QCD Matter in Neutron Star Environments

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Workshop of Recent Developments in QCD and QFT

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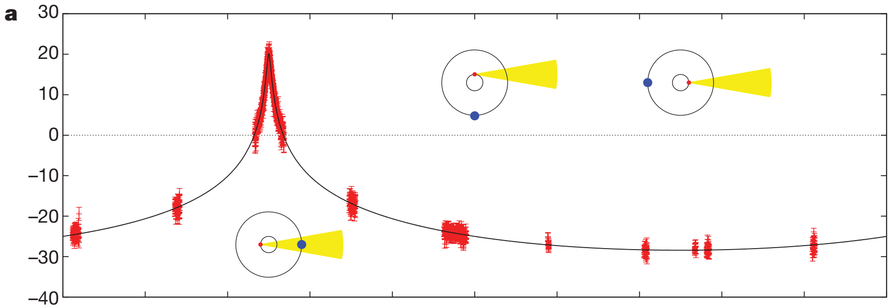
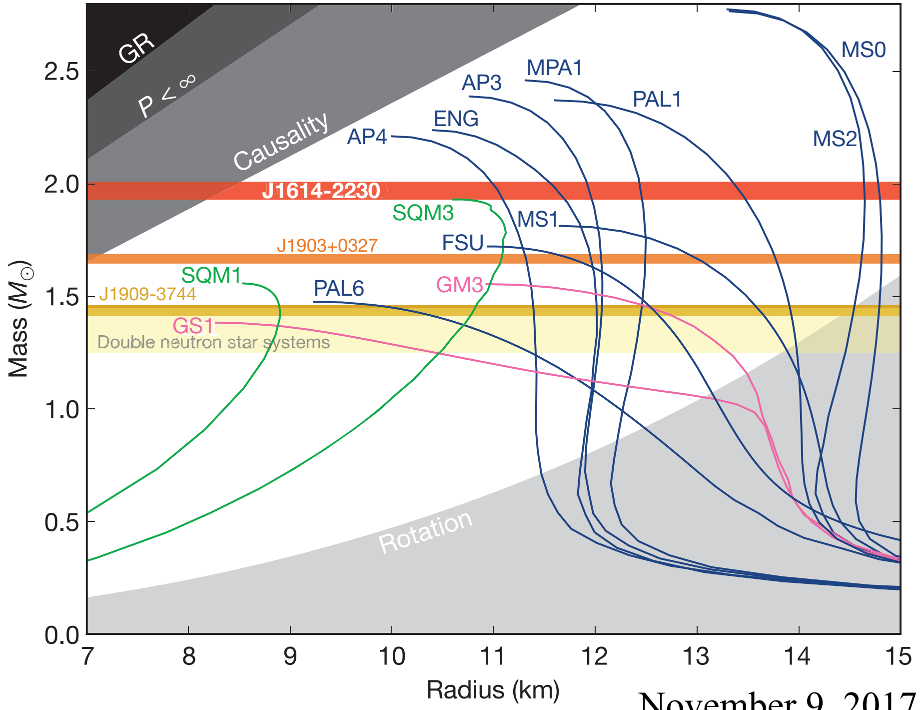
Neutron Star (NS) Constraint(s)

Neutron Star Constraint





Demorest et al. (2010)

Precise determination of
NS mass using Shapiro delay

1.928(17) M_{sun} (J1614-2230)

(slightly changed in 2016)

Antoniadis et al. (2013)

2.01(4) M_{sun} (PSRJ0348+0432)

Neutron Star Constraint

Neutron stars are composed of the densest form of matter known to exist in our Universe, the composition and properties of which are still theoretically uncertain. Measurements of the masses or radii of these objects can strongly constrain the neutron star matter equation of state and rule out theoretical models of their composition[1,2]. The observed range of neutron star masses, however, has hitherto been too narrow to rule out many predictions of 'exotic' non-nucleonic components[3–6]. The Shapiro delay is a general-relativistic increase in light travel time through the curved space-time near a massive body[7]. For highly inclined (nearly edge-on) binary millisecond radio pulsar systems, this effect allows us to infer the masses of both the neutron star and its binary companion to high precision[8,9]. Here we present radio timing observations of the binary millisecond pulsar J1614-2230[10,11] that show a strong Shapiro delay signature. We calculate the pulsar mass to be $(1.97 \pm 0.04)M_{\odot}$, which rules out almost all currently proposed[2–5] hyperon or boson condensate equations of state ($M_{\odot}$, solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks[12].

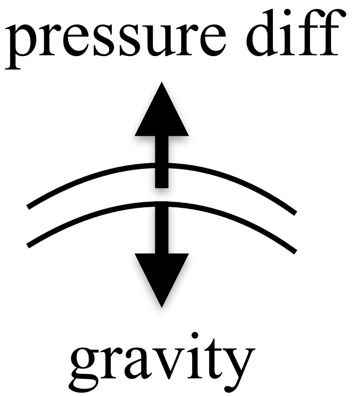
Neutron Star Constraint

Equation of State (unknown)

Pressure : p
Mass density : ρ
(Energy density : $\varepsilon = \rho c^2$)

$$\boxed{p = p(\rho)}$$

Tolman-Oppenheimer-
-Volkoff (TOV) Eqs



M-R Relation (observed)

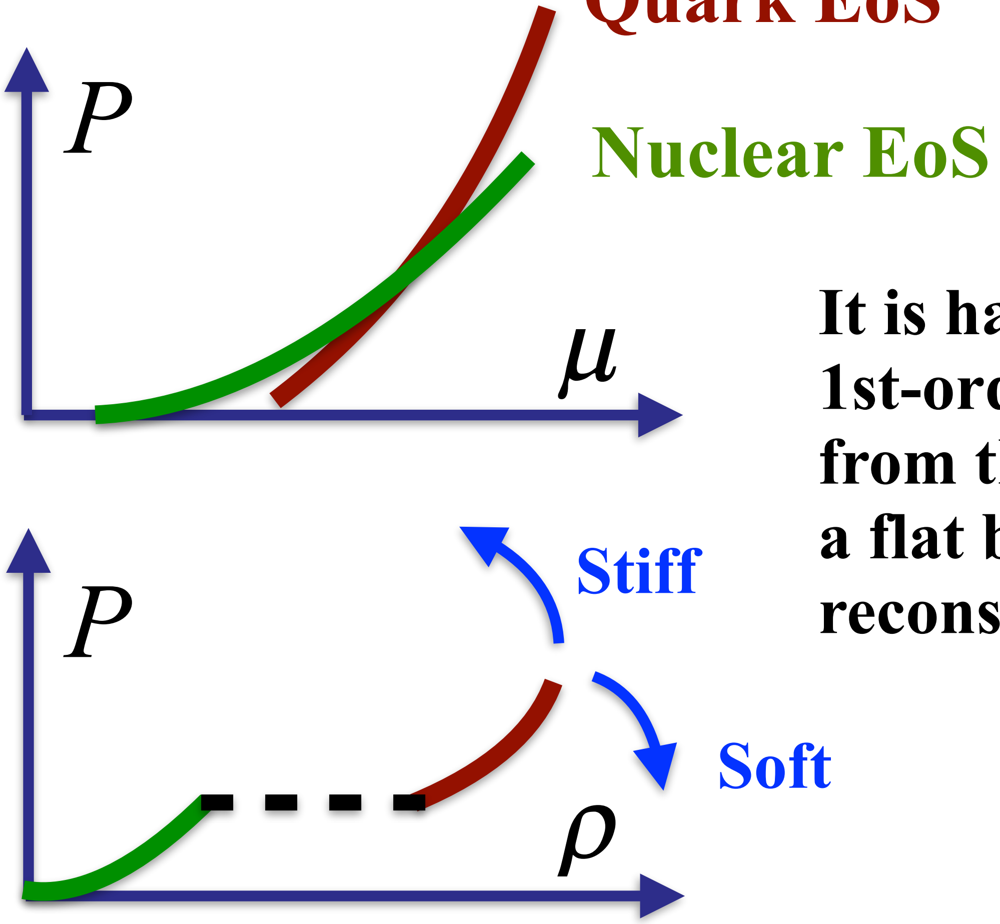
NS mass : M
NS radius : R

$$\boxed{\begin{gathered} M = M(\rho_{\max}) \\ R = R(\rho_{\max}) \end{gathered}}$$

Mathematically one-to-one correspondence

Neutron Star Constraint

Old Picture

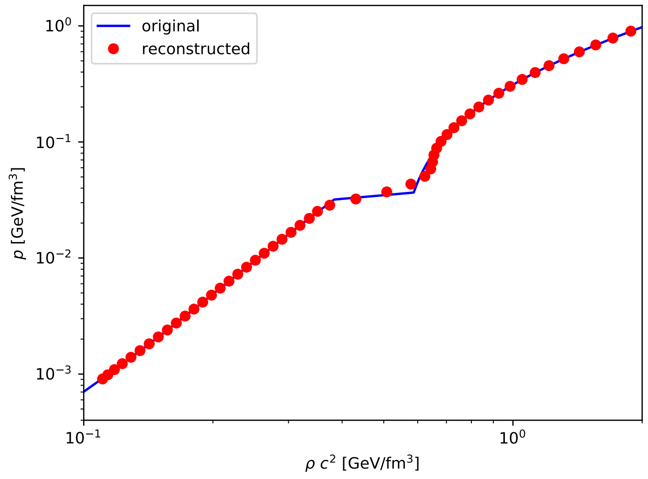


It is hard to see if there is a 1st-order transition or not from the *M*-*R* relation, but a flat behavior can be reconstructed mathematically

Neutron Star Constraint

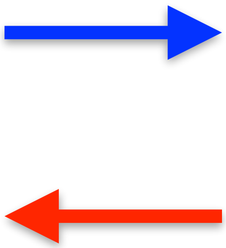
Lindblom (1992)

Some simple test cases : useful for a 1st-order transition?



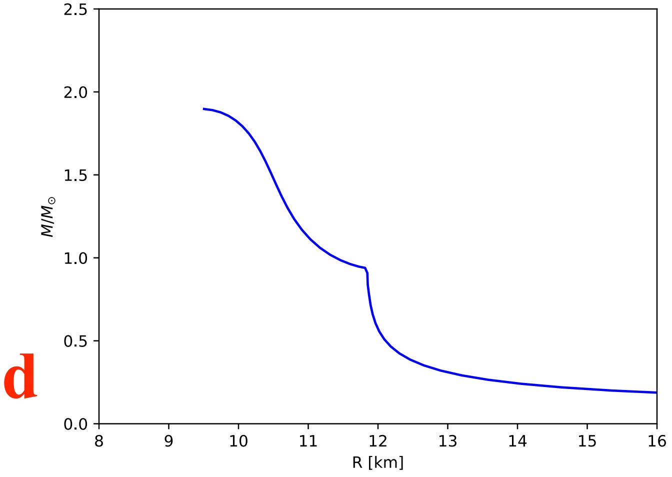
Test data set by hand

Solve TOV



Reconstructed

Thanks to Y. Fujimoto



Yes, it is useful, *in principle*

Neutron Star Constraint

IF there is a 1st-order phase transition with large density gap (i.e. strong 1st-order) at small densities,

EoS cannot be stiff enough to support massive NS

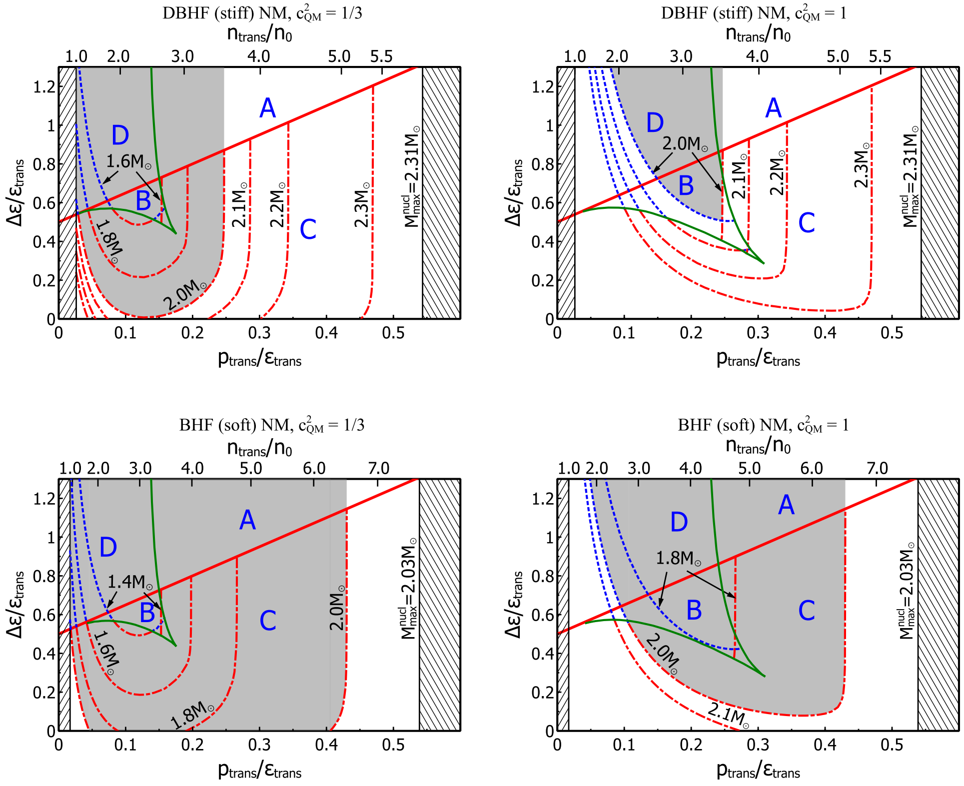
Remember: the slope is bounded by causality, and cannot exceed the speed of light.

Strong 1st-order transition excluded, which means…

Neutron Star Constraint

Alford et al. (2015) $$\varepsilon(p) = \begin{cases} \varepsilon_{\rm NM}(p) & p < p_{\rm trans} \\ \varepsilon_{\rm NM}(p_{\rm trans}) + \Delta\varepsilon + c_{\rm QM}^{-2}(p - p_{\rm trans}) & p > p_{\rm trans} \end{cases}$$

Parameters (choices) : Nuclear EoS, $c_{\rm QM}$, $\Delta\varepsilon$, $p_{\rm trans}$



Okay if…

QM only at very high density

1st trans. at very high density

1st trans. very weak

NM EoS very stiff

etc, etc

Looks generic, but
a bit misleading to say…

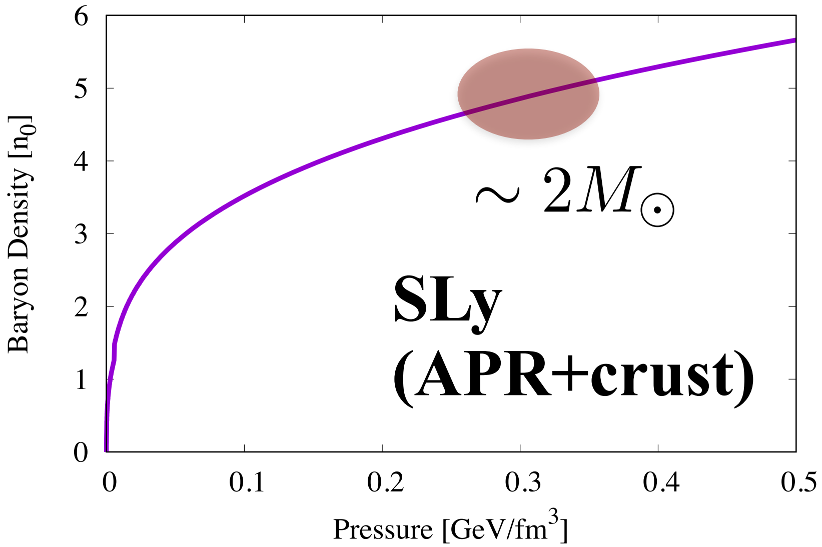
Neutron Star Constraint

Caveats

Based on the old picture of 1st-order transition to QM

Is there any reason to require 1st-order transition? **NO!**

Based on the extrapolation of NM EoS to high densities



Can it be extrapolatable?

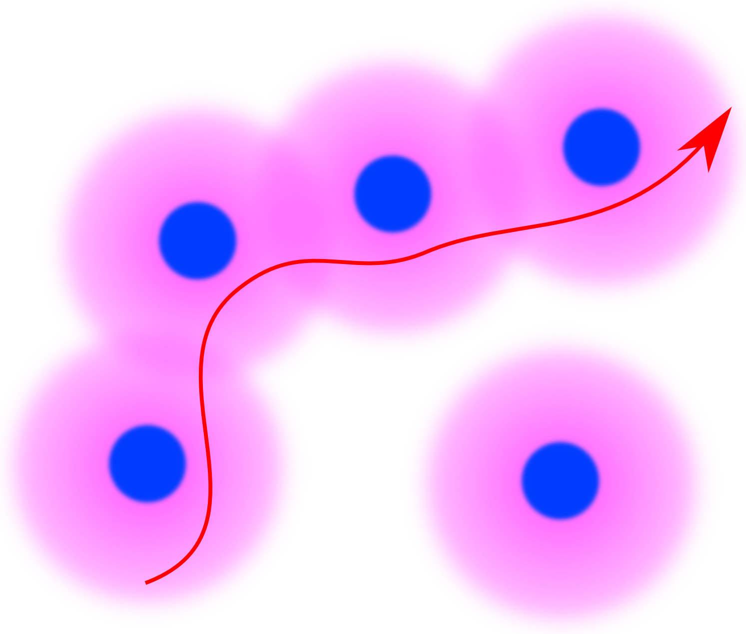
NO!

apart from an infamous
causality problem…

Neutron Star Constraint

IF nucleons are surrounded by interaction clouds of pions, such clouds undergo a classical percolation transition at

$1.4\ n_0$



Percolation transition allows for mobility enhancement of quarks?

(Picture of H. Satz)

Quantum fluctuations (Anderson localization) induce "confinement" (quantum percolation)

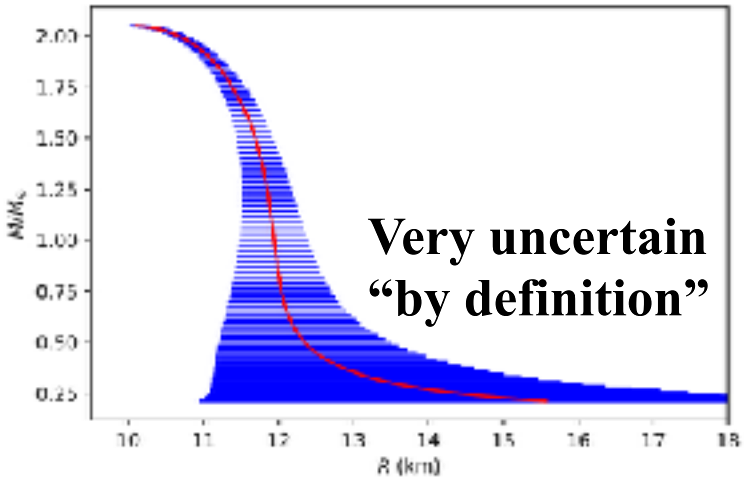
Neutron Star Constraint

One may think that the constraint may be strong for light NS BUT…

***R* is fixed by TOV with *p*(*R*)=0 and interestingly…**

$$dp/dr(r = R) = 0$$
$$d^2p/dr^2(r = R) \propto M^2/R^2$$

If *M* is small or *R* is large, uncertainty becomes huge.



Very uncertain "by definition"

People do not care assuming that NS mass > 1.2 M_{sun}

Neutron Star Constraint

Here, NS-NS merger will not be discussed, but another constraint is already available:

Hinderer et al. (2009)

$$-\frac{(1+g_{tt})}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3}n^i n^j + \ldots + \frac{\mathcal{E}_{ij}}{2}r^2 n^i n^j + \ldots,$$

$$Q_{ij} = -\lambda\mathcal{E}_{ij}$$

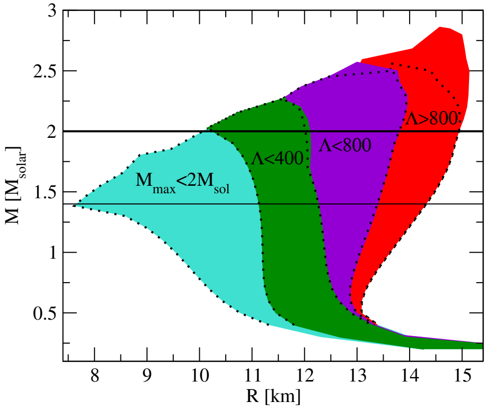
$$\boxed{\lambda = \frac{\text{quadrupole moment}}{\text{external tidal field}}}$$

(~ Love number)

Often divided by M^5 to make it dimensionless → Λ

$$\text{(tidal deformability)}\ \Lambda(1.4M_\odot) < 800$$

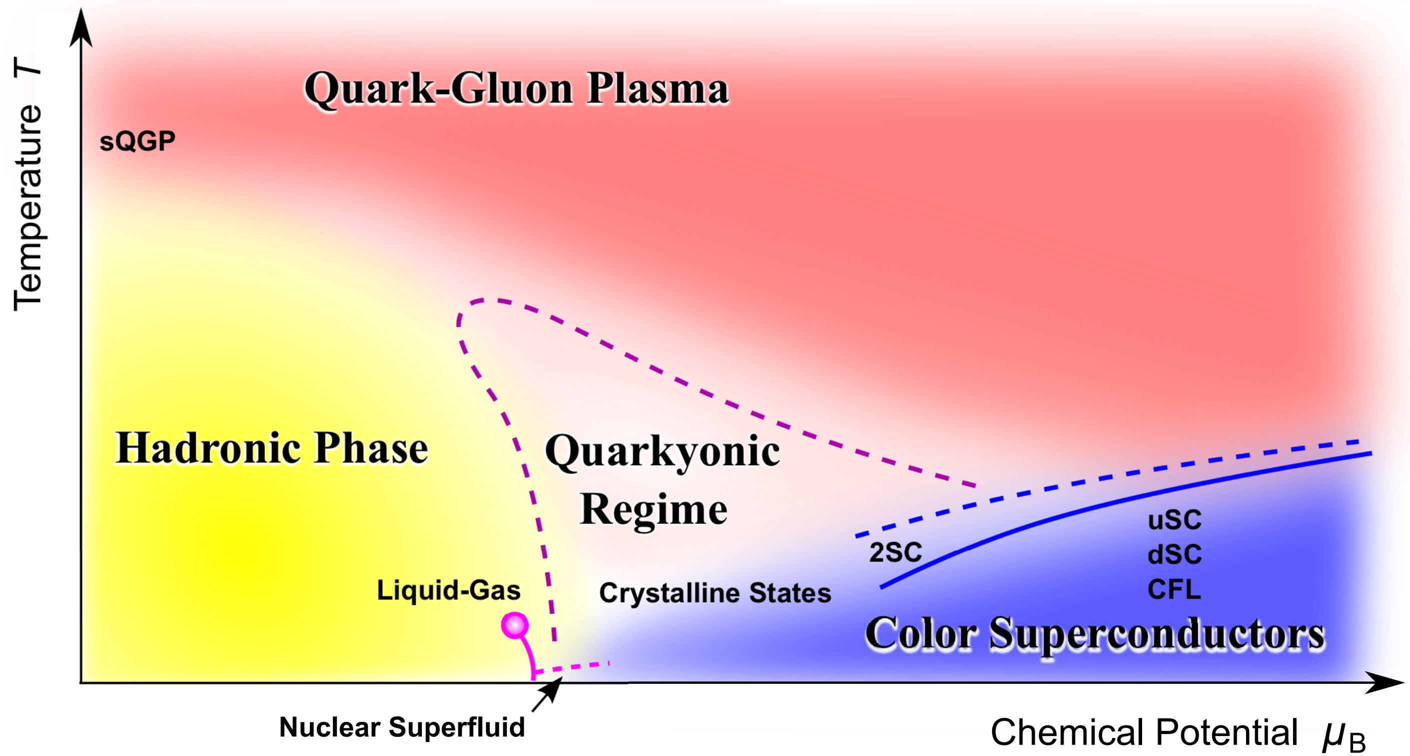
See: Annala-Gorda-Kurkela-Vuorinen (2017)



What is Known from Theory ?

What is known from theory?

Fukushima-Sasaki (2013)



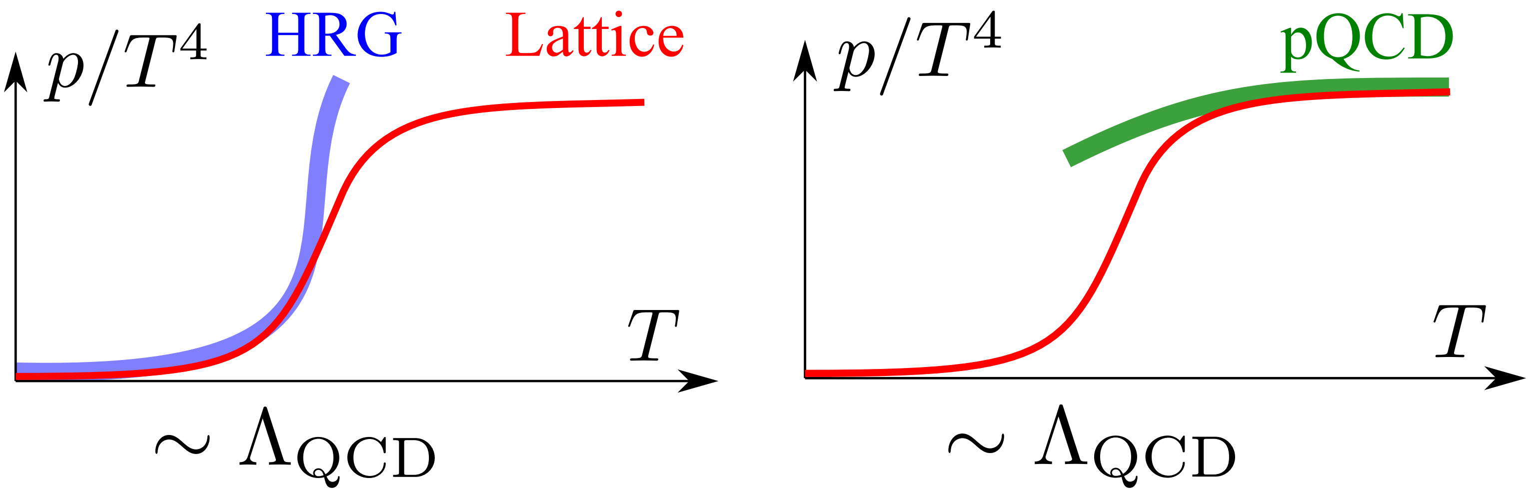
Almost nothing…

What is known from theory?

Most important lesson from high-T low-ρ QCD matter

QCD transition from hadronic to quark-gluon matter is a continuous crossover with an overlapping region (dual region) of hadrons and of quarks and gluons

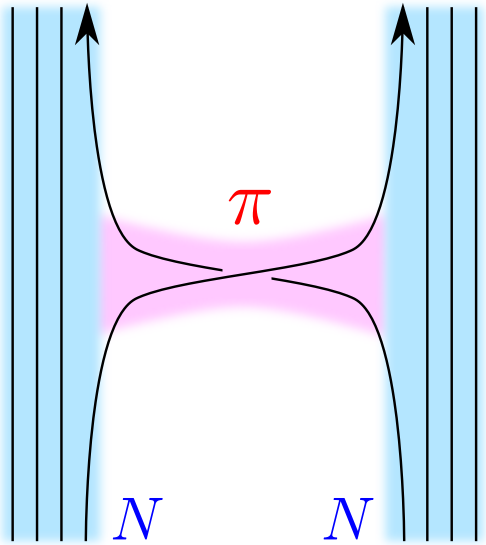
Quark Matter 2014 (Fukushima)



What is known from theory?

A hint to understand a crossover

Baryon int. at large N_c



$$\sim \mathcal{O}(N_c)$$

Pressure of large-N_c NM scales as ~ $O(N_c)$ as if it were QM.

Quark d.o.f. perceived through interactions even in baryonic matter

Quarkyonic Matter

McLerran-Pisarski (2007) Hidaka, Kojo, etc…

NM and QM indistinguishable !?

What is known from theory?

Another hint to understand a crossover

Chiral symmetry more broken at higher density

Nuclear Matter $\langle \bar{q}q \rangle \neq 0 \qquad \langle NN \rangle \neq 0$

Quark Matter $\langle q_R q_R \rangle \neq 0 \qquad \langle q_L q_L \rangle \neq 0$

breaks $SU(N_f)_R$ breaks $SU(N_f)_L$

Vectorial rotation can be canceled by color rot.

$$SU(N_f)_R \times SU(N_f)_L \times U(1)_V \to SU(N_f)_V$$

Color superconducting QM has the same symmetry as NM

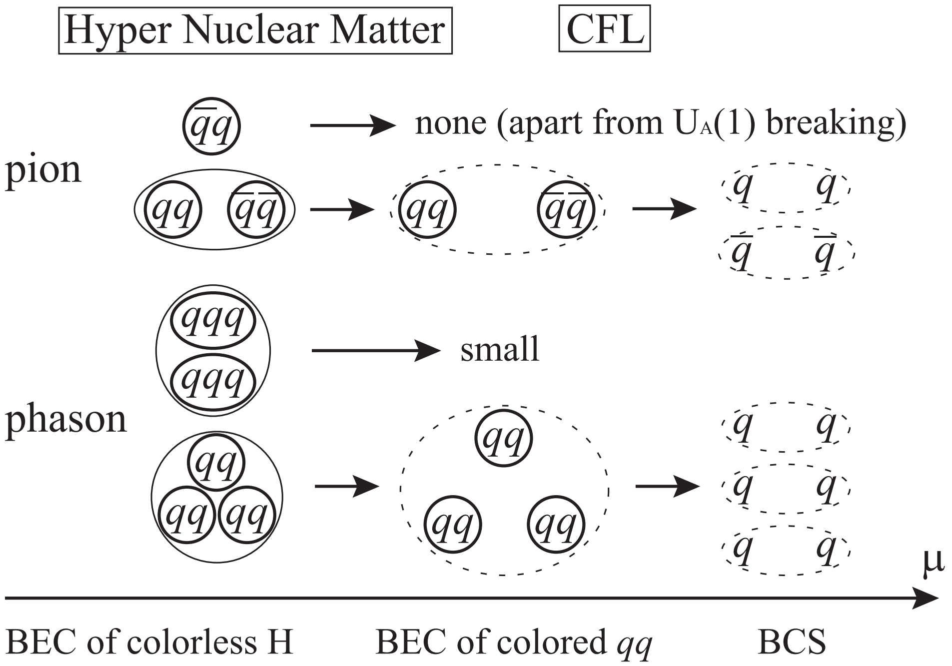
NM and QM indistinguishable indeed

Schaefer-Wilczek (1998)

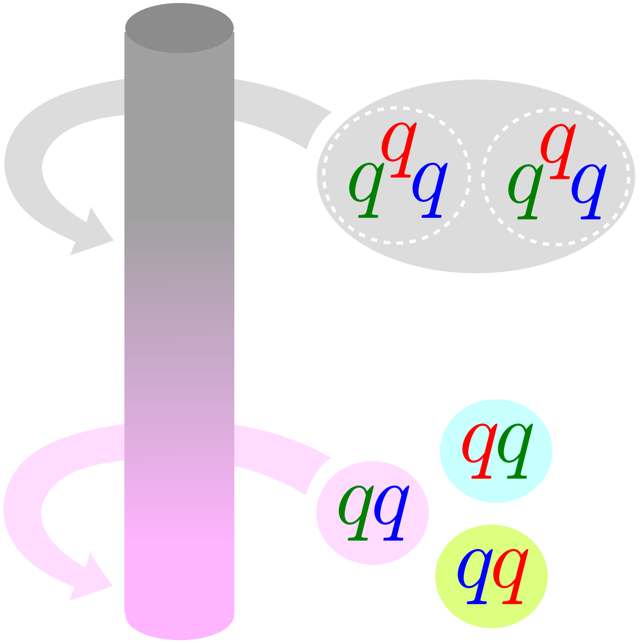
What is known from theory?

All excitations must be continuously connected…

Fukushima (2003)



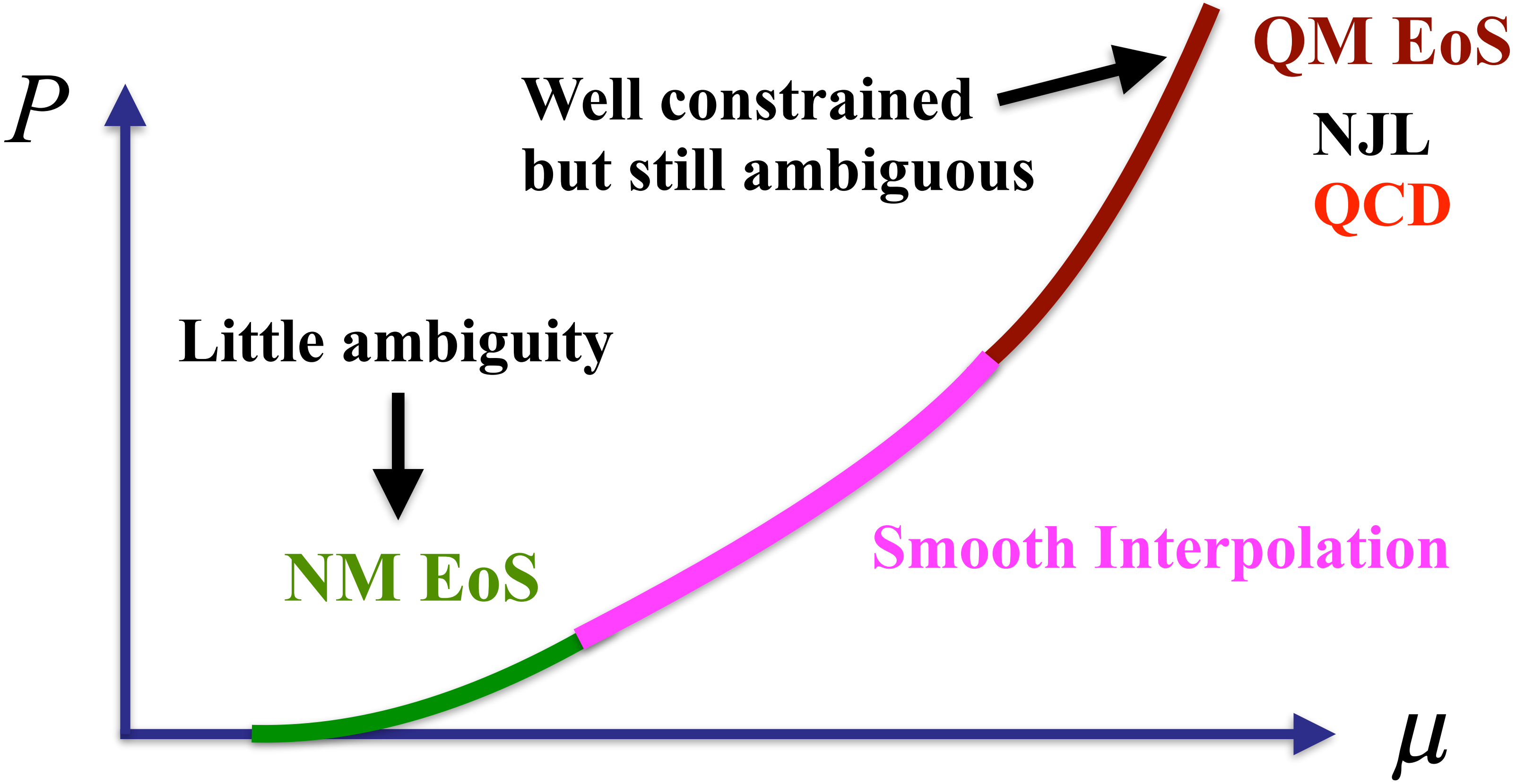
Alford-Baym-Fukushima-
-Hatsuda-Tachibana (2017)



What is known from theory?

(So far best) Bottom-up Approach

Masuda, Hatsuda, Takatsuka, Kojo, Baym, …



What is known from theory?

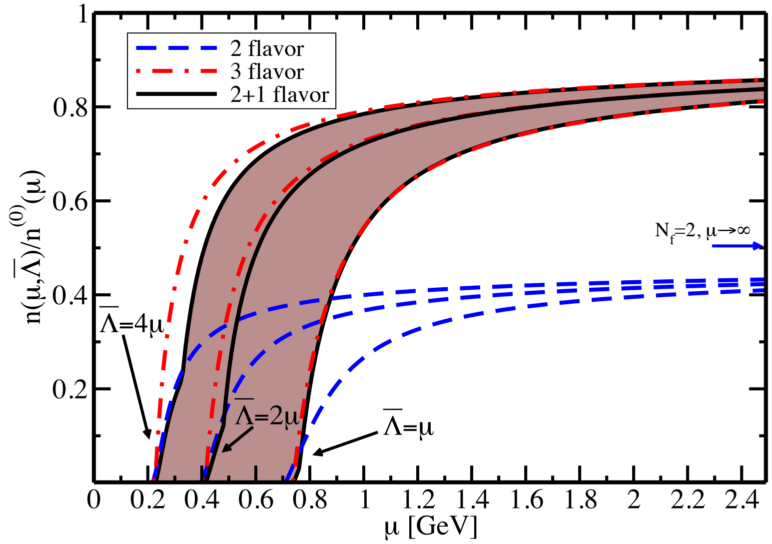
You may wonder if pQCD works at high density?

Freedman-McLerran (1977)

Baluni (1978)

$\sim \mathcal{O}(\alpha_s^2)$

Kurkela-Romatschke-Vuorinen (2009) $\sim \mathcal{O}(\alpha_s^2) + m_s$



Convergence seems to be good (as compared to high-*T*)

This is not resummed perturbation but very naive expansion

From Experiment to EoS

From Experiment to EoS

Bayesian Analysis

B : M-R Observation
A : EoS Parameters

(Bayes' theorem)

$$\underbrace{P(A|B)}_{\text{Want to know}}\overset{\text{Normalization}}{\cancel{P(B)}} = \underbrace{P(B|A)}_{\text{Likelihood}}\underbrace{P(A)}_{\text{prior}}$$

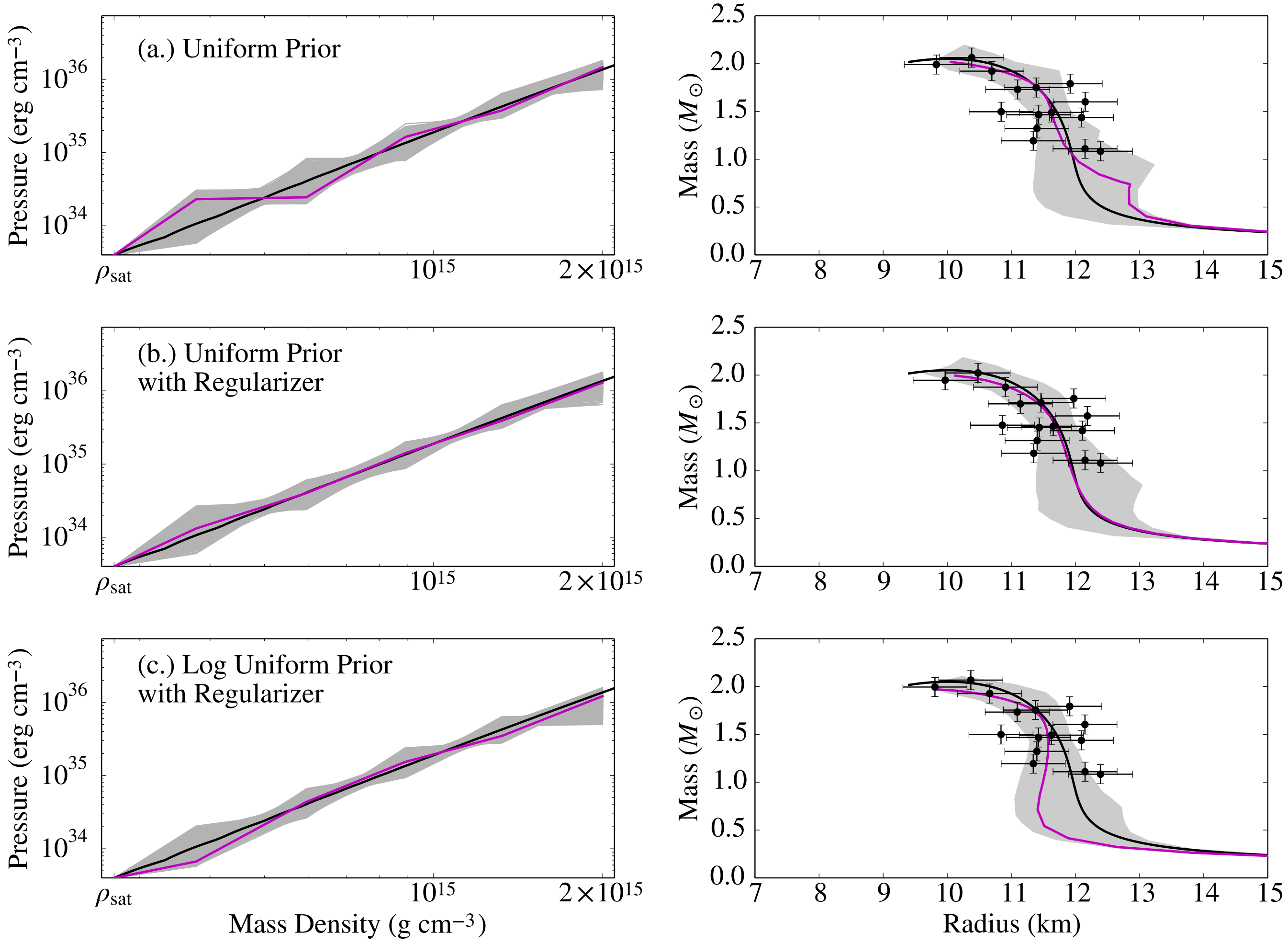
Calculable by TOV   Model

Model must be assumed.
EoS parametrization must be introduced.
Integration in parameter space must be defined.

From Experiment to EoS

Raithel-Ozel-Psaltis (2017) Mock data (SLy + Noises)

Prior Dep.



From Experiment to EoS

What we want to have *ideally* is…

Several *M*-*R* observation points with errors

Optimized Mapping

Several parameters to characterize EoS

$$\{M_i, R_i\} \qquad \{P_i\} = F(\{M_i, R_i\}) \qquad \{P_i\}$$

Generate many random EoSs $\{P\}$ and solve TOV to have $\{M,R\}$
Assume an Ansatz for F with sufficiently many fitting parameters
Tune parameters to fit {EoS, MR} correspondance
Test the validity of F with independent {EoS, MR} data

From Experiment to EoS

This process is precisely how we develop our intuition!

If we see many (input → output) data, we will eventually have good intuition to guess input by looking at output

(Supervised) Learning = Parameter Tuning

What should be the Ansatz for the fitting function?

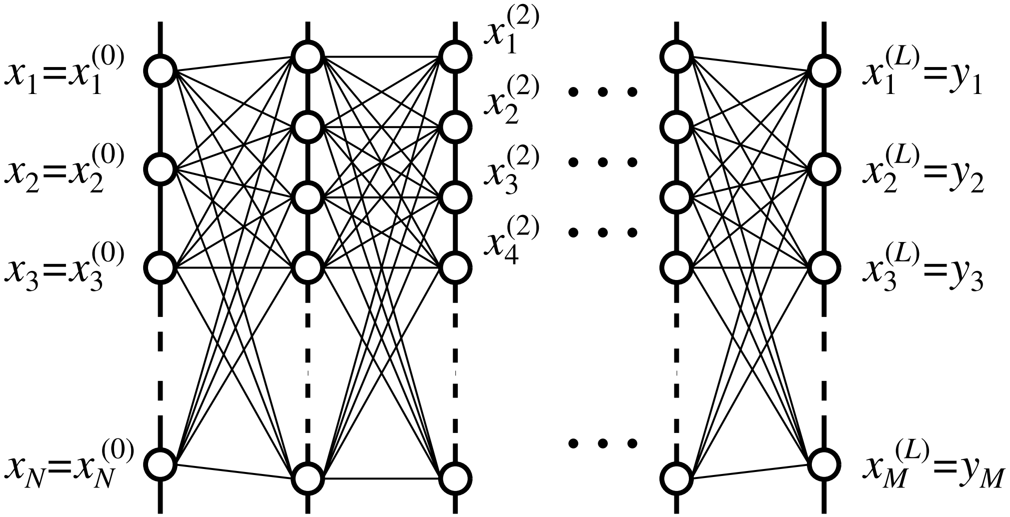
Simple "activation" functions are layered (like our brains)

In principle, any non-linear mapping can be represented

From Experiment to EoS

Input

$\{M_i, R_i\}$



Output

$\{P_i\}$

$$x_i^{(k+1)} = \sigma^{(k+1)}\left(\sum_{i=1}^{N_k} \underline{W_{ij}^{(k+1)}}\, x_j^{(k)} + \underline{a_i^{(k+1)}}\right)$$

Backpropagation

Parameters to be tuned

~~sigmoid func.~~

$\cancel{\sigma(x) = 1/(e^x + 1)}$

ReLU

$\sigma(x) = \max\{0, x\}$

tanh

$\sigma(x) = \tanh(x)$

From Experiment to EoS

For good learning, the "textbook" choice is important…

Training data (200000 sets in total)

Randomly generate **5** sound velocities → EoS × 2000 sets

Solve TOV to identify the corresponding *M*-*R* curve

Randomly pick up **15** observation points × ($n_s = 100$) sets

(with $\Delta M = 0.1 M_\odot$, $\Delta R = 0.5\,\mathrm{km}$)

(The machine learns the *M*-*R* data have error fluctuations)

Validation data (200 sets)

Generate independently of the training data

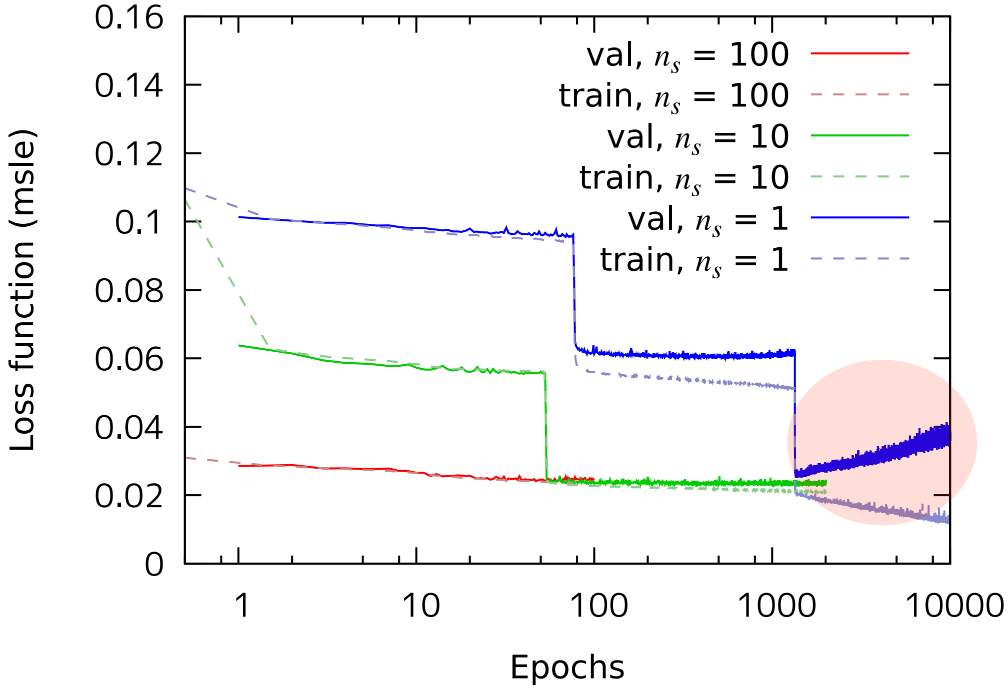
From Experiment to EoS

Our Neural Network Design

Layer index	Nodes	Activation
1	30	N/A
2	60	ReLU
3	40	ReLU
4	40	ReLU
5	5	tanh

Probably we don’t need such many hidden layers and such many nodes… anyway, this is one working example…

From Experiment to EoS



“Loss Function"
= deviation from the
true answers

Monotonically decrease for the training data, but not necessarily so for the validation data

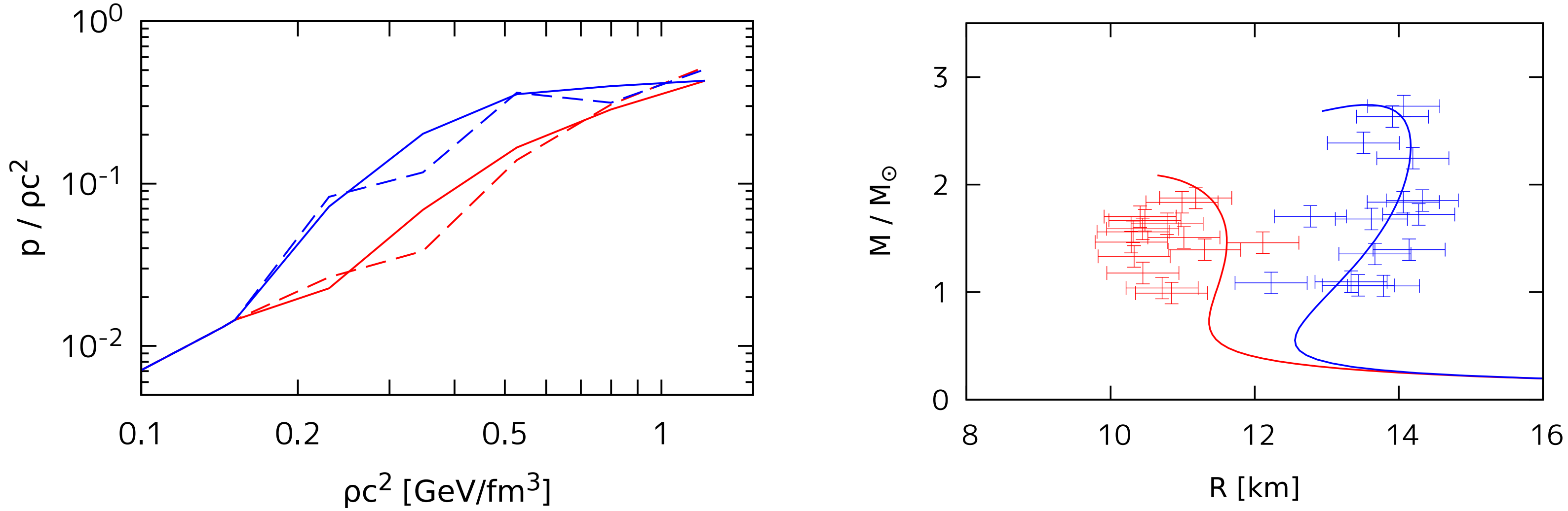
With fluctuations in the training data, the learning is quick!

Once the overlearning occurs, the performance gets worse!

From Experiment to EoS

Test with the validation data **Fujimoto-Fukushima-Murase (2017)**

(parameters not optimized to fit the validation data)



Dashed lines : randomly generated original data

Solid lines : reconstructed EoS and associated *M*-*R* rel.

From Experiment to EoS

Overall performance test

Mass ($M_\odot$)	Raw RMS (km)	Filtered RMS (km)
0.8	0.90	0.15
1.0	0.90	0.21
1.2	0.90	0.23
1.4	0.91	0.25
1.6	1.06	0.25
1.8	1.10	0.27

Sometimes, due to random unphysical EoS, the reconstruction completely fails ← very easy to exclude from the analysis

Excluding such abnormal data (~15%), the agreement is remarkable (remember, input data involve ΔR=0.5km)

Summary

Neutron Star Constraint

- ☐ Mass constraint
- ☐ Radius — symmetry energy, tidal deformability

Theoretical Approach

- ☐ Smooth interpolation (no phase transition)
- ☐ Perturbative QCD calculations need more upgrade

Experimental Data Analysis

- ☐ Bayesian analysis (hidden assumptions)
- ☐ Machine (deep) learning; easy and practical
  How to estimate confidential levels?