Stress and energy distribution in quark–anti–quark systems using gradient flow

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for FlowQCD Collaboration :
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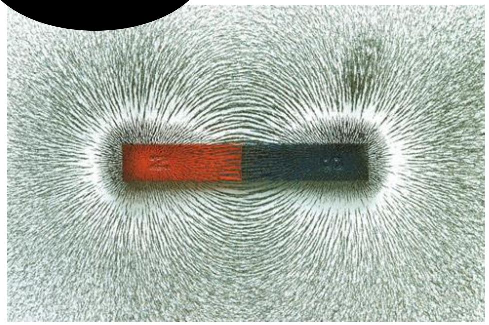
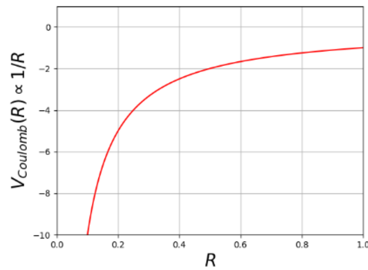
Masakiyo Kitazawa, Tetsuo Hatsuda

QED vs QCD

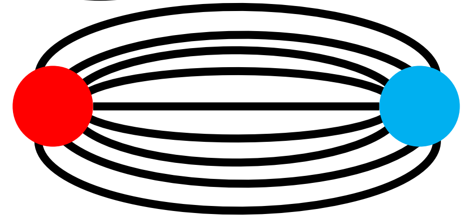
QED

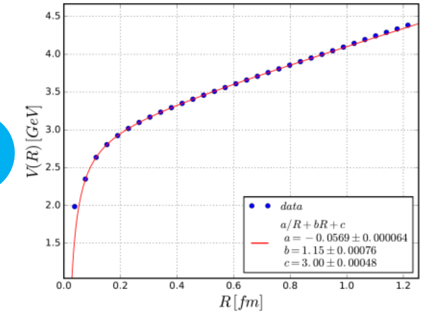




- ✓ Electric field spreads all over the space
- ✓ Coulomb potential

QCD

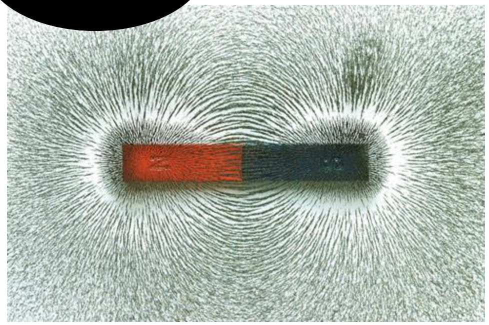


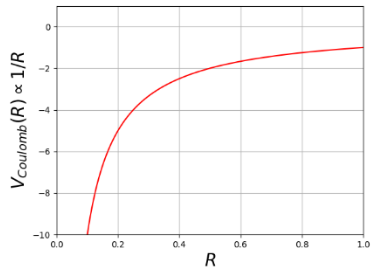


- ✓ flux tube, squeezed one-dimentionally !
- ✓ confinement potential

QED vs QCD

QED





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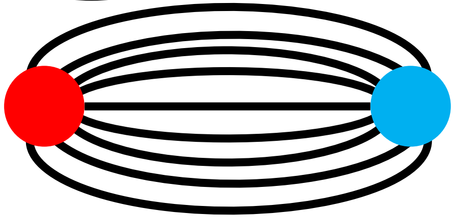
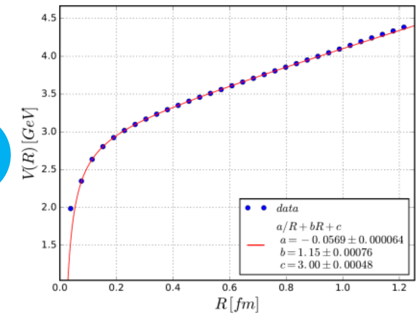
QCD





- ✓ flux tube, squeezed one-dimentionally !
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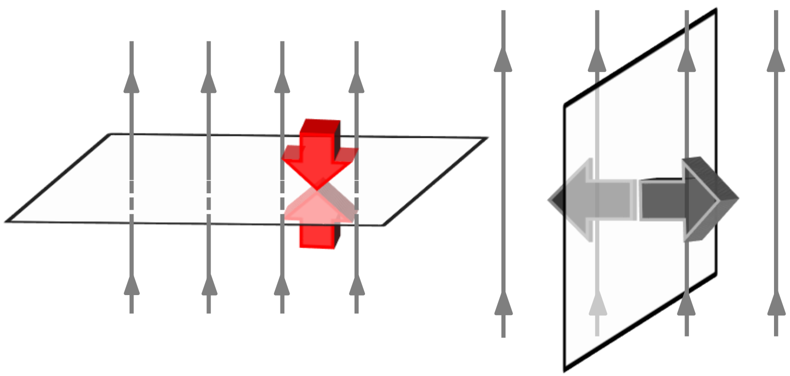
Action through medium

Maxwell stress



- ✓ Perpendicular plane : attractive
- ✓ Parallel plane : repulsive

Energy Momentum Tensor (EMT)

goal

Physics around flux tube in terms of energy and stress

Energy density

$$T_{\mu\nu} = \begin{pmatrix} \boxed{T_{00}} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

Momentum density

Stress tensor

- ✓ **Gauge invariant !**
- ✓ Determine **absolute values** of all components

Action through medium

Measurement of the Stress on the Lattice

To Do

①prepare $q\bar{q}$ on the lattice and ②measure EMT around $q\bar{q}$

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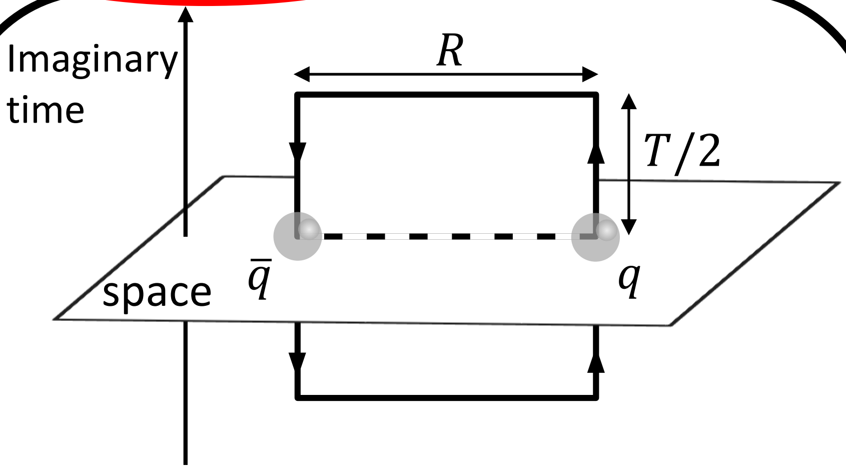
Wilson Loop

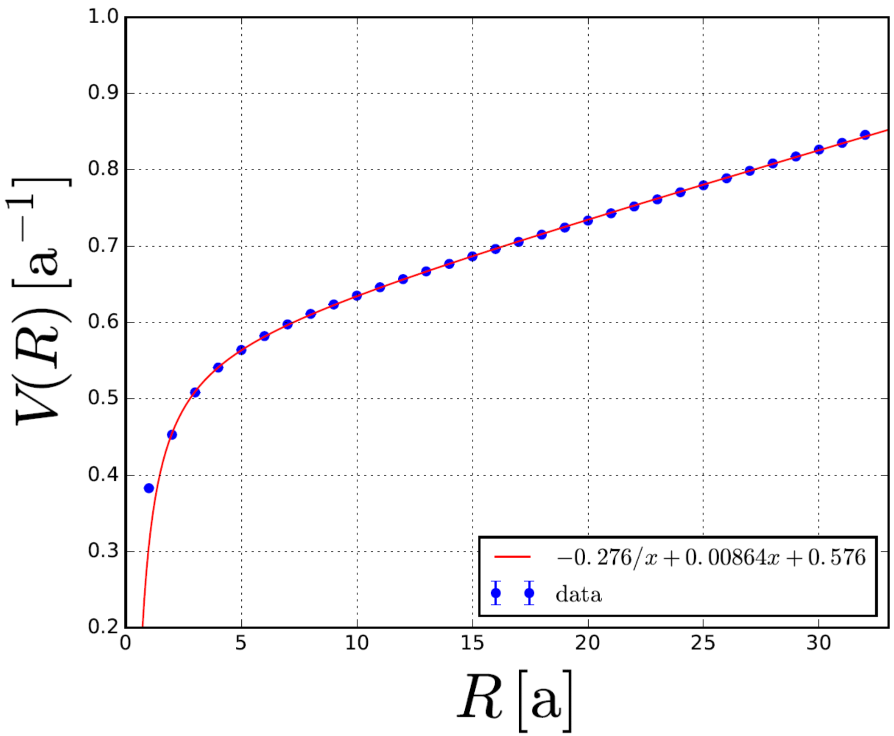


$$W(R,T)$$
$$= C_0 \exp[-V_0(R)T] + C_1 \exp[-V_1(R)T] + \cdots$$

$$V_0(R) = -\lim_{T\to\infty} \frac{1}{T} \log W(R,T)$$

Ground state potential

Confinement potential



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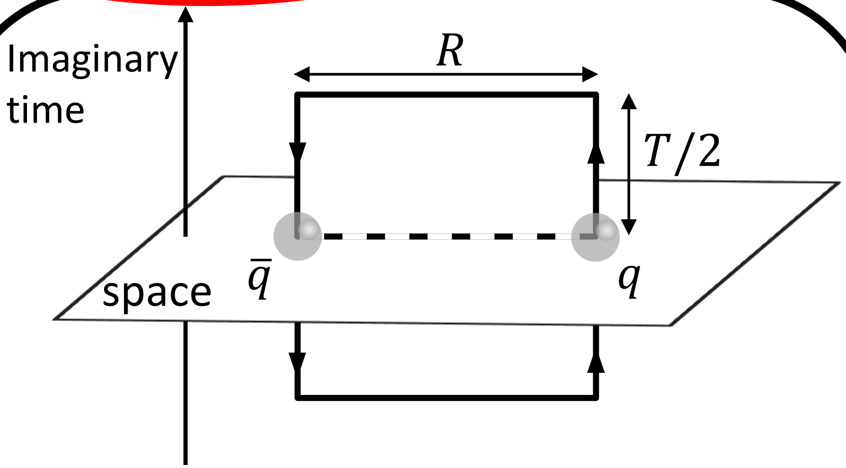

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$$V_0(R) = -\lim_{T\to\infty} \frac{1}{T} \log W(R,T)$$

Ground state potential

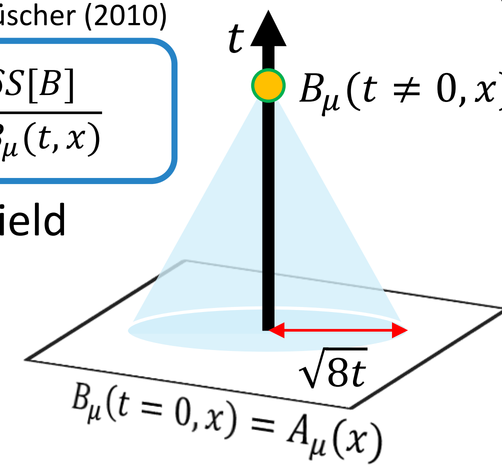
Gradient flow

Flow eq.

Lüscher (2010)

$$\frac{\partial B_\mu(t,x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t,x)}$$

B_μ : smeared field



EMT defined via gradient flow

Suzuki (2013)

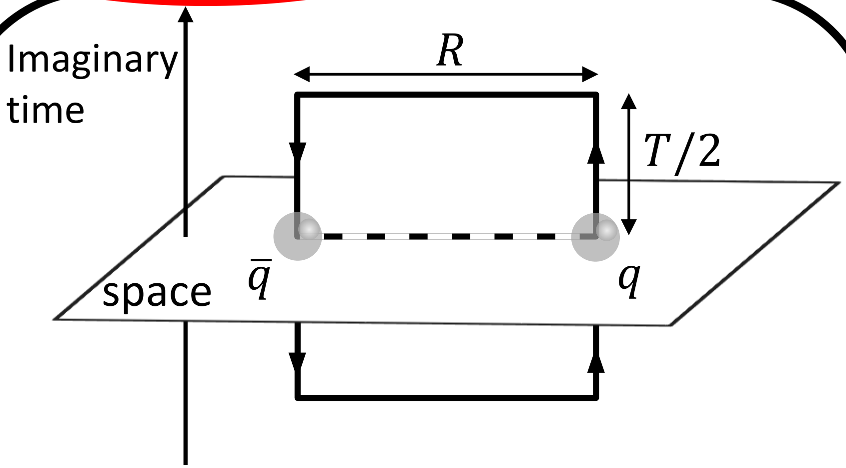
$$T_{\mu\nu}(t,x) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} [E(t,x) - \langle E(t,x) \rangle] + O(t)$$

Measurement of the Stress on the Lattice

To Do

①prepare $q\bar{q}$ on the lattice and ②measure EMT around $q\bar{q}$

Wilson Loop



$W(R,T)$
$= C_0 \exp[-V_0(R)T] + C_1 \exp[-V_1(R)T] + \cdots$

$V_0(R) = -$

Gradient flow

Flow eq.

Lüscher (2010)

$$\frac{\partial B_\mu(t,x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t,x)}$$

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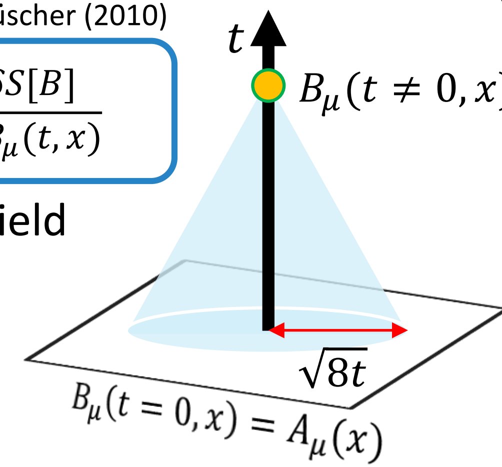


EMT defined via gradient flow

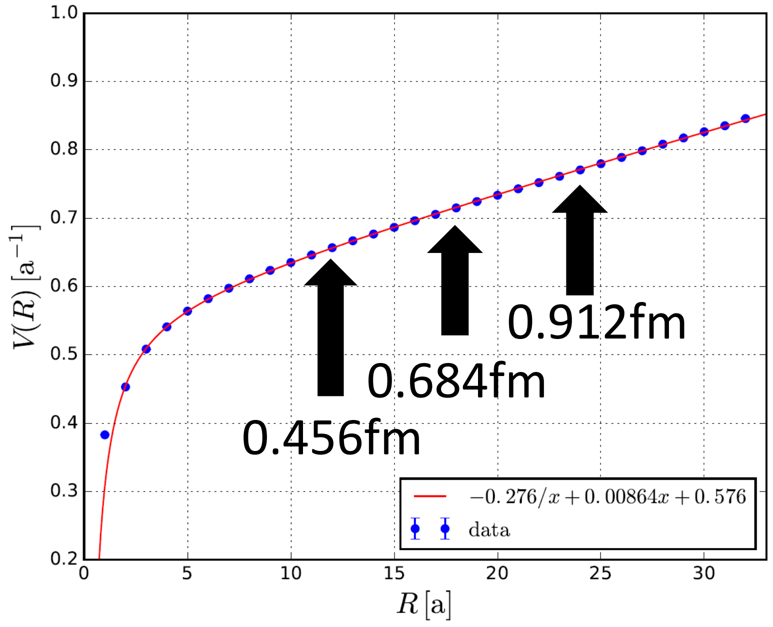
Suzuki (2013)

$) - \langle E(t,x) \rangle] + O(t)$

$$\langle T_{\mu\nu}(t,x) \rangle_W = \frac{\langle T_{\mu\nu}(t,x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle T_{\mu\nu}(t,x) \rangle$$

Setup

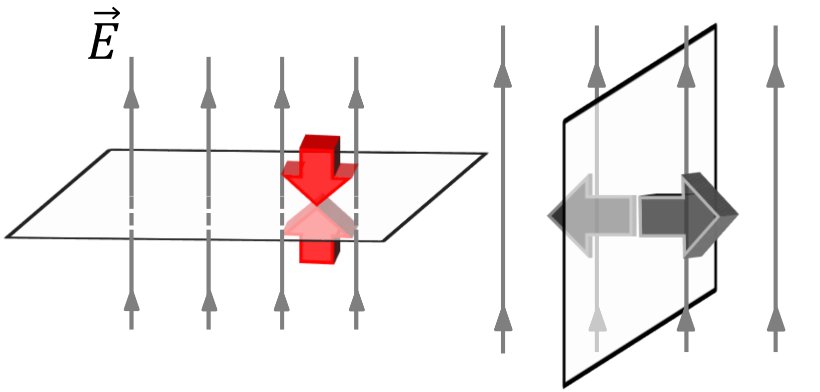
- ✓ Quenched SU(3)
- ✓ Wilson gauge action
- ✓ Clover operator
- ✓ APE smearing for spatial links
- ✓ Multihit improvement in temporal link
- ✓ Simulation using BlueGene/Q @ KEK



β	lattice spacing	ratio	lattice size	# of statistics
6.304	0.057 fm	4	48^4	140
6.465	0.046 fm	5	48^4	440
6.600	0.038 fm	6	48^4	1500
6.819	0.029 fm	8	64^4	1000

Stress Distribution in Maxwell Theory

$$T_{ij} = \epsilon_0\left(E_iE_j - \frac{\delta_{ij}}{2}E^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{\delta_{ij}}{2}B^2\right)$$

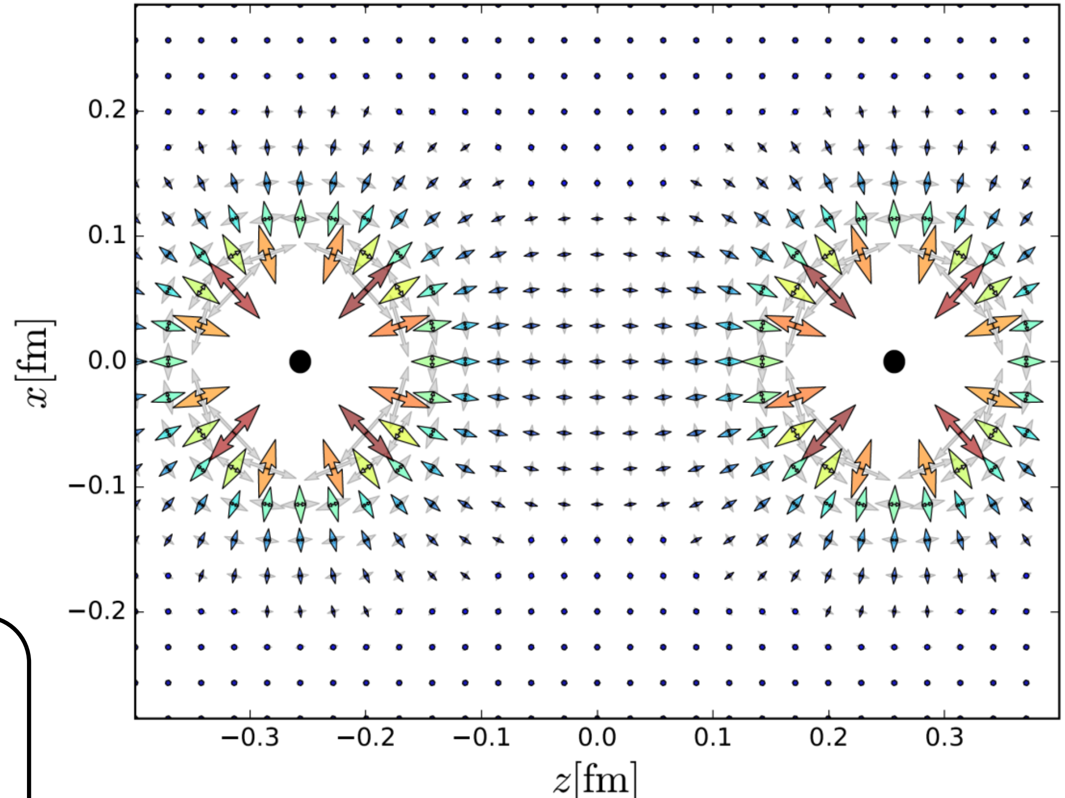


- ✓ Perpendicular plane : attractive
- ✓ Parallel plane : repulsive

✓ stress tensor

$$T_{ij}n_j^{(\lambda_k)} = \lambda_k n_i^{(\lambda_k)}$$

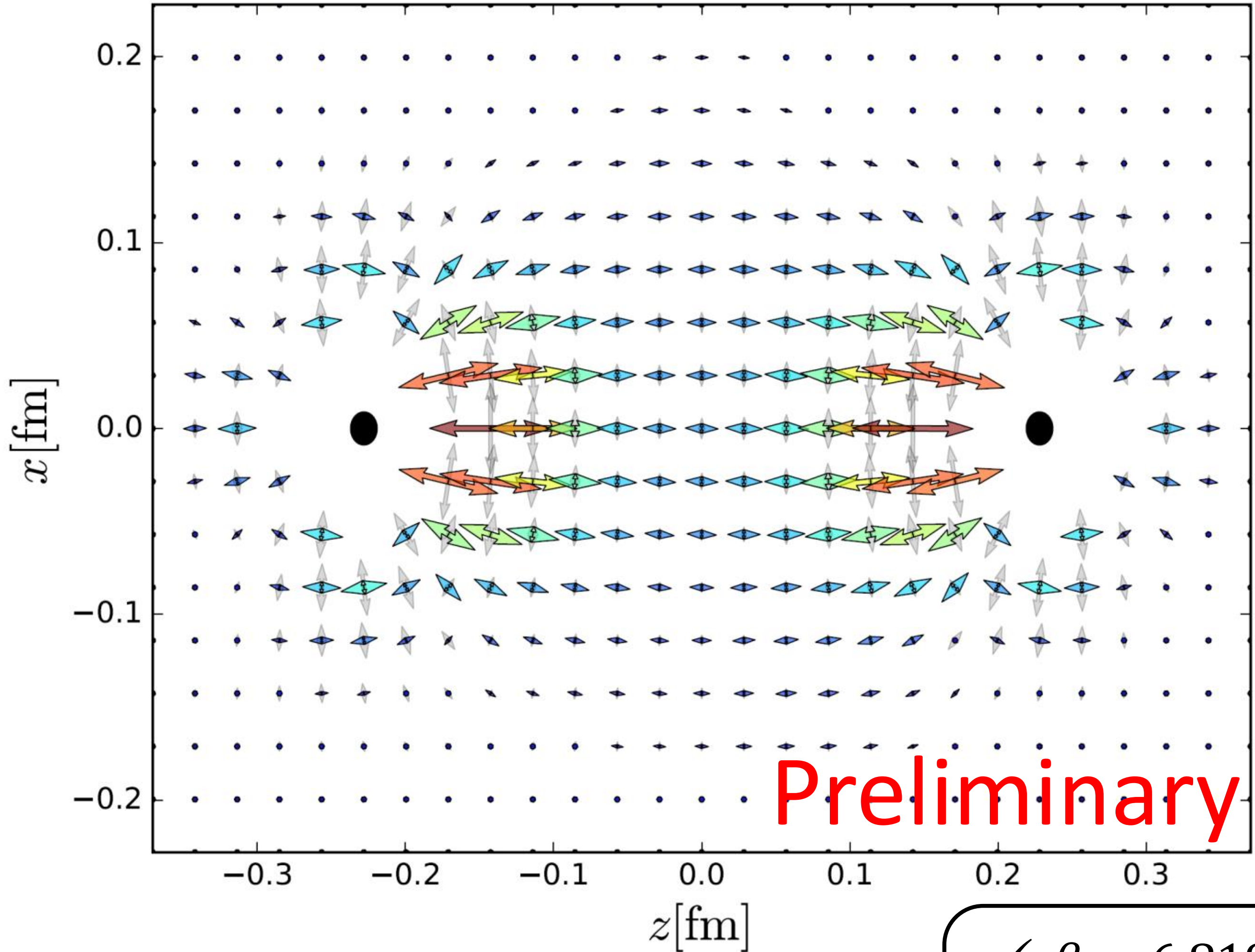
$$(i,j = x,y,z; k = x,y,z)$$

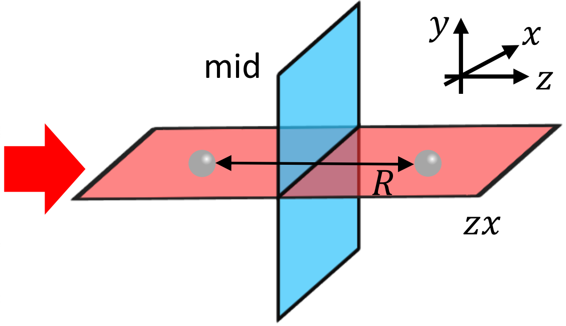


Colored : attractive

Gray : repulsive

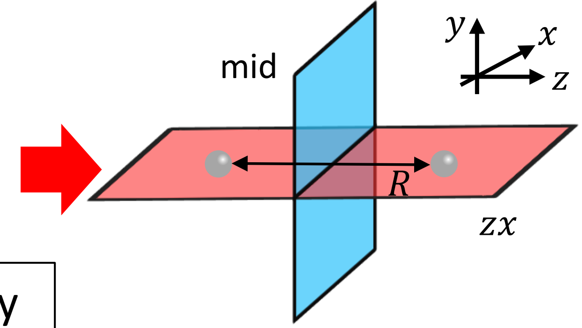
Stress Distribution in SU(3) YM Theory



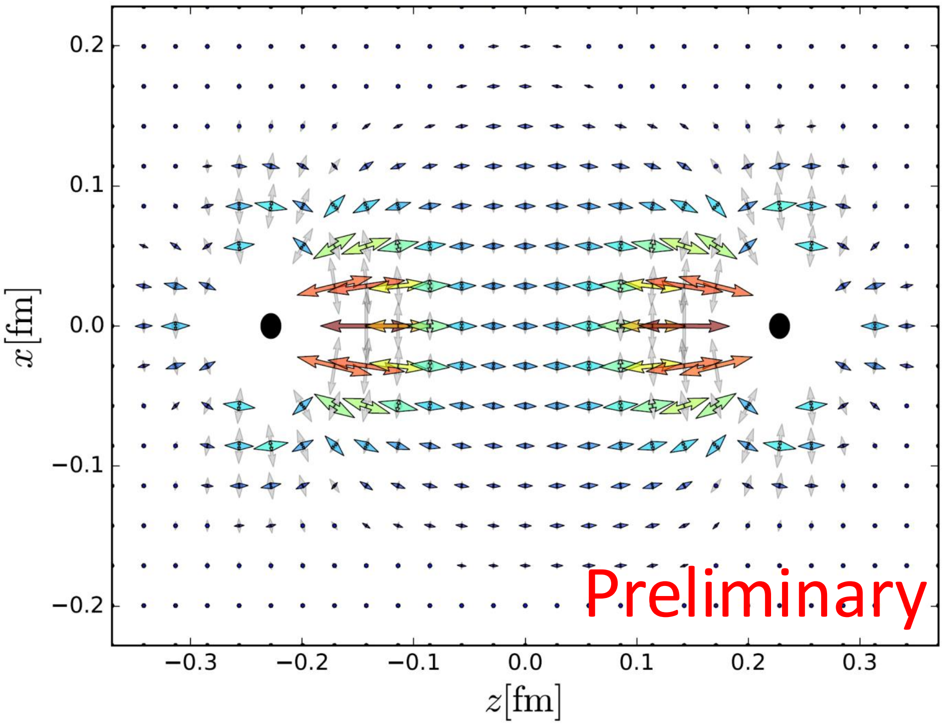


- ✓ $\beta = 6.819$ (no continuum limit)
- ✓ $R = 0.456[\mathrm{fm}]$, $a = 0.029[\mathrm{fm}]$
- ✓ Only $t \to 0$(linear fit)

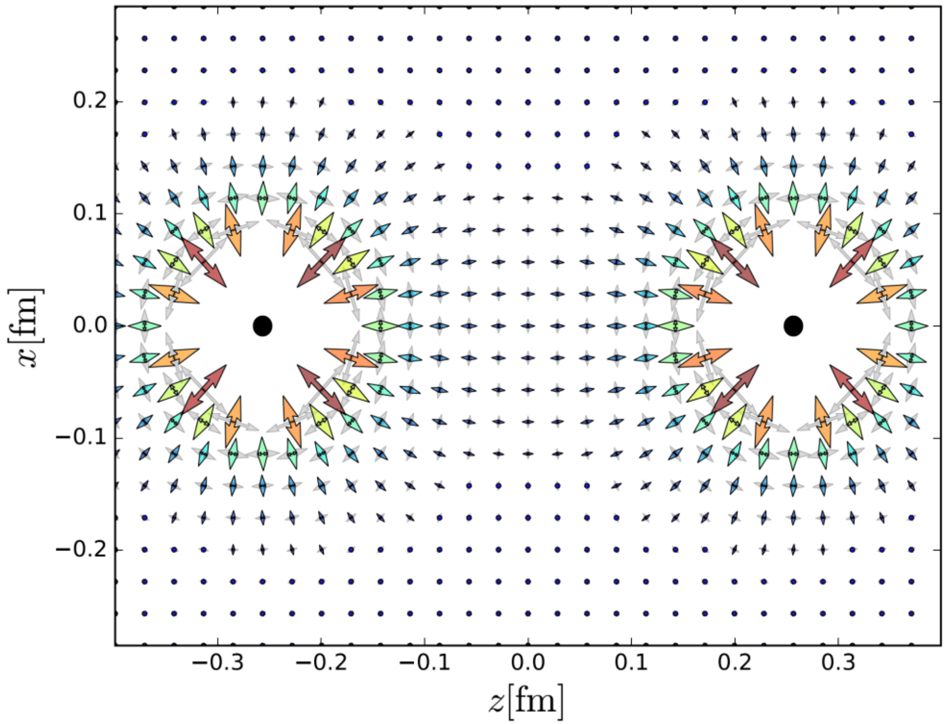
SU(3) YM Theory vs Maxwell Theory



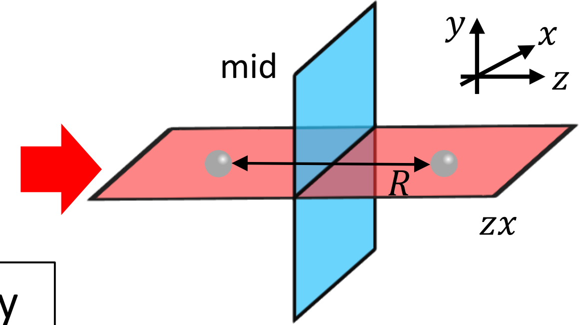
SU(3) YM Theory



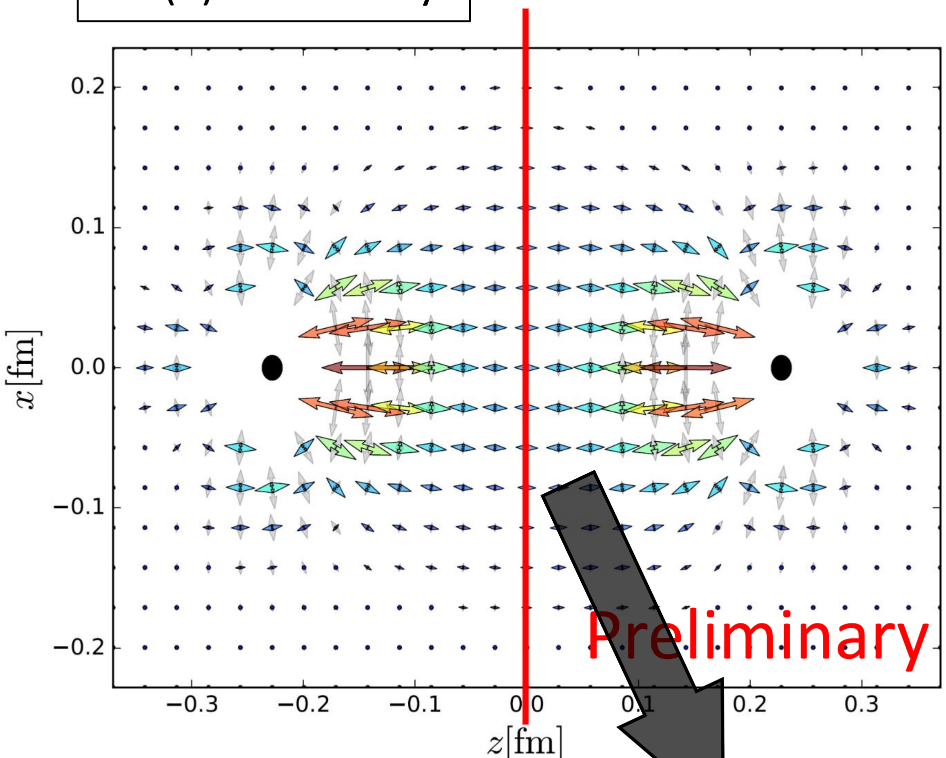
Maxwell Theory



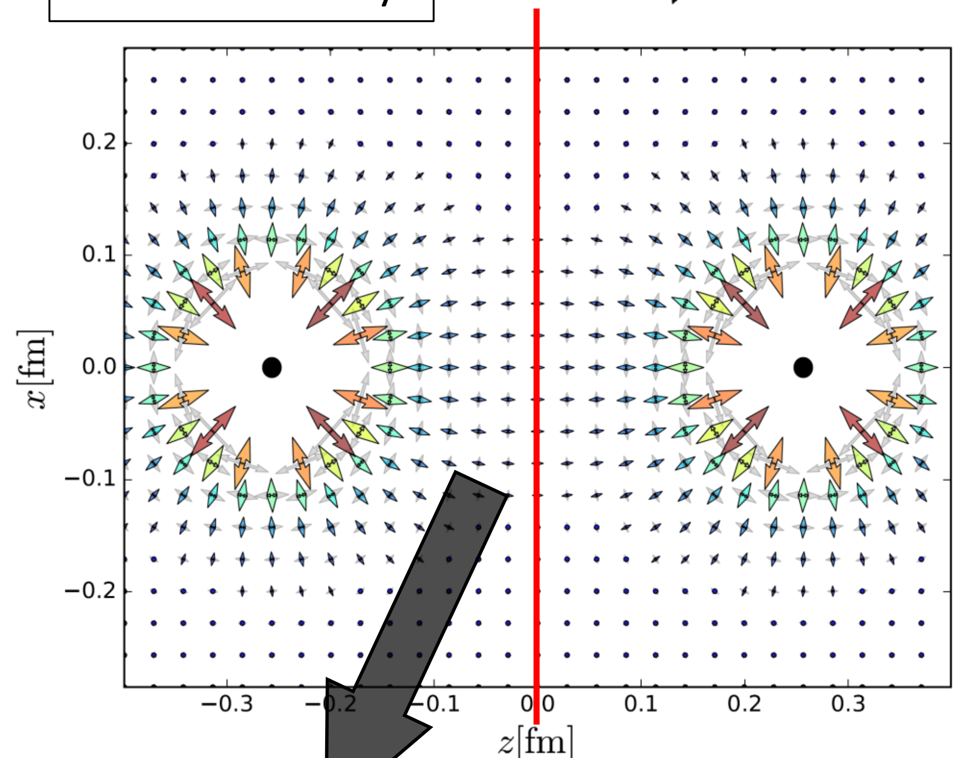
SU(3) YM Theory vs Maxwell Theory



SU(3) YM Theory



Maxwell Theory

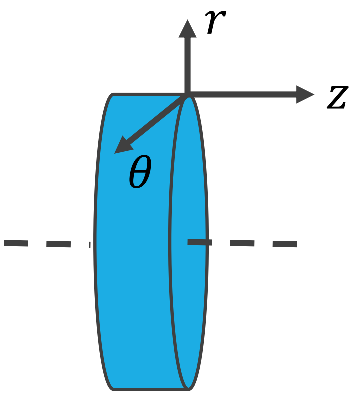


next

We focus on mid-plane.

EMT in Maxwell Theory (revisit)

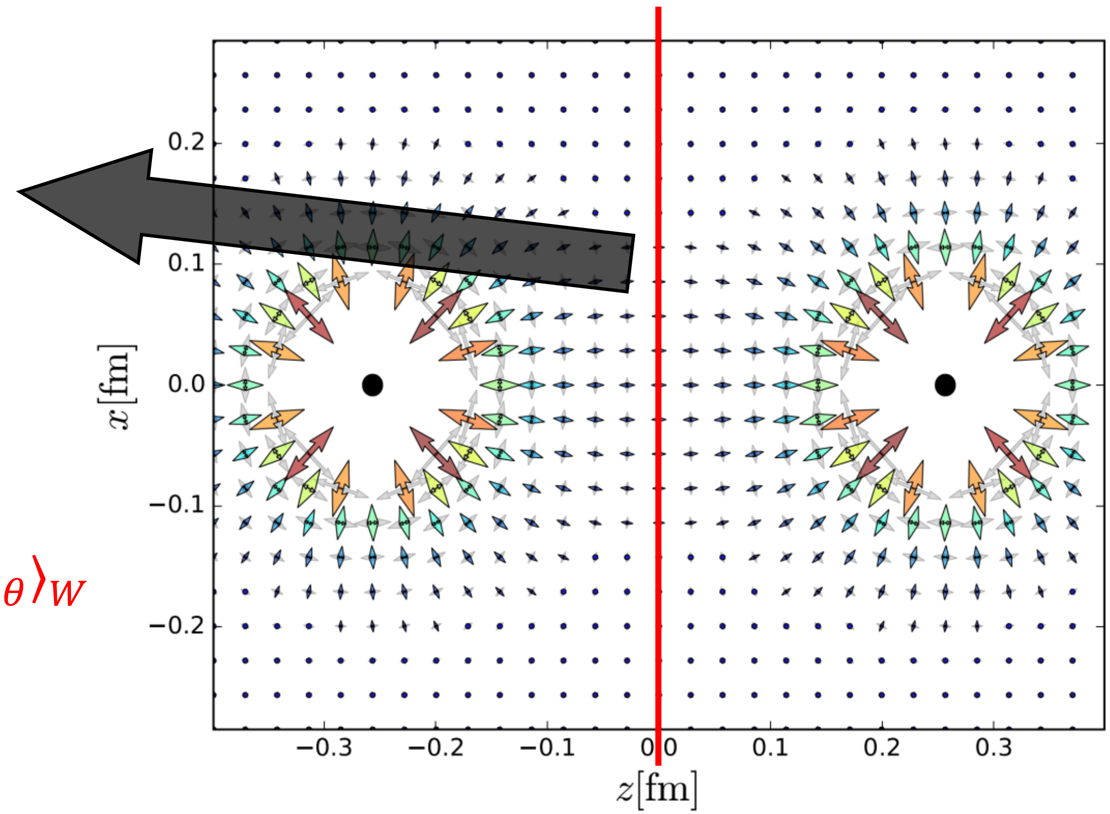
$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{\delta_{ij}}{2} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{\delta_{ij}}{2} B^2 \right)$$



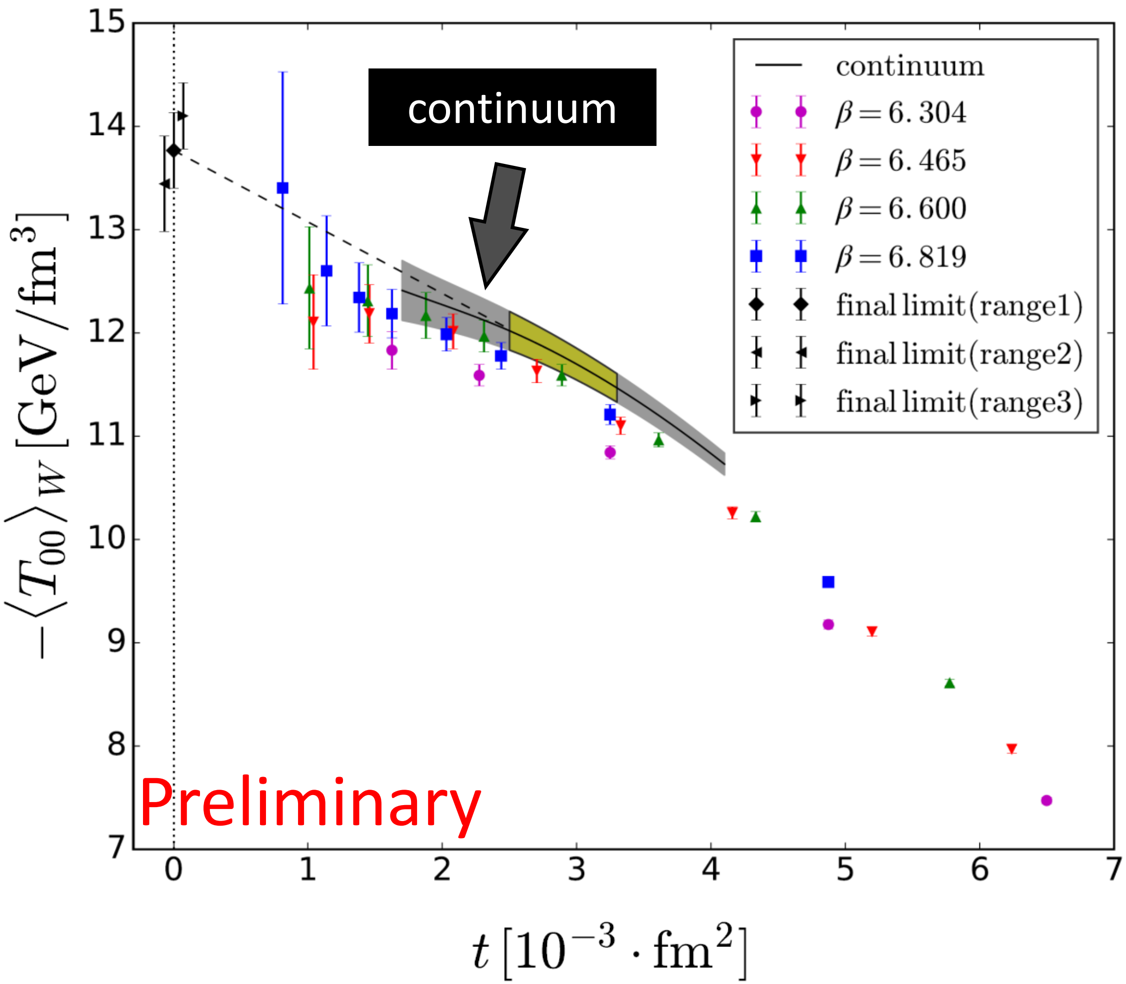
(In this case, $\vec{E} = (0,0,E), \vec{B} = \vec{0}$)

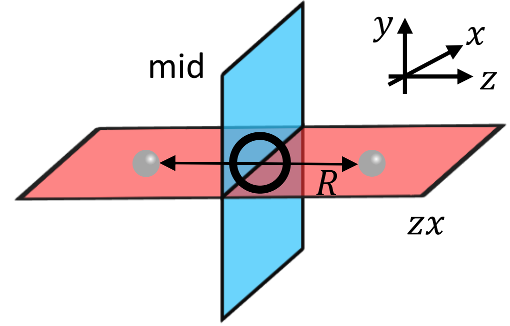
$-\langle T_{00} \rangle_W = -\langle T_{zz} \rangle_W = \langle T_{rr} \rangle_W = \langle T_{\theta\theta} \rangle_W$

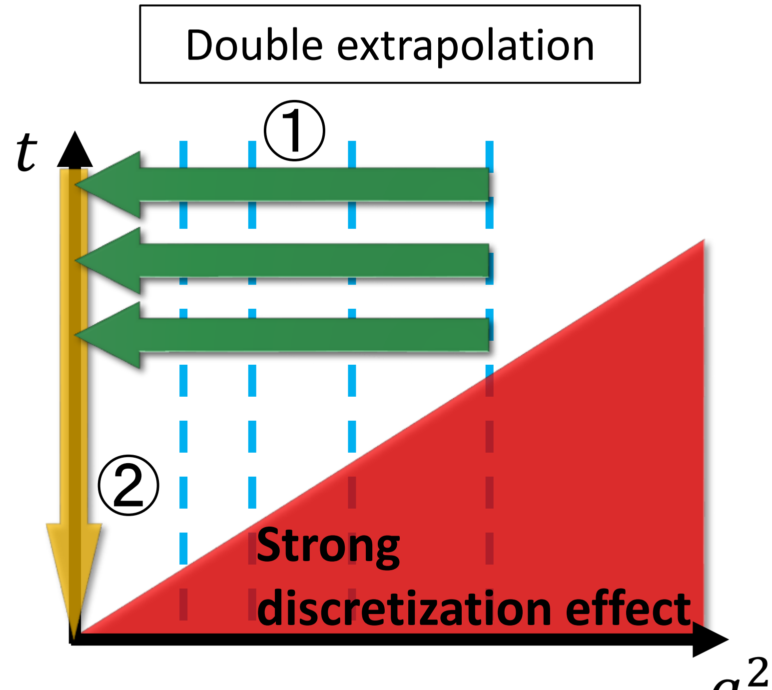
degenerate



Double Extrapolation @ mid point



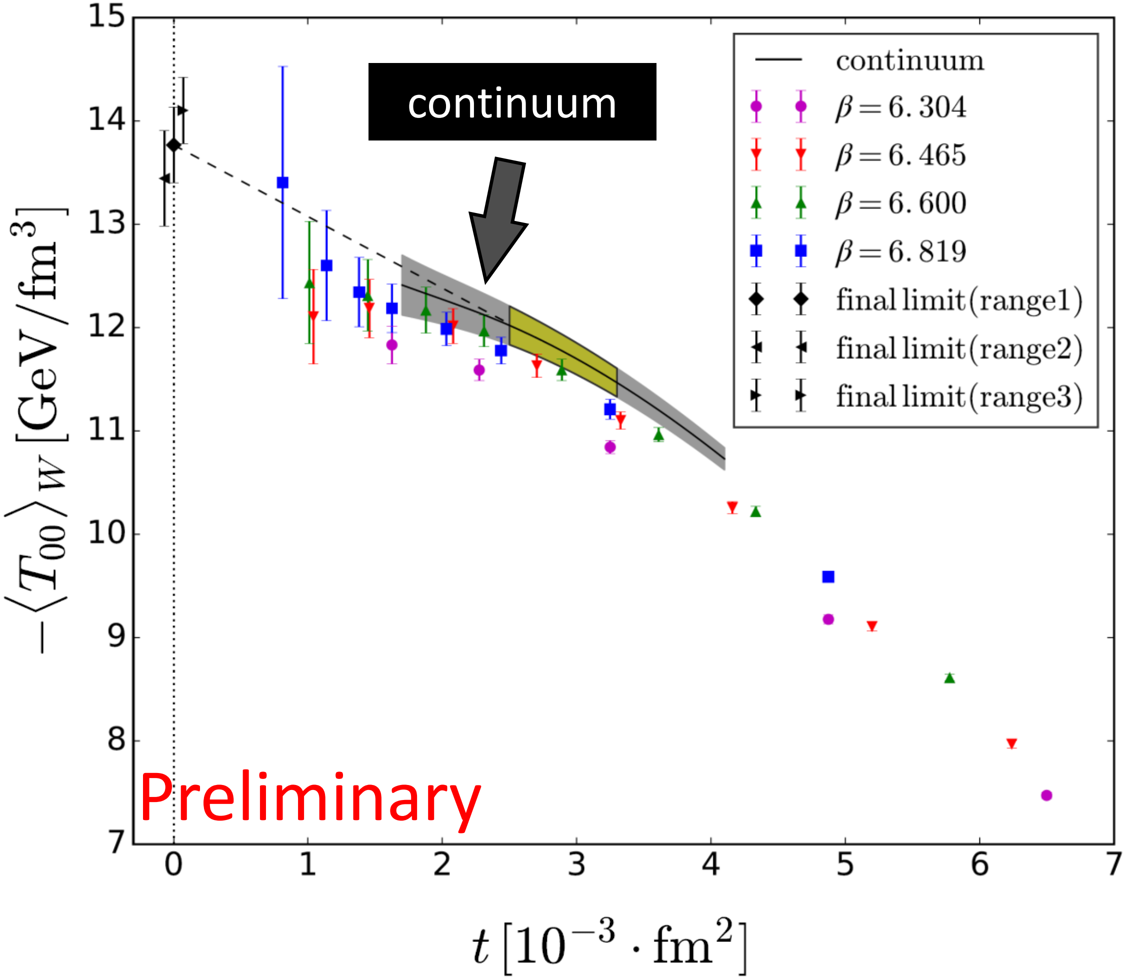


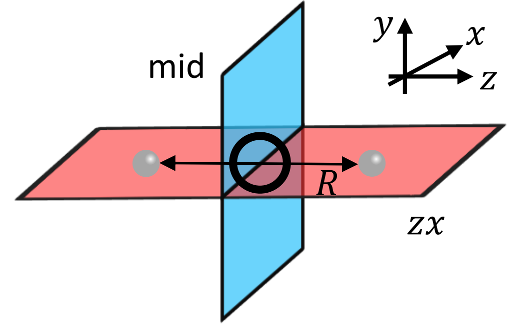


$$O_{\text{lat}} = O_{\text{cont}} + \cancel{C_0 t} + \cancel{C_1 \frac{a^2}{t}} + \cdots$$

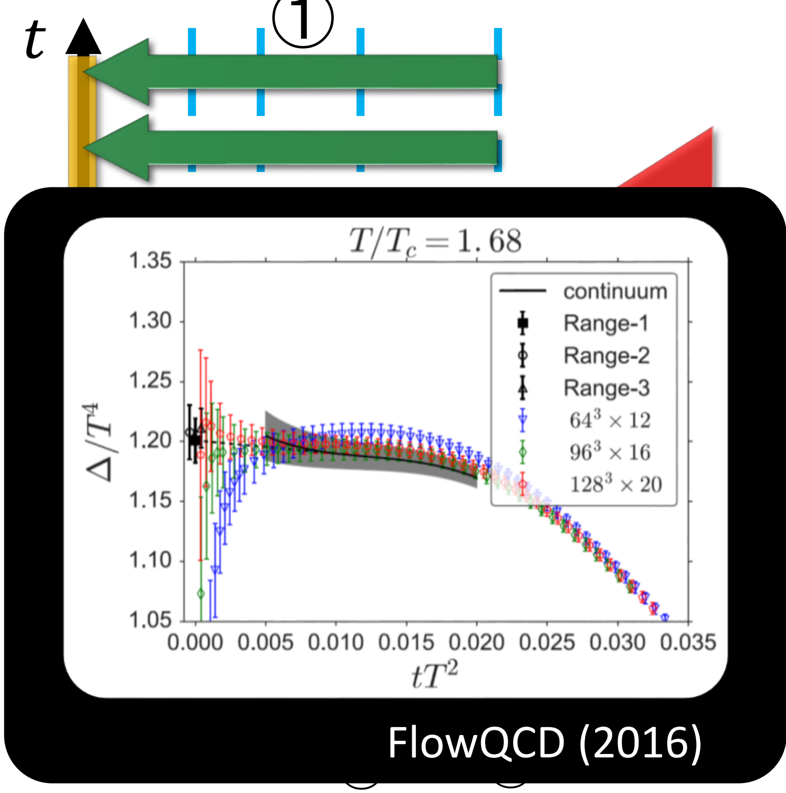
② ①

Double Extrapolation @ mid point

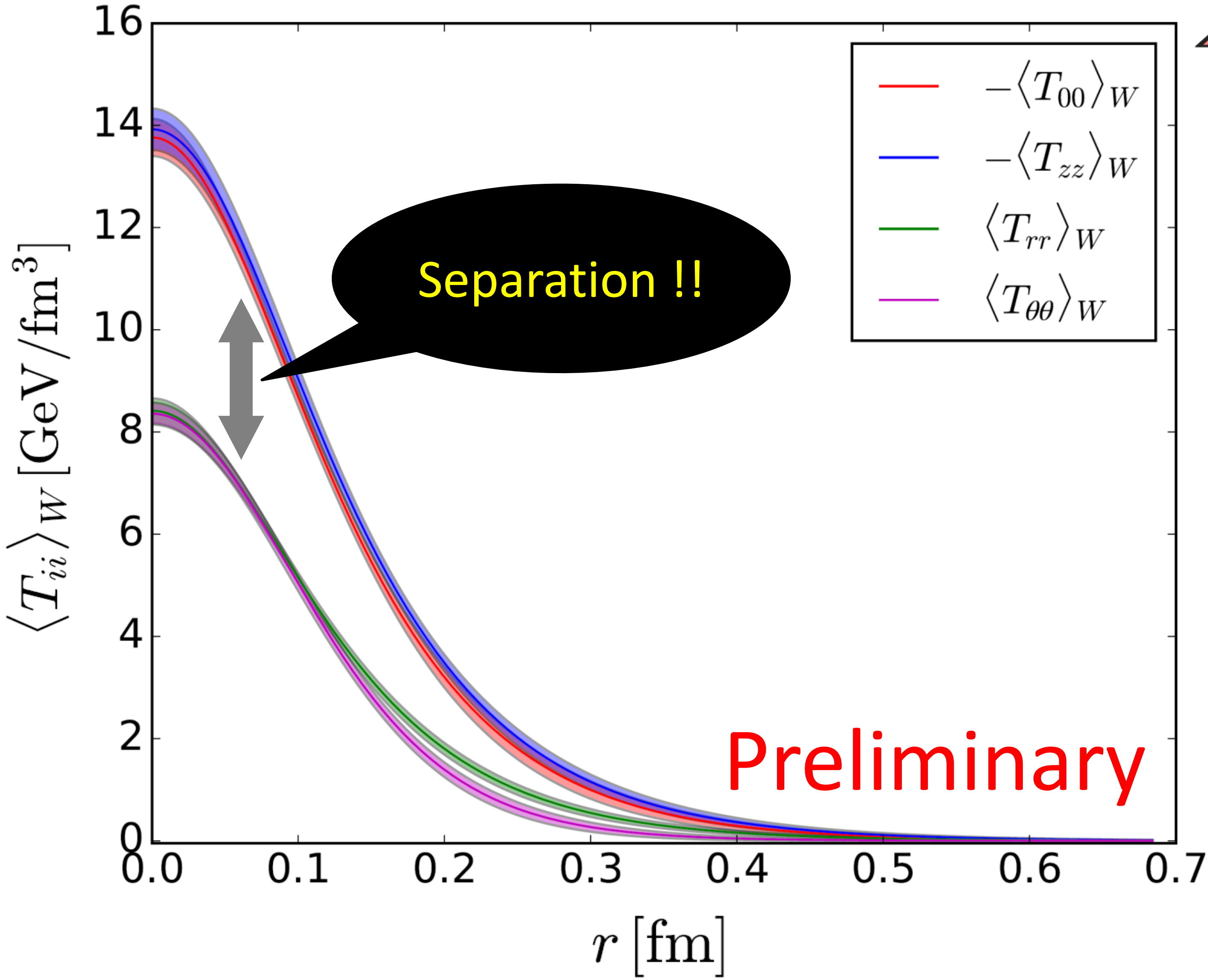


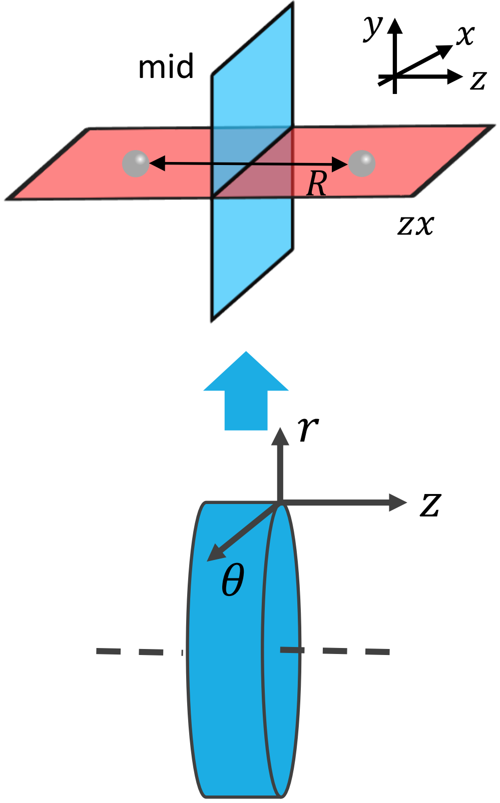


Double extrapolation



Profile of $\langle T_{ii} \rangle_W (i = 0, z, r, \theta)$ (mid plane)





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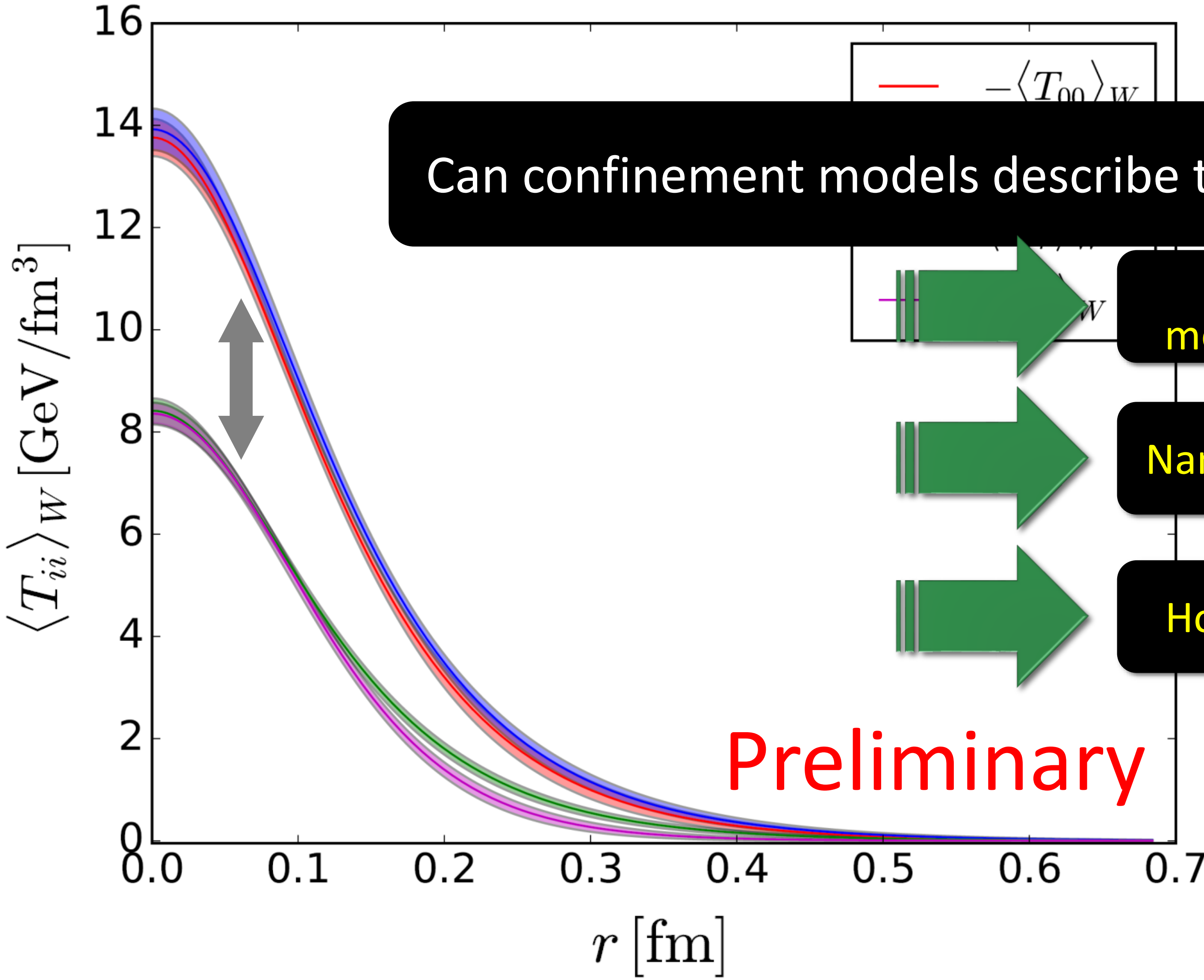


Can confinement models describe this separation ?

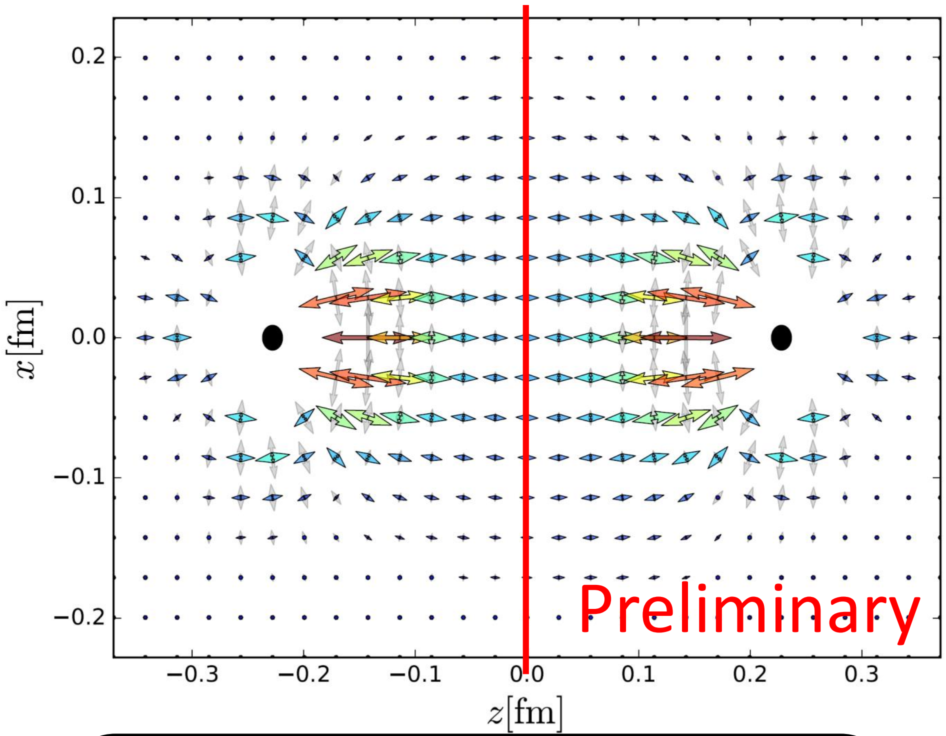
Abelian dual monopole model

Nambu-Goto string

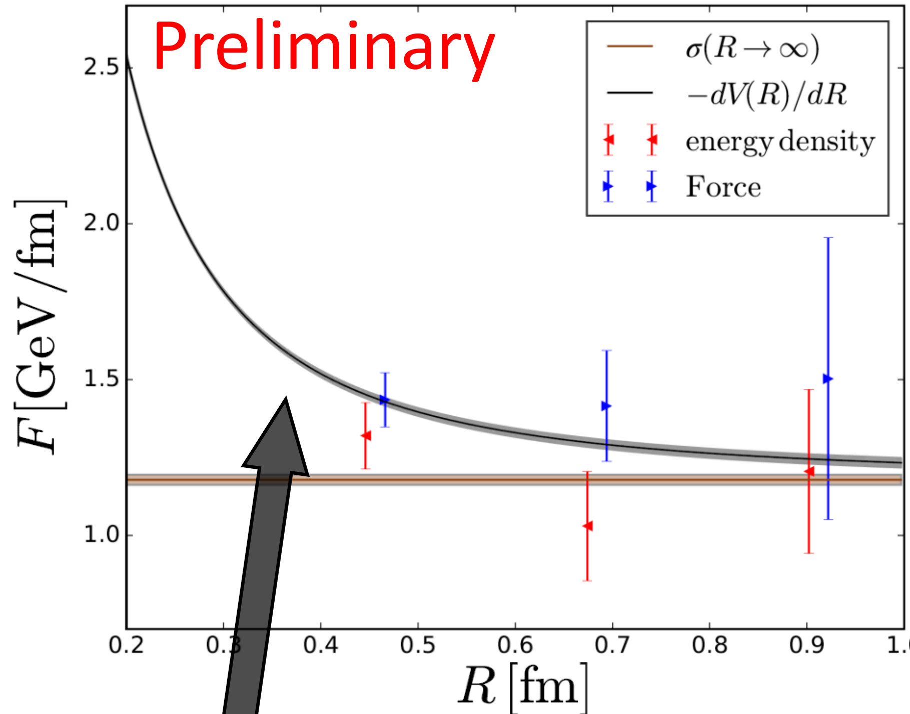
Holographic QCD

Potential vs EMT (mid plane)

From EMT



From confinement potential



Energy density$= \int_{mid} T_{00}\, d^2x$

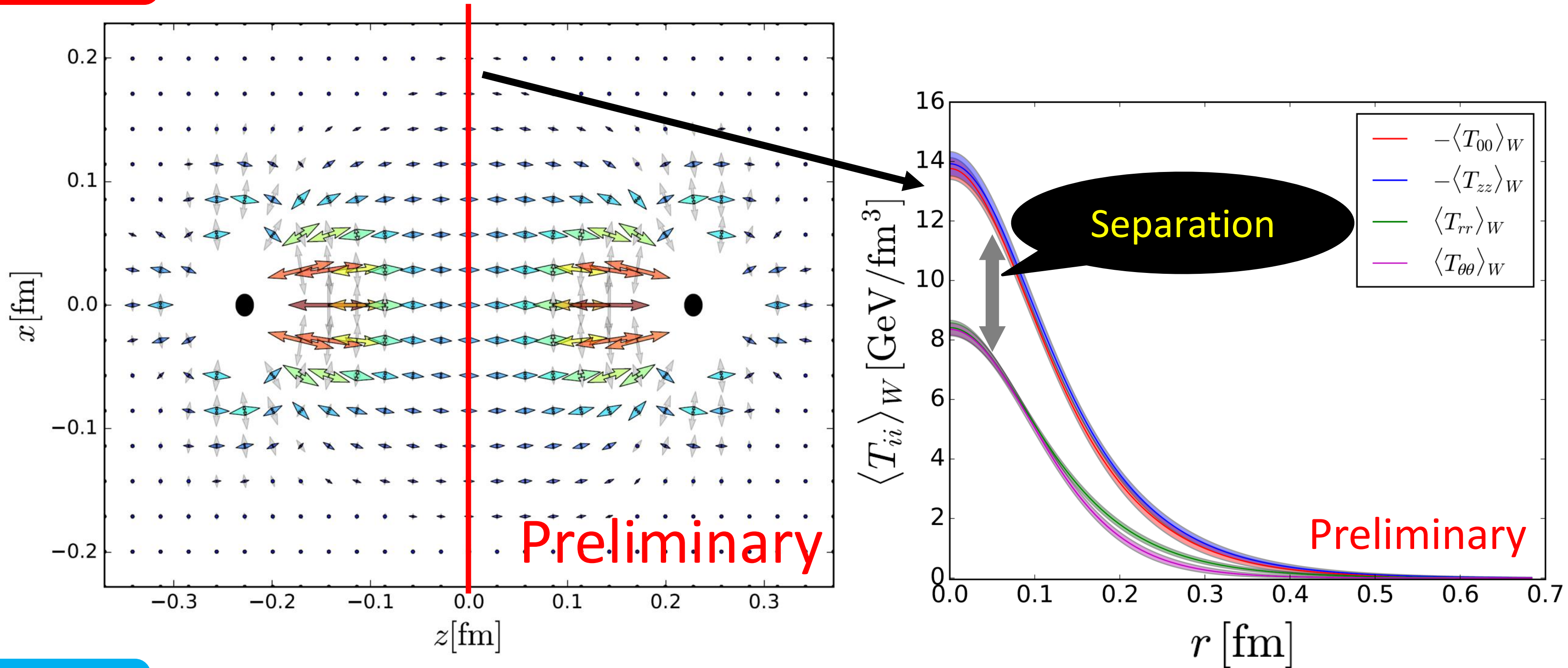
Force$= \int_{mid} T_{11}\, d^2x$

$$V(R) = a + \sigma R + c/R$$

$$F = -\frac{dV(R)}{dR}$$

Summary and Outlook

summary First measurements of stress distribution on the lattice !!



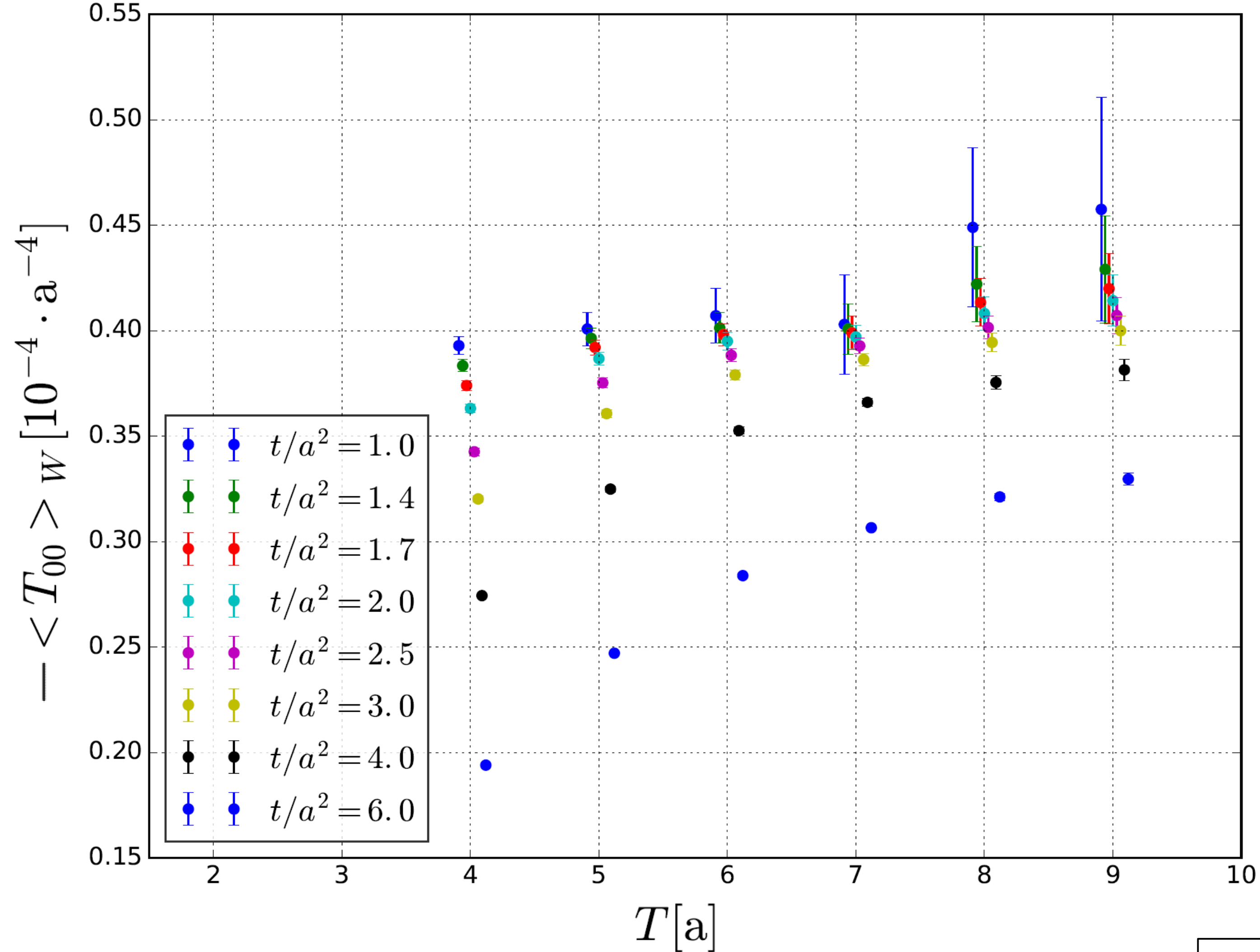
outlook

✓ We need to explain the stress distribution
using abelian dual monopole model model, NG string…

✓ Application : two flux tube, finite temperature, excited states…

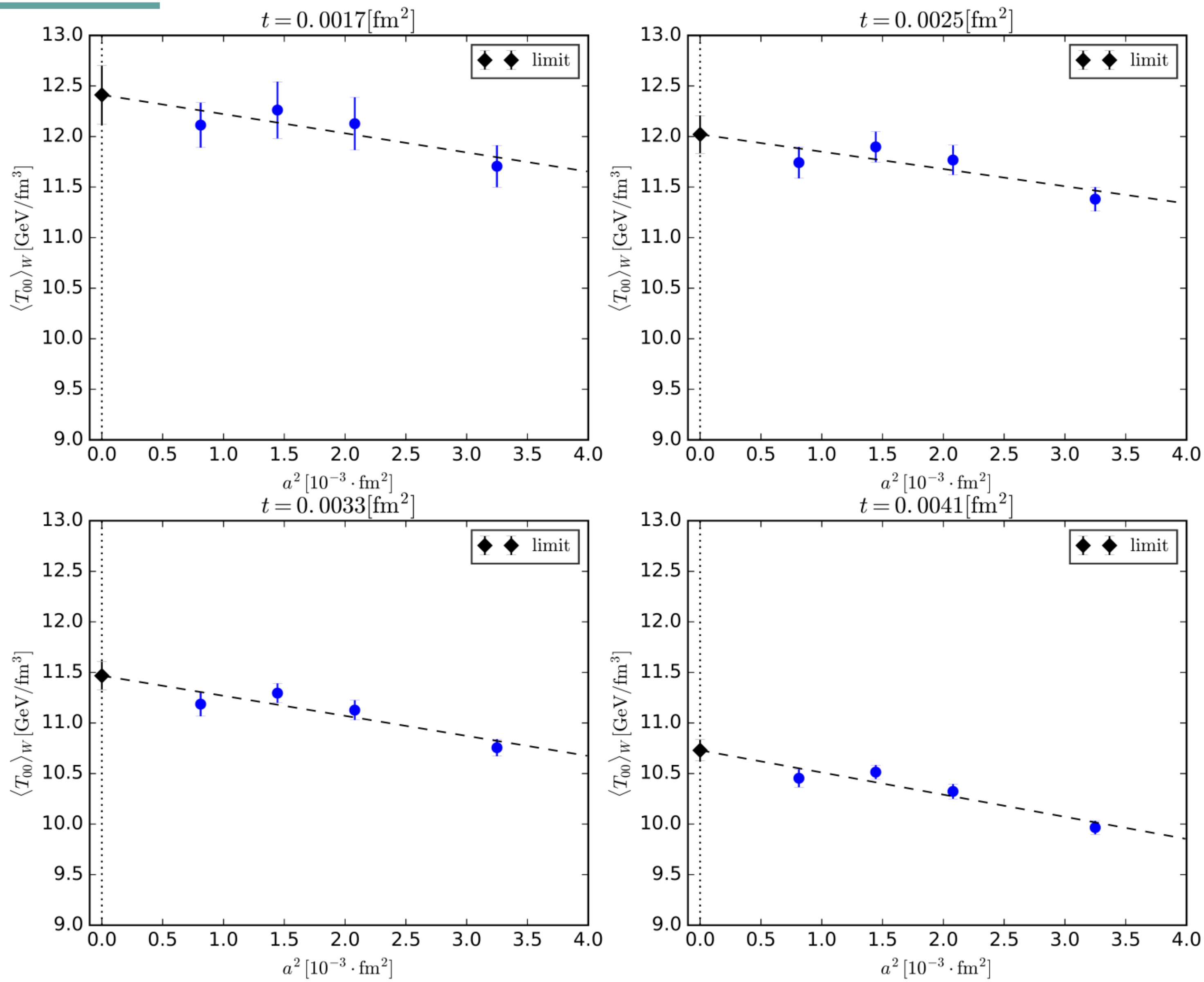
backup

Enhancement of ground state



$\beta = 6.819$

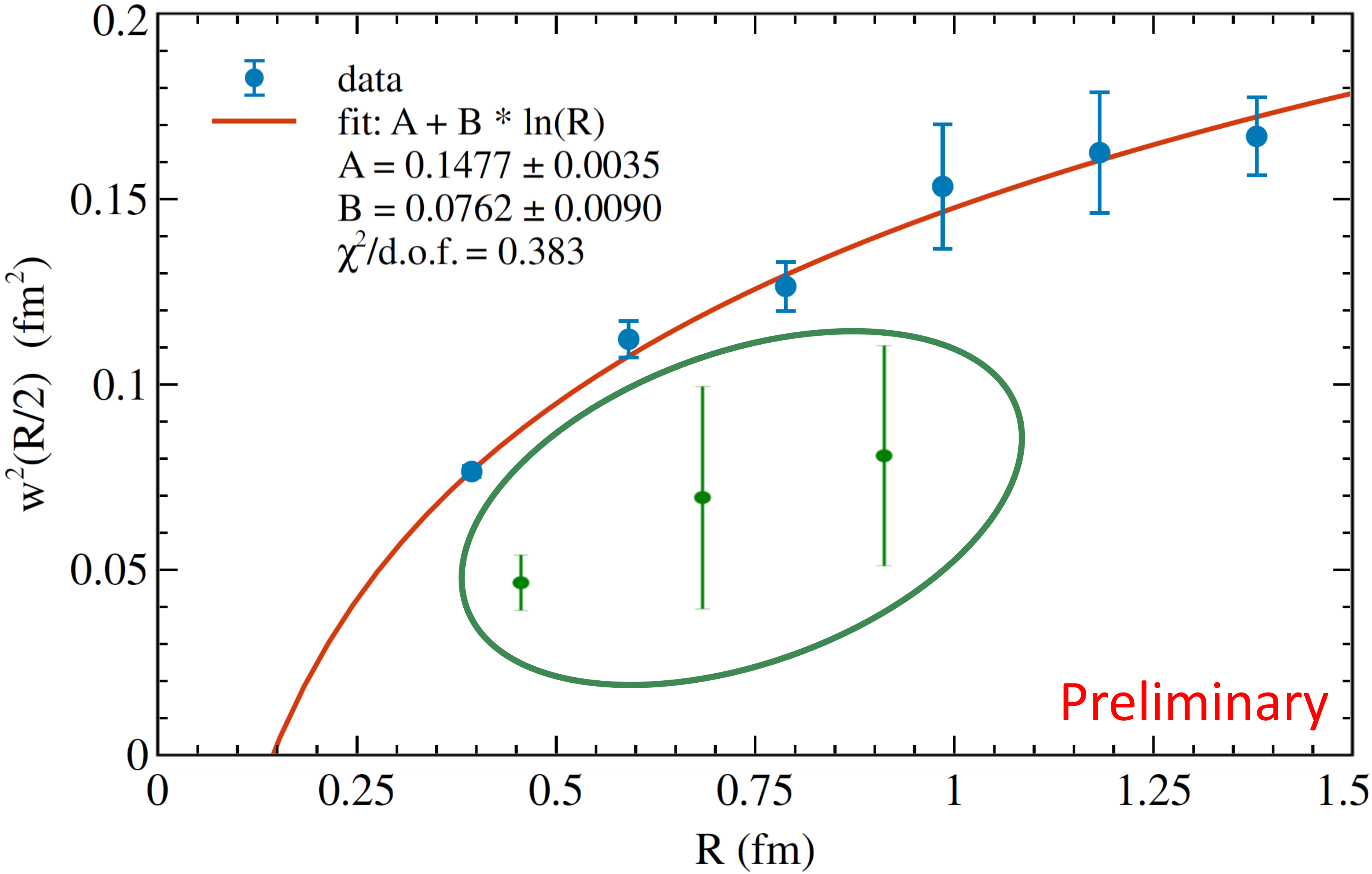
$a \to 0$ limit



$t = 0.0017 [\mathrm{fm}^2]$
$t = 0.0025 [\mathrm{fm}^2]$
$t = 0.0033 [\mathrm{fm}^2]$
$t = 0.0041 [\mathrm{fm}^2]$
limit
$\langle T_{00} \rangle_W$ [GeV/fm^3]
$a^2\,[10^{-3} \cdot \mathrm{fm}^2]$

Fat flux tube

Cardoso *et al.* (2013)



Stress asymmetry