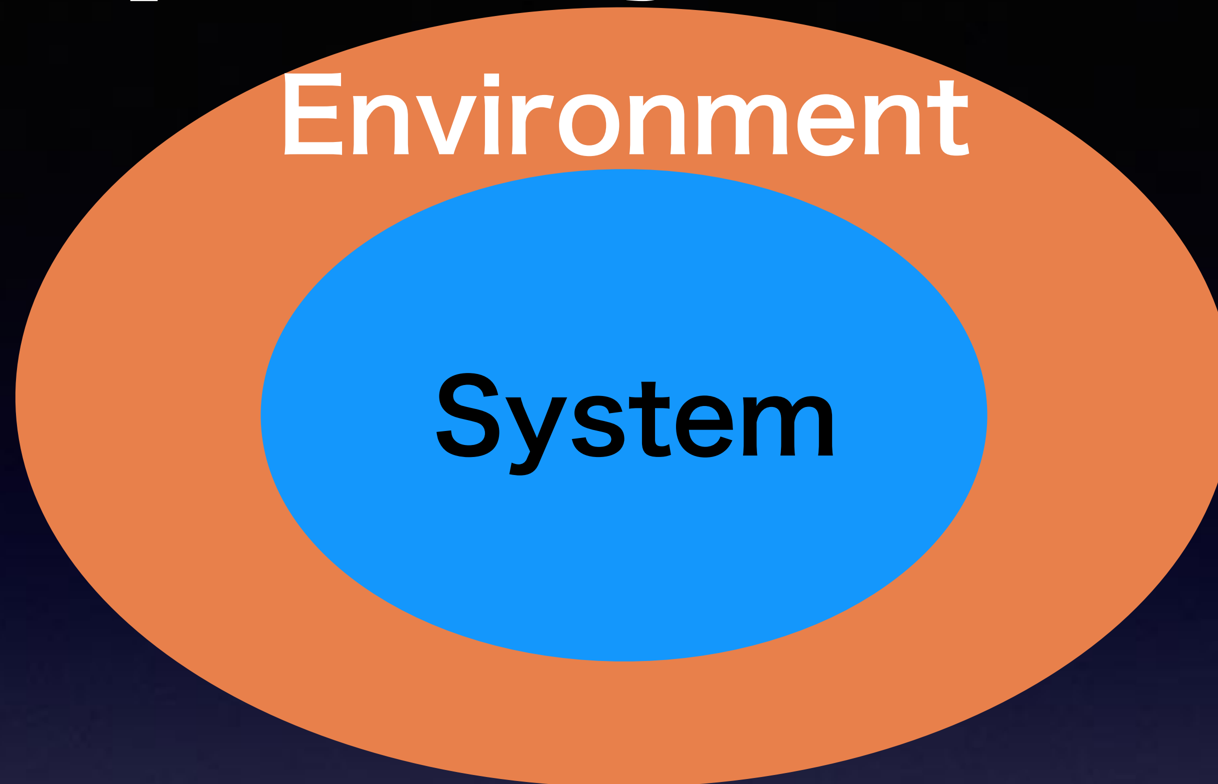
1509.05042[cond-mat.stat-mech], 1712.xxxx

Open systems

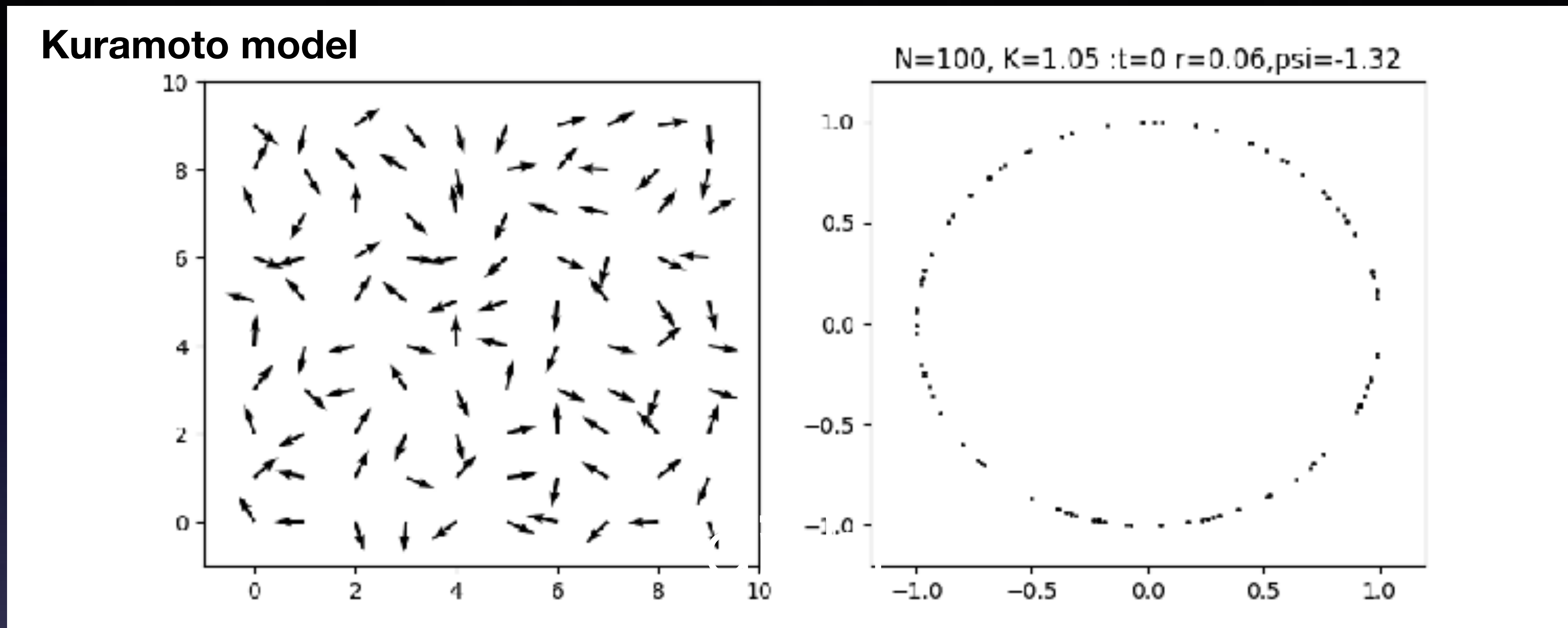


Environment: QGP
System: Heavy quarks

Environment: Air
System: flock of birds

Symmetry breaking in open systems

Synchronization



Metronome, fireflies, …

Driven dissipative condensate

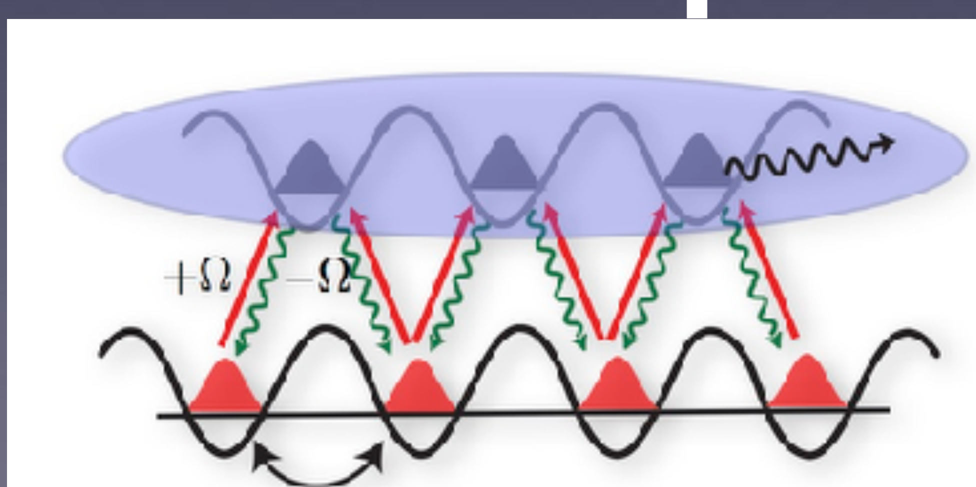


figure is taken from Diehl's website

Driving force and dissipative causes a condensate.

Diehl, Micheli, Kantian, Kraus, Büchler, Zoller, Nature Physics 4, 878 (2008);
Kraus, Diehl, Micheli, Kantian, Büchler, Zoller, Phys. Rev. A 78, 042307 (2008).

Questions

Hamiltonian systems

Continuum symmetry ⟺ $\partial_\mu J^\mu = 0$

Open systems

$\partial_\mu J^\mu \neq 0$ because of friction

What is the symmetry?

Is there any symmetry breaking?

Does a NG mode appear?

Nambu-Goldstone theorem

Nambu('60), Goldstone(61), Nambu Jona-Lasinio('61), Goldstone, Salam, Weinberg('62).

For Lorentz invariant vacuum
Spontaneous breaking of global symmetry

⬇

$$N_{NG} = N_{BS}$$

# of NG modes — # of broken symmetries

Dispersion relation: $\omega = c|\boldsymbol{k}|$

Gapless modes in nature

Need generalization

We focus on open systems

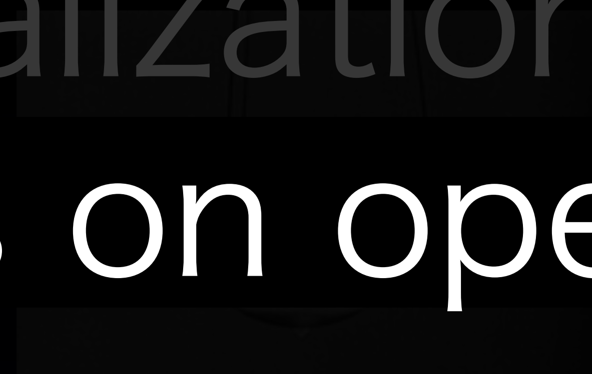
NG theorem OK

pion

chiral symm.

superfluid phonon

U(1) symm.



spin waves

spin symm.

modes

photon

U(1) 1-form symm.

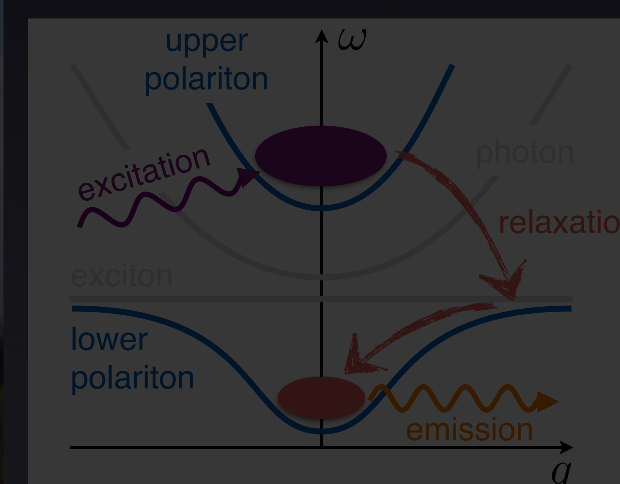
surface waves

translation symm.

diffusive modes

rotation symm.

U(1) symm.



Classification of Nambu-Goldstone modes in Hamiltonian system

Exception of NG theorem

NG modes with $N_{\rm BS} \neq N_{NG}$ and $\omega \neq k$ exist

NG modes in Kaon condensed CFL phase

Miransky, Shovkovy ('02) Schafer, Son, Stephanov, Toublan, and Verbaarschot ('01)

$$SU(2)_I \times U(1)_Y \to U(1)_{\rm em}$$

$$N_{\rm BS} = 3, \quad N_{\rm NG} = 2$$

Dispersion: $\omega \propto k$ and $\omega \propto k^2$

Magnon

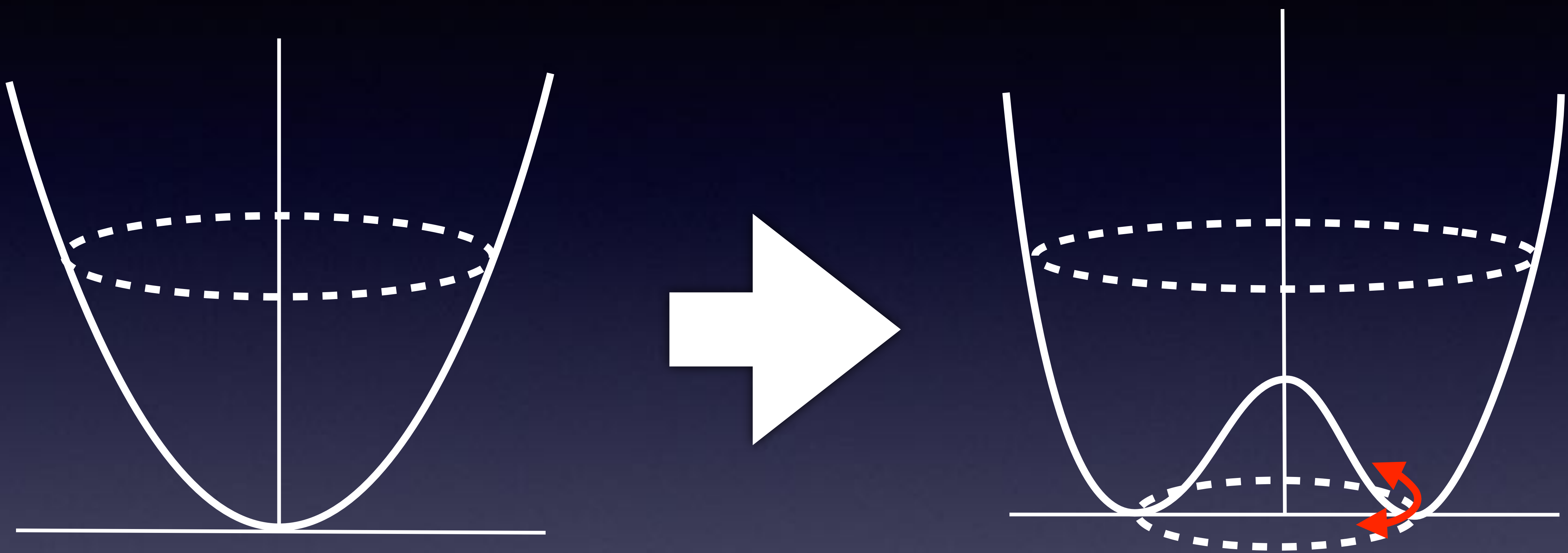
spin rotation $SO(3) \to SO(2)$

$$N_{\rm BS} = \dim(G/H) = 2 \quad N_{\rm NG} = 1$$

Dispersion: $\omega \propto k^2$

Internal symmetry breaking

Symmetry group $G \Rightarrow H$



# of flat direction

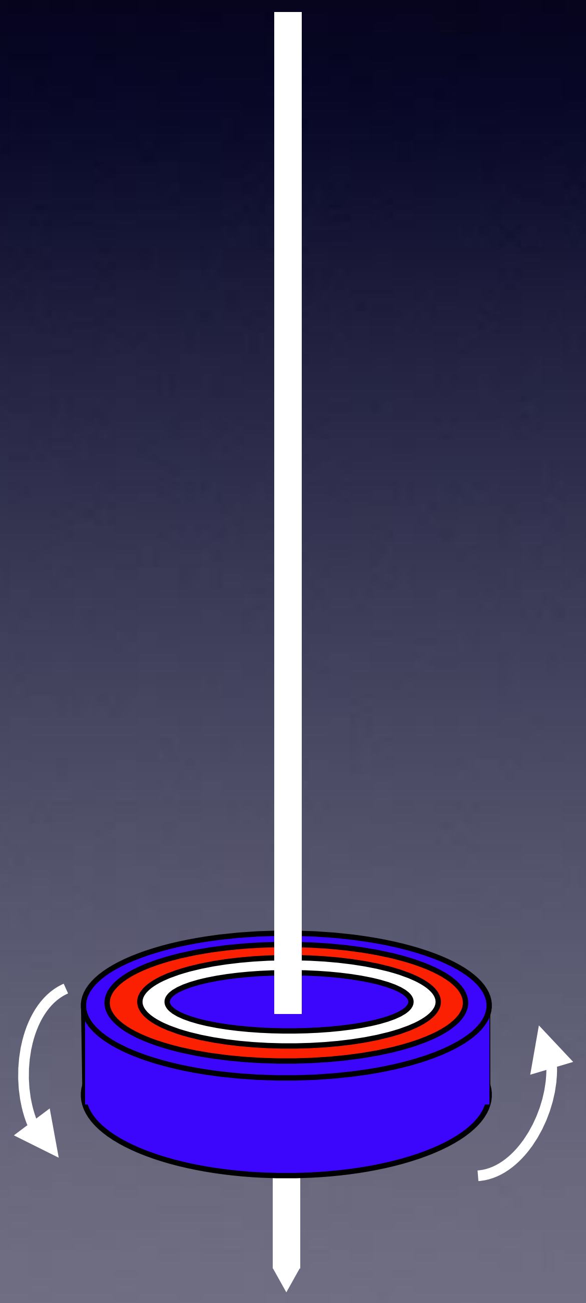
$$N_{\mathrm{BS}} = \dim(G/H)$$

This does work in nonrelativistic system at zero and finite temperature

Intuitive example for type-B NG modes

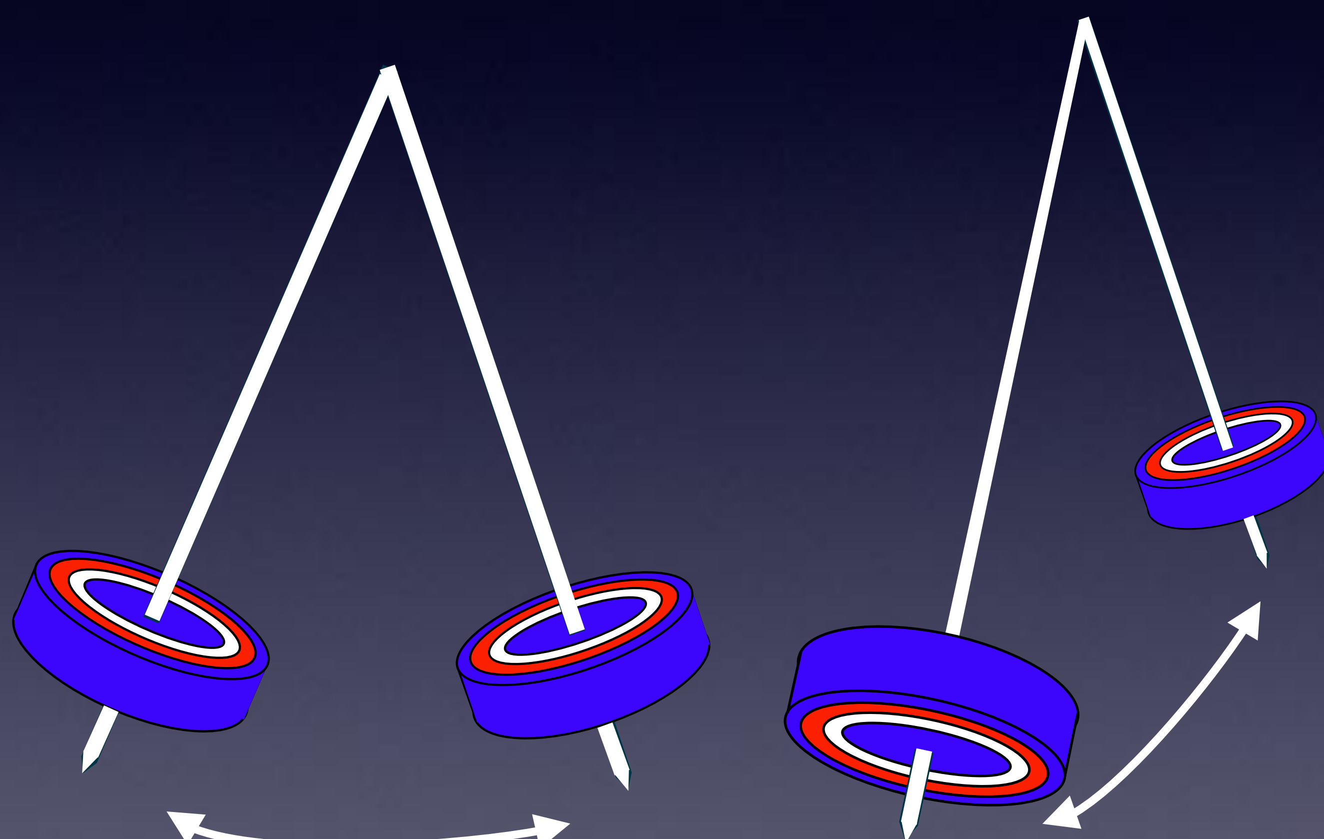
Pendulum with a spinning top

- Rotation symmetry is explicitly broken by a weak gravity
- Rotation along with z axis is unbroken.
- Rotation along with x or y is broken.
- The number of broken symmetry is two.



Intuitive example for type-B NG modes

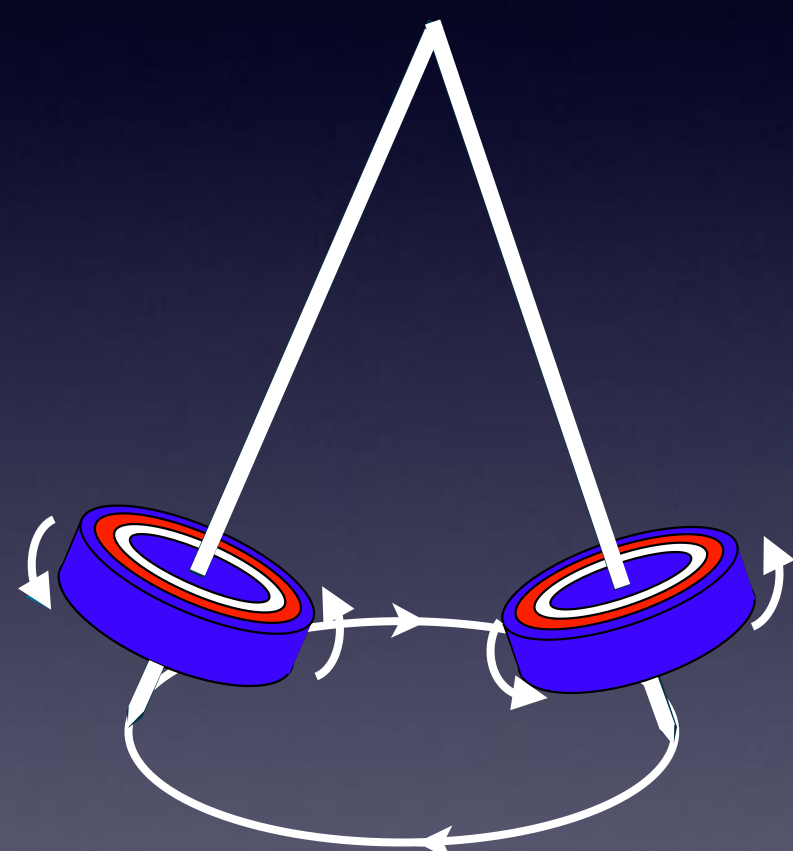
Pendulum has two oscillation motions



if the top is not spinning.

Intuitive example for type-B NG modes

If the top is spinning,



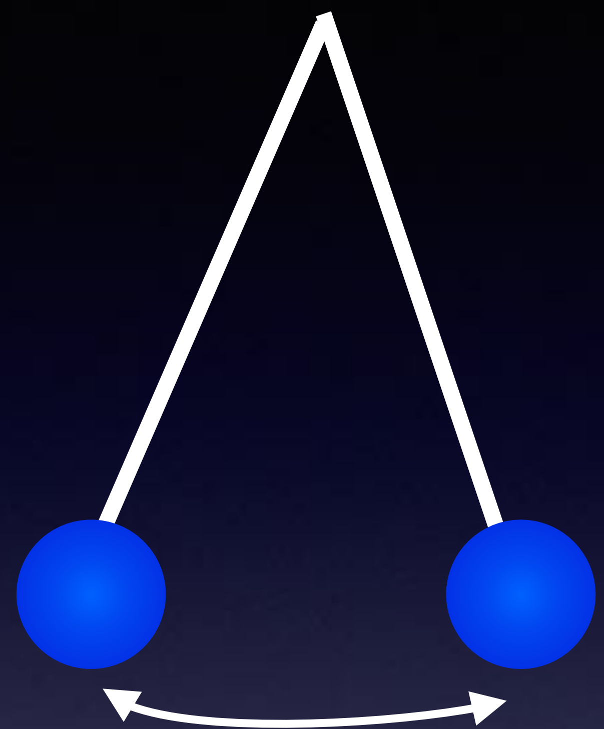
the only one rotation motion (Precession) exists.

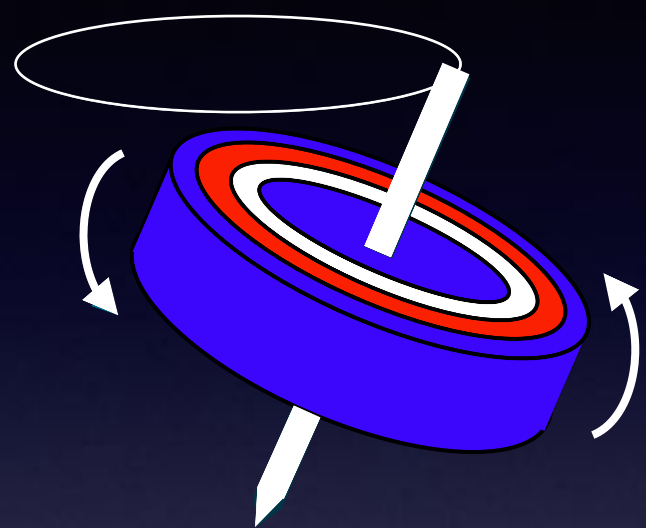
In this case, $\{L_x, L_y\}_P = L_z \neq 0$

Classification of NG modes

Watanabe, Murayama ('12), YH ('12)

cf. Takahashi, Nitta ('14), Beekman ('14)





Type-A

Harmonic oscillation

$$N_A = N_{\rm BS} - \mathrm{rank}\langle [iQ_a, Q_b]\rangle$$

Ex.) superfluid phonon

Type-B

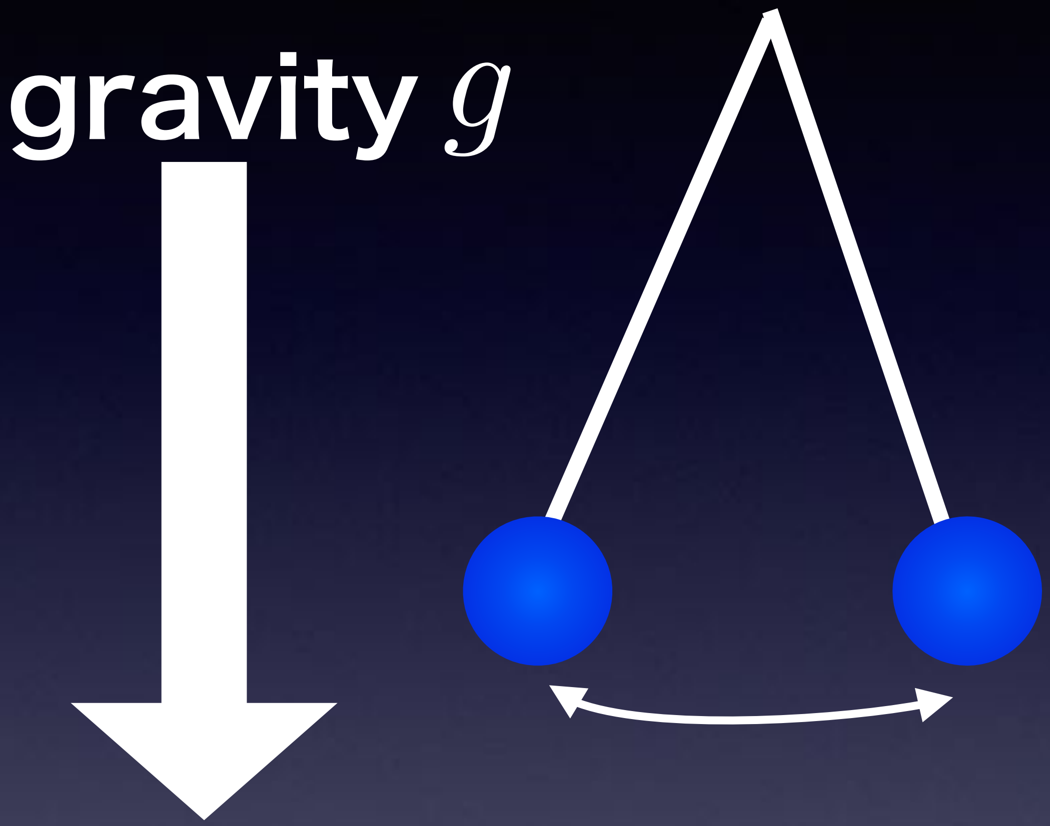
Precession

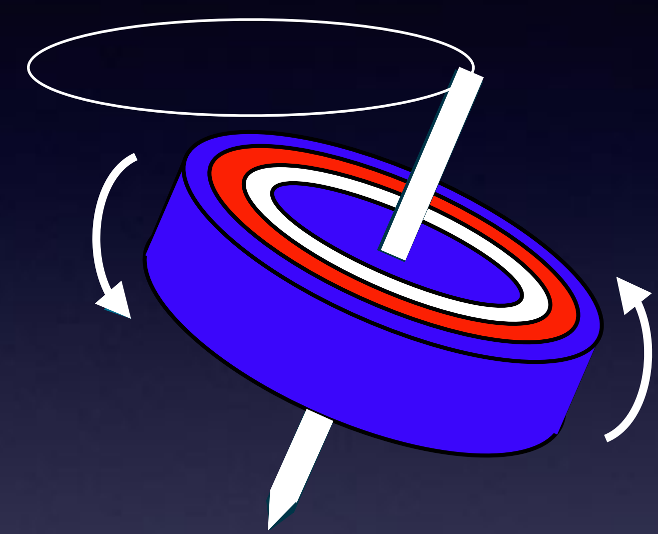
$$N_B = \frac{1}{2}\mathrm{rank}\langle [iQ_a, Q_b]\rangle$$

Ex.) magnon

$$\Rightarrow \quad N_{\rm NG} = N_{\rm BS} - \frac{1}{2}\langle i[Q_a, Q_b]\rangle$$

Dispersion relation





Type-A

$$\omega \sim \sqrt{g} \sim \sqrt{k^2}$$

Type-B

$$\omega \sim g \sim k^2$$

Examples of Type-B NG modes

	N_{BS}	$N_{\text{type-A}}$	$N_{\text{type-B}}$	$\frac{1}{2}\text{rank}\langle[Q_a,Q_b]\rangle$	$N_{\text{type-A}}+2N_{\text{type-B}}$
Spin wave in ferromanget SO(3)→SO(2)	2	0	1	1	2
NG modes in Kaon condensed CFL $SU(2)\times SU(1)_Y \to U(1)_{em}$	3	1	1	1	3
Spinor BEC SO(3)xU(1)→U(1)	3	1	1	1	3
nonrelativistic massive CP^1 model $U(1)\times\mathbf{R}^3\to\mathbf{R}^2$	2	0	1	1	2

$$N_{\text{type-A}}+2N_{\text{type-B}}=N_{\text{BS}} \qquad N_{\text{BS}}-N_{\text{NG}}=\frac{1}{2}\text{rank}\langle[Q_a,Q_b]\rangle$$

At finite temperature

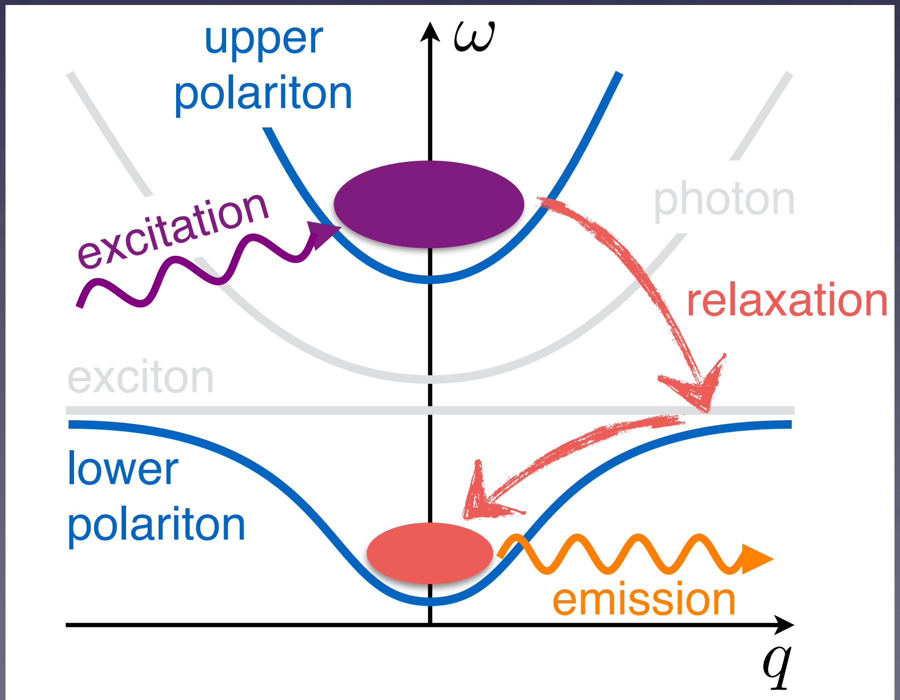
Hayata, YH ('14)

The interaction with thermal particles modifies the dispersion relation

Type-A: $\omega = ak - ibk^2$

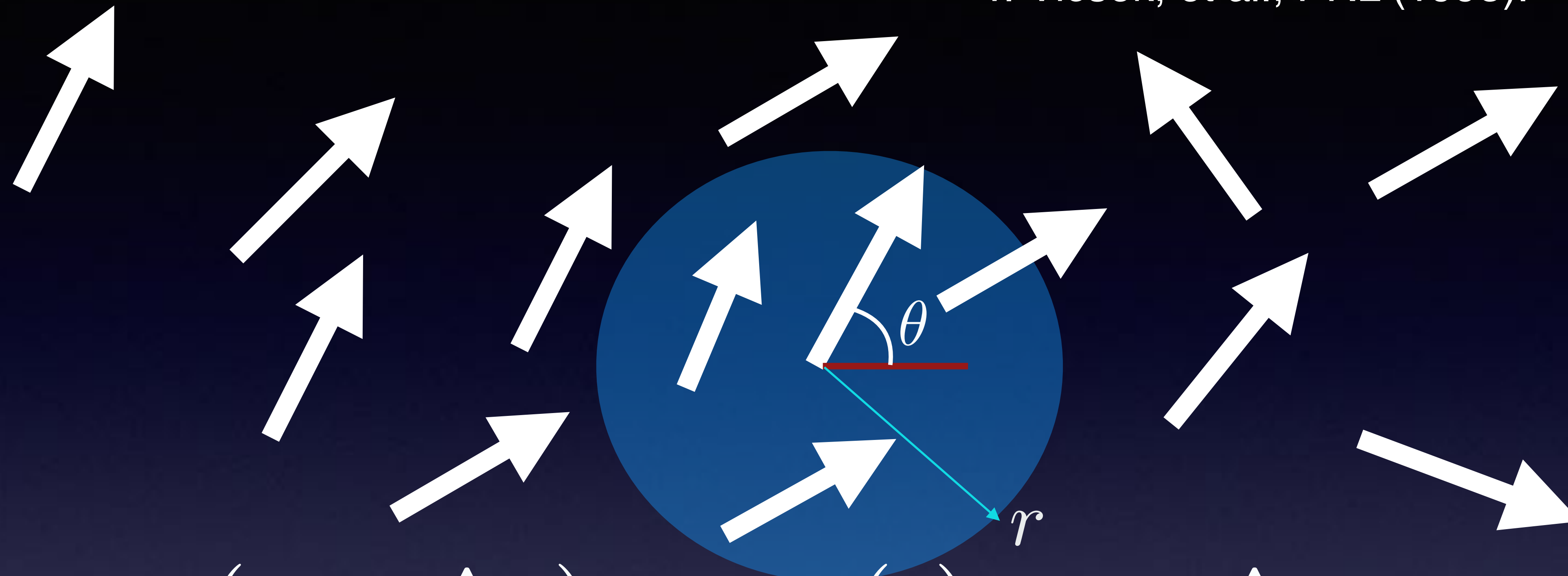
Type-B: $\omega = a'k^2 - ib'k^4$

Open system


upper
polariton
ω
photon
excitation
relaxation
exciton
lower
polariton
emission
q

Ex2) Vicsek model

T. Vicsek, et al., PRL (1995).



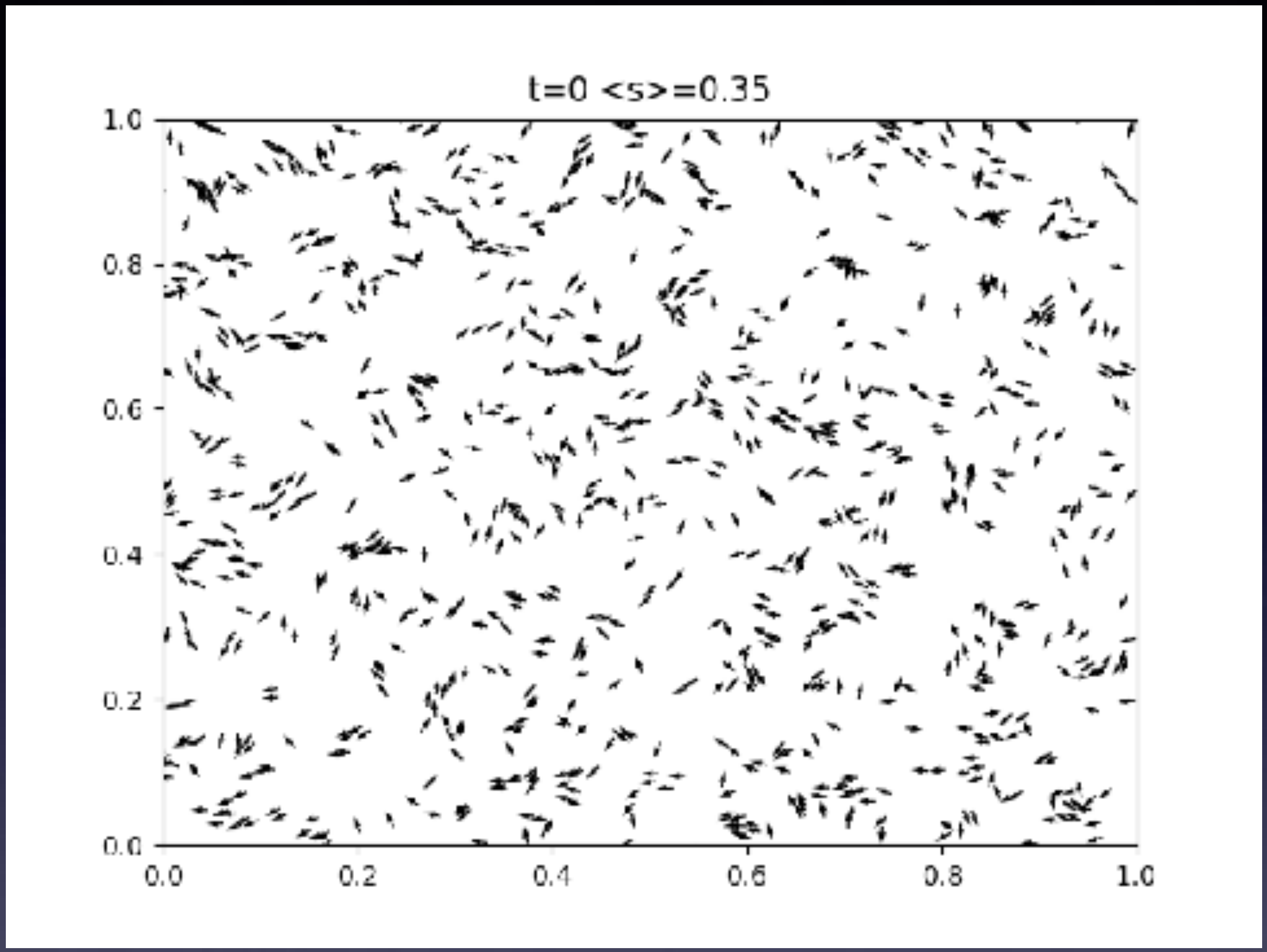
$$\boldsymbol{x}_i(t+\Delta t) = \boldsymbol{x}_i(t) + \boldsymbol{v}_i \Delta t$$

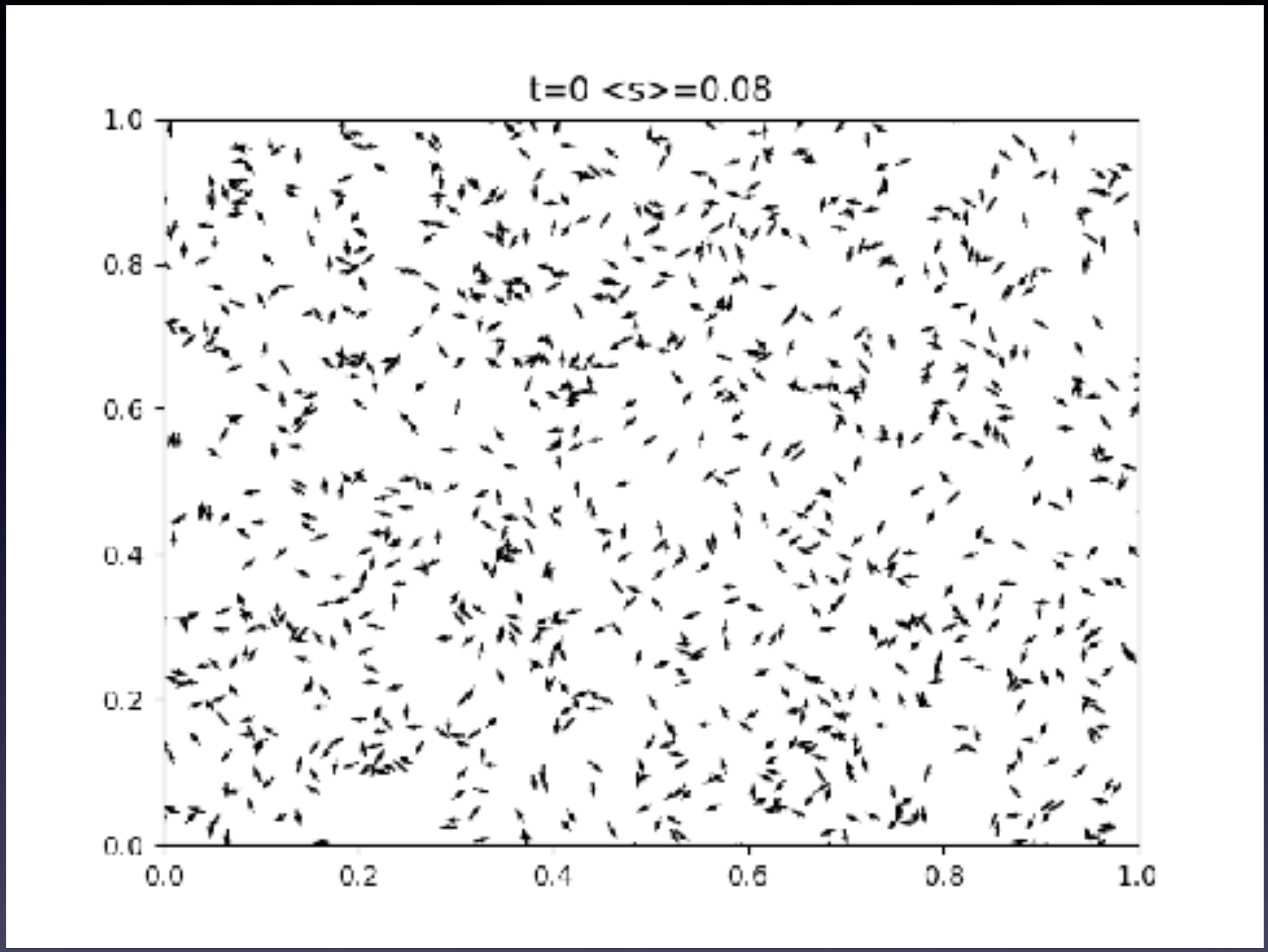
velocity

$$\boldsymbol{v}_i = v_0(\cos\theta_i, \sin\theta_i)$$

$$\theta_i(t+\Delta t) = \langle \theta_i(t) \rangle_r + \xi_i$$

angle of velocity **average angle** **noise**

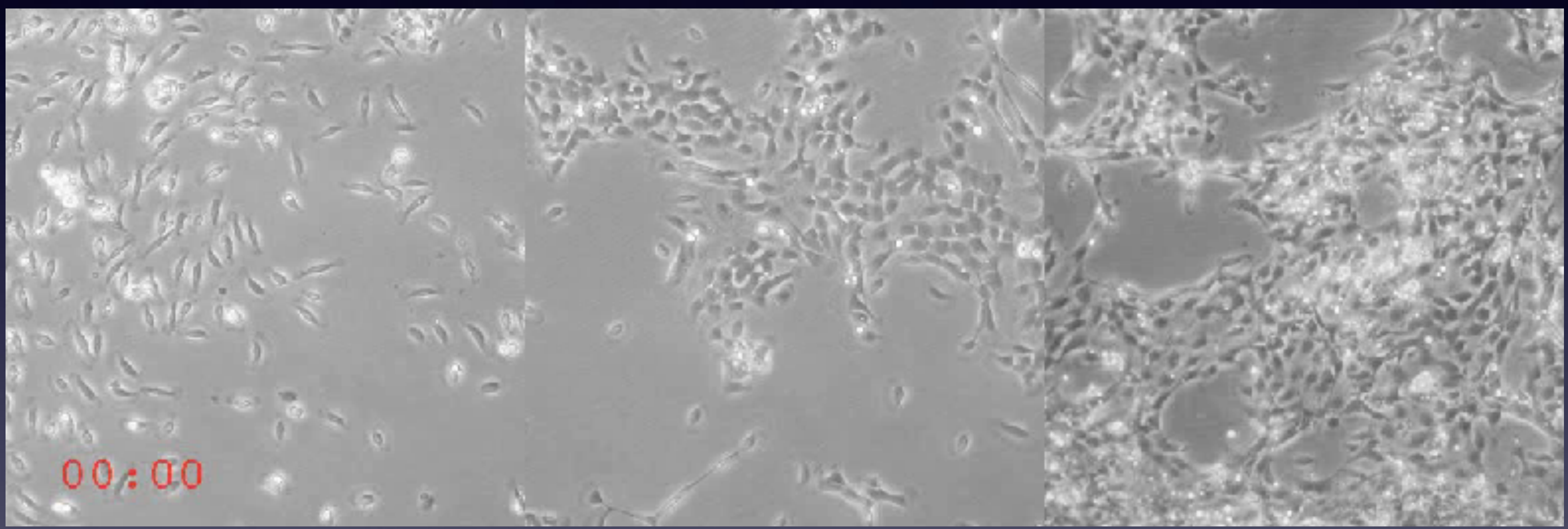

t=0 <s>=0.35
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


t=0 <s>=0.08
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

An example of SSB in open systems

Some cells on fish skin

low density **high density**



B. Szabo, et al., Phys. Rev. E 74, 061908 (2006)

Model of active matter: **Vicseck model, Active hydrodynamics, ….**

T. Vicsek, et al., PRL (1995). J. Toner, and Y. Tu, PRL (1995).

Field theoretical model

Ex.) NG mode in Active hydrodynamics

J. Toner, and Y. Tu, PRE (1998)

$$\partial\rho + \nabla\cdot(\rho\boldsymbol{v}) = 0$$

$$\partial_t\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \alpha\boldsymbol{v} - \beta\boldsymbol{v}^2\boldsymbol{v} - \nabla P + D_L\nabla(\nabla\cdot\boldsymbol{v}) + D_l(\boldsymbol{v}\cdot\nabla)^2\boldsymbol{v} + \boldsymbol{f}$$

nonconserved term

noise

Steady state solution: $\boldsymbol{v}^2 = \alpha/\beta \equiv v_0^2$

Symmetry breaking: $O(3) \to O(2)$

Fluctuation: $\boldsymbol{v} = (v_0 + \delta v_x, \delta v_y, \delta v_z)$

⇒ $\omega = ck$ (**propagating**) $\omega = i\Gamma k^2$ (**diffusive**) **NG modes**

Can we discuss symmetry breaking without ordinary conservation law?

Ex) Symmetry of Brownian motion

Langevin equation

$$\frac{d}{dt}\boldsymbol{x}(t) = \boldsymbol{u}(t)$$

$$\frac{d}{dt}\boldsymbol{u}(t) = -\gamma\boldsymbol{u}(t) + \boldsymbol{\xi}(t)$$

$$\langle \xi_i(t)\xi_j(t')\rangle = 2\delta_{ij}\gamma T\delta(t-t')$$

Angular momentum $\boldsymbol{L} = \boldsymbol{x}\times\boldsymbol{u}$

$$\frac{d}{dt}\langle \boldsymbol{L}(t)\rangle = -\gamma\langle \boldsymbol{x}\times\boldsymbol{u}(t)\rangle \neq 0$$

not conserved

Langevin equation

$$\frac{d}{dt}\boldsymbol{u}(t) = -\gamma \boldsymbol{u}(t) + \boldsymbol{\xi}(t)$$

⇕

Fokker-Planck equation

$$\partial_t P(t,u) = \frac{\partial}{\partial u_i}\Big(\gamma T \frac{\partial}{\partial u_i} + \gamma u_i\Big) P(t,u)$$

⇕

Path integral

Martin-Siggia-Rose formalism

$$Z = \int \mathcal{D}\chi \mathcal{D}u e^{iS[\chi,u]}$$

Dynamic action: $iS = \int dt \Big[i\chi_i \Big(\frac{d}{dt}u_i + \gamma u_i\Big) - T\gamma\chi_i^2 \Big]$

Symmetry of Dynamic action

$$iS = \int dt \Big[i\chi_i \Big(\frac{d}{dt} u_i + \gamma u_i \Big) - T\gamma \chi_i^2 \Big]$$

O(3) symmetry

$\chi_i \to R_{ij}\chi_j \quad u_i \to R_{ij} u_j$ with $R_{ik}R_{kj} = \delta_{jk}$

Noether charge

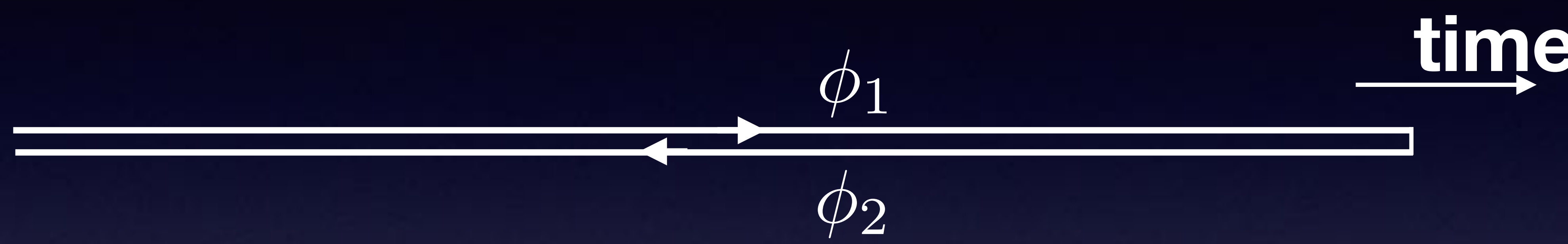
$$\boldsymbol{L}_{\mathrm{MSR}} = \boldsymbol{\chi} \times \boldsymbol{u} \qquad \boldsymbol{L} = \boldsymbol{x} \times \boldsymbol{u}$$

$$\boldsymbol{L}_{\mathrm{MSR}} \neq \boldsymbol{L}$$

Open quantum system

cf. for review, Sieberer, Buchhold, Diehl, 1512.00637

Schwinger-Keldysh Path integral



$$Z = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp\left[iS[\phi_1] - iS[\phi_2] + iS_{12}[\phi_1, \phi_2] \right]$$

complex

$S[\phi_1]$: **forward evolution**

$S[\phi_2]$: **backward evolution**

$S_{12}[\phi_1, \phi_2]$: Interaction with environment

Example

Lindblad equation

$$\partial_t \rho = -i[H,\rho] + \gamma\Big(L\rho L^\dagger - \frac{1}{2}(L^\dagger L\rho + \rho L^\dagger L)\Big)$$

fluctuation and dissipation

Action

$$S[\phi] = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$iS_{12}[\phi_1,\phi_2] = \gamma L(\phi_1)L^\dagger(\phi_2) - \frac{\gamma}{2}\Big(L(\phi_1)L^\dagger(\phi_1) + L(\phi_2)L^\dagger(\phi_2)\Big)$$

R/A basis

$$\phi_R = \frac{1}{2}(\phi_1 + \phi_2) \qquad \phi_A = \phi_1 - \phi_2$$

classical field **fluctuation**

$$S[\phi_1] - S[\phi_2] = \int d^4x \phi_A \left(-\partial_\mu^2 \phi_R - m^2 \phi_R - \frac{\lambda}{3!}\phi_R^3 \right) + \int d^4x \frac{\lambda}{24}\phi_A^3 \phi_R$$

classical Equation of motion

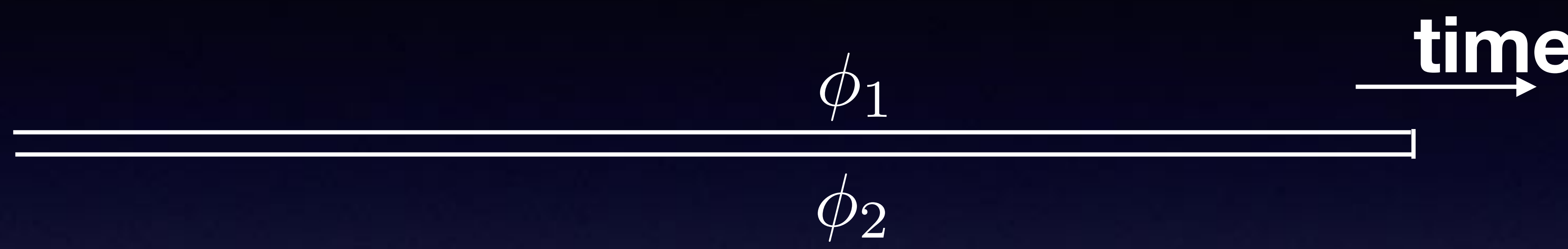
For example,

$$iS_{12}[\phi_R, \phi_A] = -\gamma \phi_A \partial_0 \phi_R - \frac{A}{2}\phi_A^2 + \cdots$$

Symmetry of Open quantum system

cf. for review, Sieberer, Buchhold, Diehl, 1512.00637

Schwinger-Keldysh Path integral



$$Z = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp\left[iS[\phi_1] - iS[\phi_2] + iS_{12}[\phi_1, \phi_2] \right]$$

complex

Q_1, Q_2 :Symmetry generators:

$S[\phi_1], S[\phi_2]$ are invariant.

Suppose $S_{12}[\phi_1, \phi_2]$ is invariant under $Q_A = \dfrac{Q_1 - Q_2}{2}$

We also define $Q_R = \dfrac{Q_1 + Q_2}{2}$

Spontaneous symmetry breaking

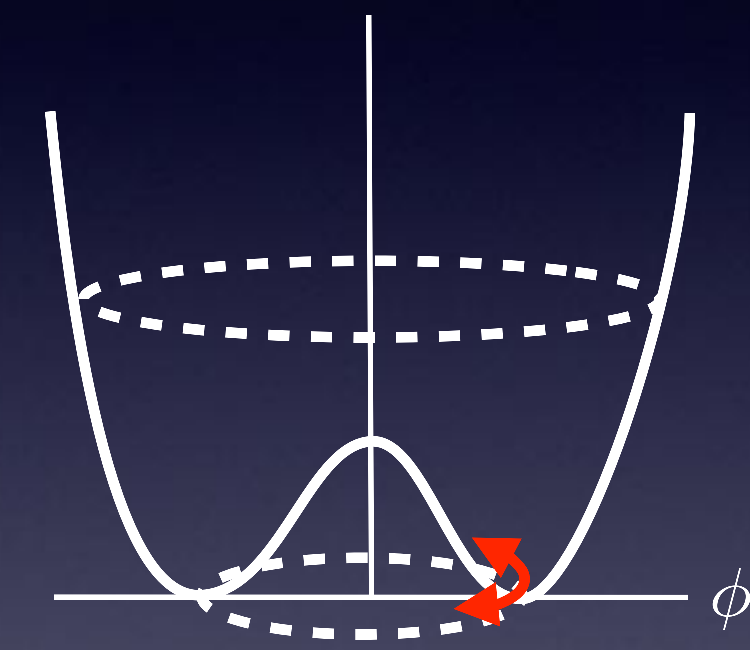
Minami, YH ('15)

Ex1) SU(2)xU(1) model Type-A

$$iS = \int d^4x \Big[\phi_A^\dagger(-\partial_0^2 + \nabla^2 - \gamma\partial_0)\varphi_R - 2\lambda|\varphi_R|^2\varphi_R - A\varphi_A^\dagger\varphi_A\Big] + \cdots$$

$\varphi_{R/A}$ **two component complex field**

$$\varphi_R = (\pi_1 + i\pi_2, v + h + i\pi_3)$$



Linear analysis

$$(\partial_0^2 + \gamma\partial_0 - \nabla^2)\pi_a = 0$$ **NG type-A mode**

$$\Rightarrow -\omega^2 - i\gamma\omega + k^2 = 0$$

$$\Rightarrow \omega = \frac{-i\gamma}{2} \pm \frac{i}{2}\sqrt{\gamma^2 - 4k^2} \sim -\frac{i}{\gamma}k^2, -i\gamma + \frac{i}{\gamma}k^2$$

diffusion mode

Spontaneous symmetry breaking

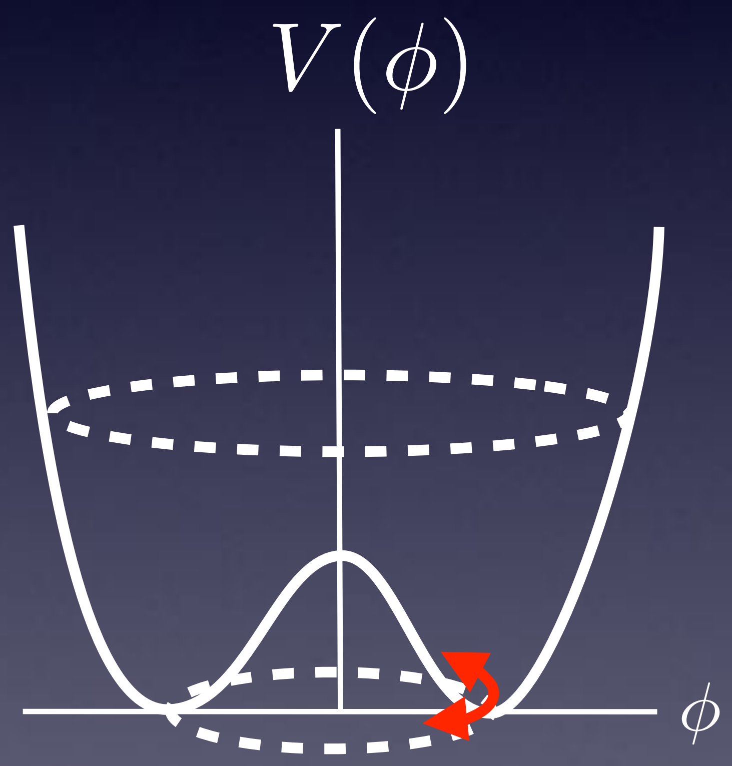
Minami, YH ('15)

Ex2) SU(2)xU(1)model Type-B

with chemical potential μ

$$iS = \int d^4x \Big[\phi_A^\dagger(-(\partial_0 + i\mu)^2 + \nabla^2 - \gamma\partial_0)\varphi_R - 2\lambda|\varphi_R|^2\varphi_R - A\varphi_A^\dagger\varphi_A\Big] + \cdots$$

$$\begin{pmatrix} -\partial_0^2 - \gamma\partial_0 + \nabla^2 & 2\mu\partial_0 \\ -2\mu\partial_0 & -\partial_0^2 - \gamma\partial_0 + \nabla^2 \end{pmatrix}\begin{pmatrix}\pi_1 \\ \pi_2\end{pmatrix} = 0,$$



$$\Rightarrow \quad \omega = \frac{k^2}{4\mu^2 + \gamma^2}(\pm 2\mu - i\gamma)$$

quadratic dispersion

$\langle [iQ_A^1, Q_R^2] \rangle \neq 0$ **Type-B**

Spontaneous symmetry breaking

Minami, YH ('17)

Ex3) SU(2)xU(1)model Type-B

with complex potential

$$iS = \int d^4x \Big(i\varphi_A^\dagger((-\partial_0^2 + \nabla^2 - (\gamma + 2i\mu)\partial_0 - m_\mathrm{r}^2 - im_\mathrm{i}^2)\varphi_R$$

$$- 2(\lambda_\mathrm{r} + i\lambda_\mathrm{i})(\varphi_R^\dagger \varphi_R)\varphi_R) - A\varphi_A^\dagger \varphi_A \Big) + \cdots$$

Assuming $\varphi_R = (0, ve^{-i\omega_0 t})$

Gap equation

$$(\omega_0^2 - 2\mu\omega_0 - m_\mathrm{r}^2 - 2\lambda_\mathrm{r} v^2 + i(\gamma\omega_0 - m_\mathrm{i}^2 - 2\lambda_\mathrm{i} v^2))v = 0$$

Symmetric phase $v = 0$

Broken phase $v \neq 0, \quad \omega_0 \neq 0$

Spontaneous symmetry breaking

Minami, YH ('17)

Ex3) SU(2)xU(1)model Type-B with complex potential

Linear analysis

$$\begin{pmatrix} -\partial_0^2-\gamma\partial_0+\nabla^2 & 2(\mu-\omega_0)\partial_0 \\ -2(\mu-\omega_0)\partial_0 & -\partial_0^2-\gamma\partial_0+\nabla^2 \end{pmatrix}\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix}=0,$$

⇨
$$\omega=\frac{k^2}{4(\mu-\omega_0)^2+\gamma^2}(\pm 2(\mu-\omega_0)-i\gamma)$$

We still have quadratic dispersion

Similarly, we find

$$\omega=-i\frac{\boldsymbol{k^2}}{\gamma+2(\mu-\omega_0)\lambda_i/\lambda_r}$$

Diffusive mode for type-A

Inverse propagator and dispersion

Minami, YH ('17)

$$[G_\pi^{-1}(k)]^{\beta\alpha} = iC^{\mu;\beta\alpha}k_\mu + C^{\mu\nu;\beta\alpha}k_\mu k_\nu + \cdots$$

Hamiltonian system

Open system

$$C^{\mu;\beta\alpha} = -\langle[iQ_R^\alpha, j_A^{\beta\mu}(0)]\rangle$$

$$-i\int d^Dx\langle[iQ_R^\alpha, \mathcal{L}_{12}(x)]j_A^{\beta\mu}(0)\rangle_{\pi c}$$

$$C^{\mu\nu;\beta\alpha} = i\int d^Dx\langle j_R^{\alpha\mu}(x)j_A^{\beta\nu}(0)\rangle_{\pi c}$$

$$-\lim_{k\to 0}\frac{\partial}{\partial k_\nu}i\int d^Dx e^{ik_\rho x^\rho}\langle[iQ_R^\alpha, \mathcal{L}_{12}(x)]j_A^{\beta\mu}(0)\rangle_{\pi c}$$

Our result is too general

Need to impose symmetry of S_{12}

Ex)'Standard' Fokker-Plank eq.

Type-A mode

Diffusive $\omega = -ik^2\Gamma$

$$N_A = N_{\rm BS} - {\rm rank}\langle [iQ_R^\alpha, Q_A^\beta]\rangle$$

Type-B mode

$$\omega = ak^2 - ik^2\Gamma'$$

$$N_B = \frac{1}{2}{\rm rank}\langle [iQ_R^\alpha, Q_A^\beta]\rangle$$

Next step: classification

Summary

Spontaneous breaking of symmetry of Dynamic action

Two-type of diffusive NG modes

Type-A mode

Diffusive $\omega = -ik^2\Gamma$

Type-B mode

$\omega = ak^2 - ik^2\Gamma'$

Next step: classification

What is the condition satisfying this table?

Type	Dispersion Re	Dispersion Im	Conserved charge	Examples
A	k	k^2	Q_A, Q_R	Superfluid, etc.
	0	k^2	Q_A	flock of birds, Exciton-polariton condensates
B $\langle[Q_A, Q_R]\rangle \neq 0$	k^2	k^4	Q_A, Q_R	Ferromagnet
	k^2	k^2	Q_A	Spinor BEC in open quantum system? Magnetotactic bacteria?

Possible active matter with type B modes?

Collective motion of Magnetotactic bacteria