Dynamical Thermalization in the Quark-Meson Model

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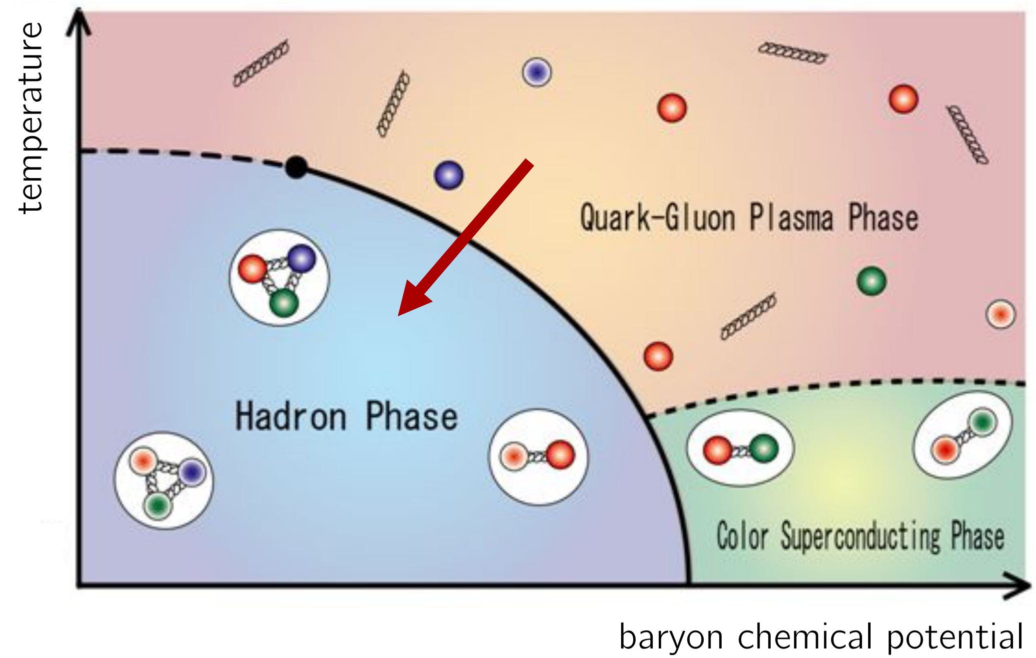
Masterthesis with J. Berges, J. Pawlowski, A. Rothkopf

From heavy ion collisions towards the QCD phase diagram: an equilibration process



- QCD phase diagram: an equilibrium concept
- deconfinement + chiral phase transition

Figures from http://wl33.web.r
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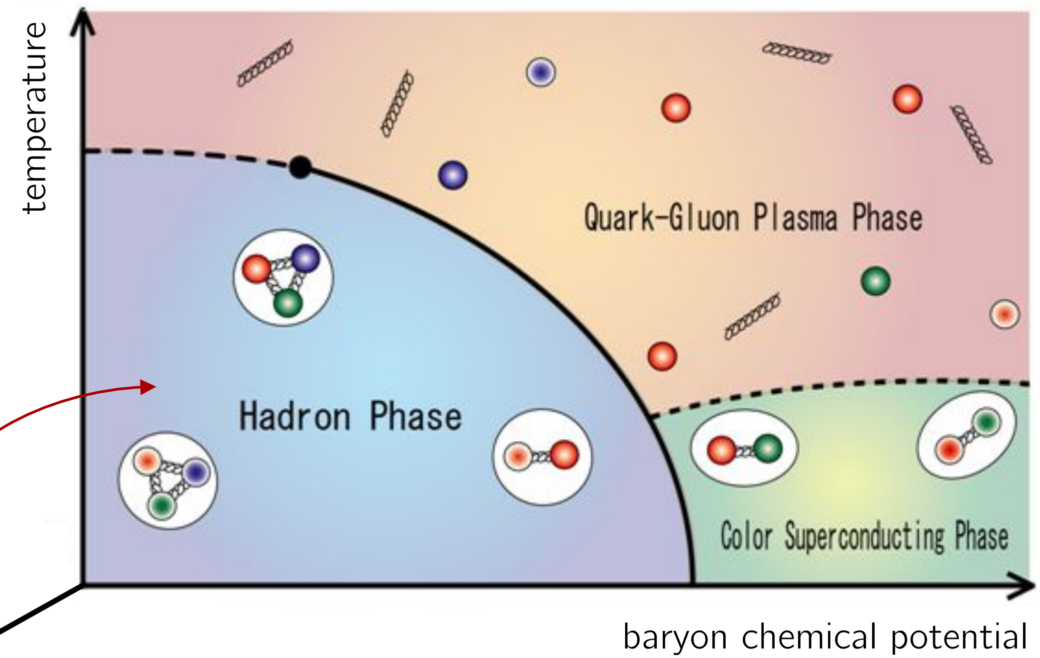
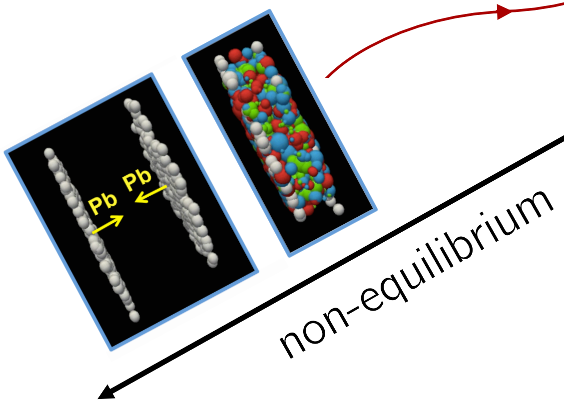
From heavy ion collisions towards the QCD phase diagram: an equilibration process





- QCD phase diagram: an equilibrium concept
- Need of out-of-equilibrium dynamics to describe initial stages of heavy ion collision.

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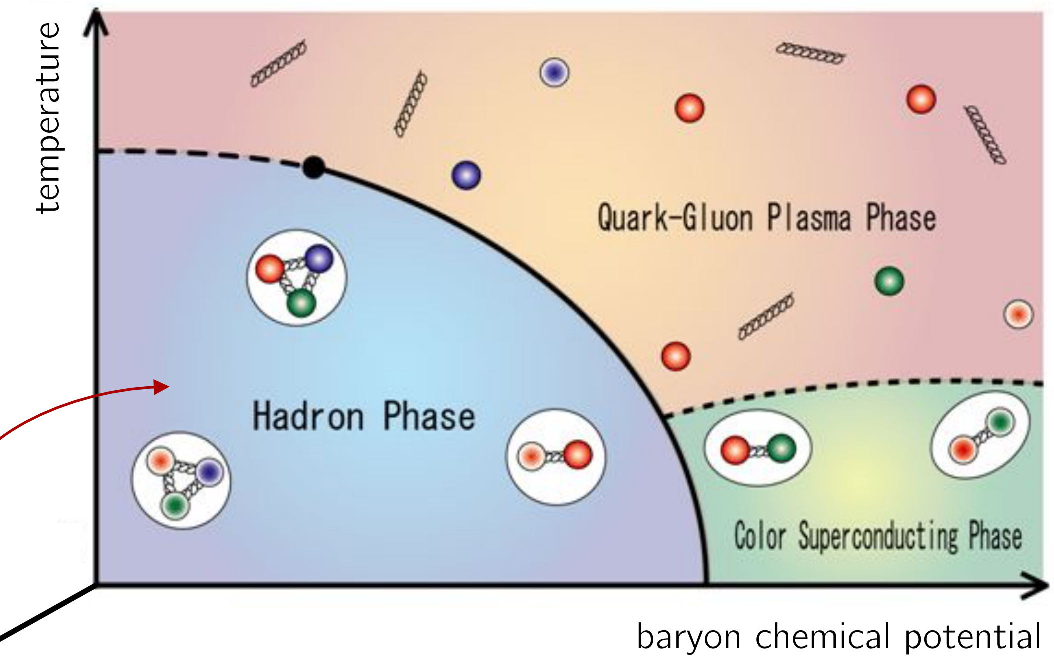
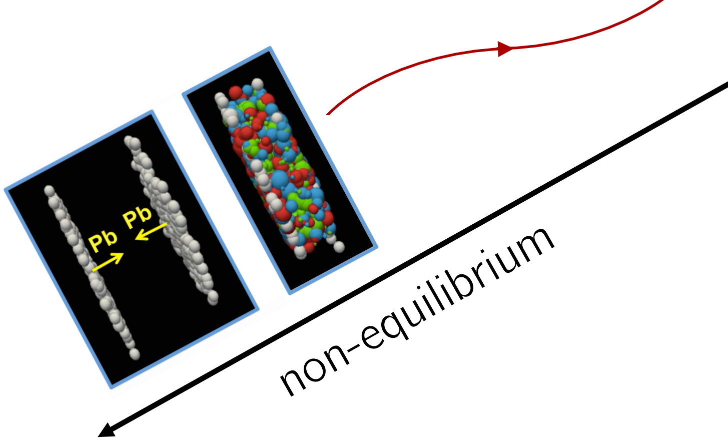
From heavy ion collisions towards the QCD phase diagram: an equilibration process

non-perturbative regime of QCD





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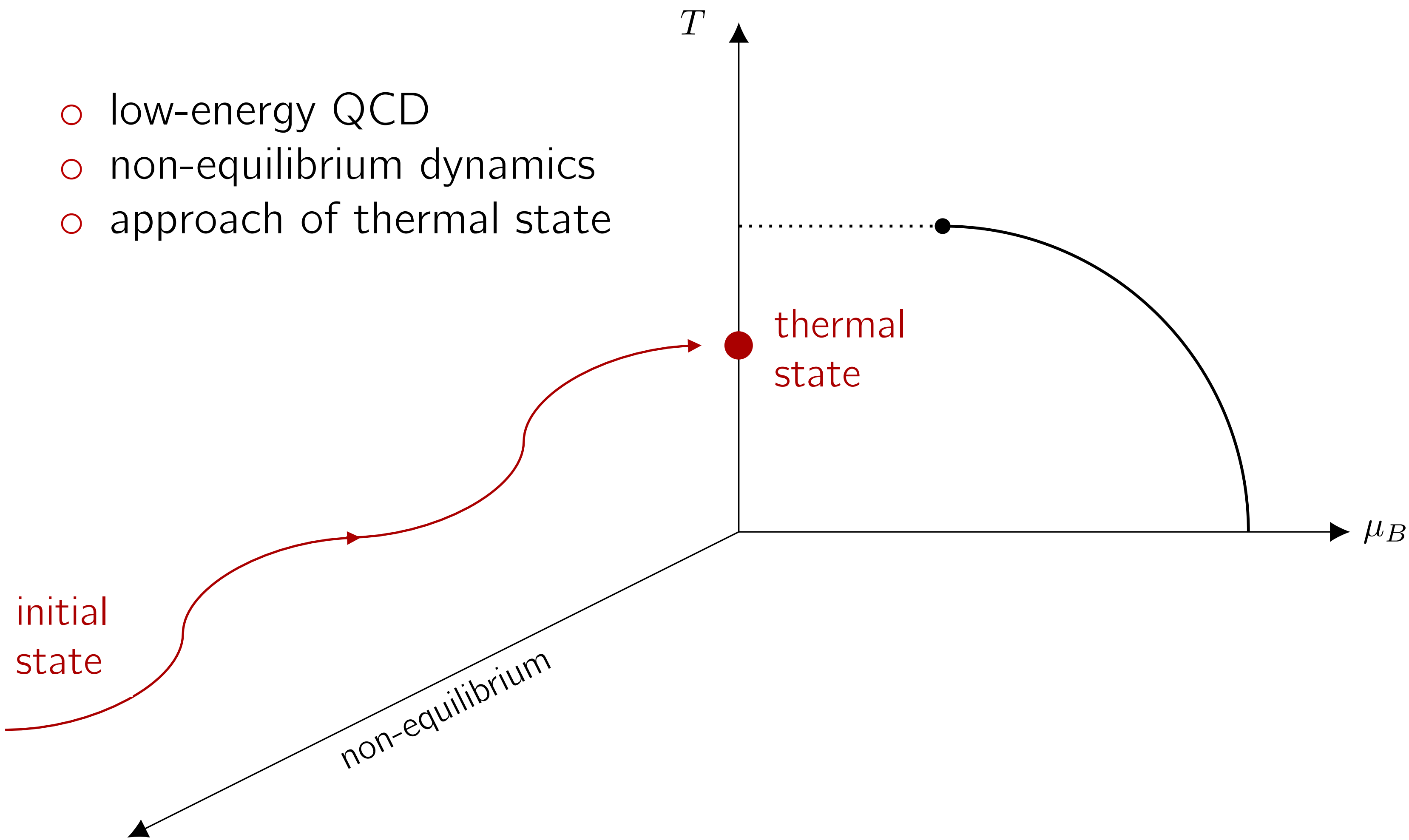
We can investigate this equilibration using effective field theories.

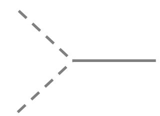
- low-energy QCD
- non-equilibrium dynamics
- approach of thermal state



The quark-meson model provides a successful formulation of QCD below scales ~1 GeV.

- model for low-energy QCD
- chiral symmetry breaking
- phase diagram with 1st and 2nd order transition

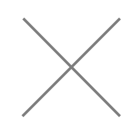
up and down quark — Yukawa coupling



$$S[\bar{\psi}, \psi, \sigma, \pi] = \int_x \Big[\underbrace{\bar{\psi}\, [i\gamma^\mu \partial_\mu - m_\psi]\, \psi}_{\text{up and down quark}} - \underbrace{\frac{g}{N_f} \bar{\psi}\, [\sigma + i\gamma_5 \tau^\alpha \pi^\alpha]\, \psi}_{\text{Yukawa coupling}}$$

$$+ \underbrace{\frac{1}{2} [\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi^\alpha \partial^\mu \pi^\alpha]}_{\sigma\text{-meson and pions}} - \underbrace{\frac{1}{2} m^2 [\sigma^2 + \pi^\alpha \pi^\alpha] - \frac{\lambda}{4! N} [\sigma^2 + \pi^\alpha \pi^\alpha]^2}_{\text{scalar potential}} \Big]$$

σ-meson and pions — scalar potential



The 2PI effective action is a practical tool to study thermalization.

classical action
$S[\bar{\psi}, \psi, \sigma, \pi]$

↓

non-equilibrium | 2PI effective action $\Gamma[\phi, G]$ | ✓ non-perturbative approximations

↓

non-equilibrium | evolution equations for ϕ and G | ✓ non-secular and universal

classical action
$S[\bar{\psi}, \psi, \sigma, \pi]$

↓ double Legendre transf.

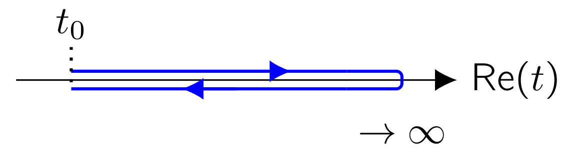
2PI effective action
$\Gamma[\phi, G]$

↓

evolution equations
for ϕ and G

non-equilibrium generating functional for $\rho^{\text{Gauss}}(t_0)$

$$Z[J,R] = e^{iW[J,R]} = \int \mathcal{D}\varphi \; e^{iS[\varphi] + iJ\cdot\varphi + \frac{i}{2}\varphi\cdot R\cdot\varphi}$$

with 

classical action
$S[\bar{\psi}, \psi, \sigma, \pi]$

double Legendre transf.

2PI effective action
$\Gamma[\phi, G]$

stationary conditions

evolution equations for ϕ and G

non-equilibrium generating functional for $\rho^{\text{Gauss}}(t_0)$

$$Z[J,R] = e^{iW[J,R]} = \int \mathcal{D}\varphi \; e^{iS[\varphi]+iJ\cdot\varphi+\frac{i}{2}\varphi\cdot R\cdot\varphi}$$

with

$= S[\phi] +$ 1-loop quantum corrections $+$ 2PI diagrams

large N expansion

$\underbrace{\text{LO diagrams}}_{\sim N^1} + \underbrace{\text{NLO diagrams}}_{\sim N^0} + \;\ldots\; +$ fermion-boson-loop

classical action
$S[\bar{\psi}, \psi, \sigma, \pi]$

double Legendre transf.

non-equilibrium generating functional for $\rho^{\text{Gauss}}(t_0)$

$$Z[J,R] = e^{iW[J,R]} = \int \mathcal{D}\varphi \; e^{iS[\varphi] + iJ\cdot\varphi + \frac{i}{2}\varphi\cdot R\cdot\varphi}$$

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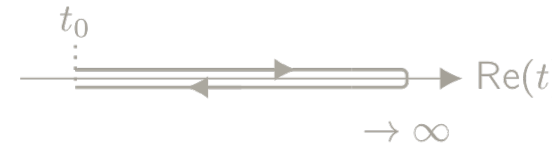


2PI effective action
$\Gamma[\phi, G]$

$= S[\phi] +$ 1-loop quantum corrections $+$ 2PI diagrams

large N expansion

$\underbrace{\text{LO diagrams}}_{\sim N^1} + \underbrace{\text{NLO diagrams}}_{\sim N^0} + \; \dots \; + \text{fermion-boson-loop}$

stationary conditions

evolution equations for ϕ and G

$$\left[\partial_t^2 + M^2(x;\phi)\right]\phi(t) = \text{fermion backreaction + 2PI corrections}$$

$$\left[\Box_x + M^2(x;\phi)\right] G(x,y) = -i\int_z \left[\Sigma(x,z;\phi,G)\right] G(z,y) - i\delta(x-y)$$

effective mass   self-energy

classical action
$S[\bar{\psi}, \psi, \sigma, \pi]$

non-equilibrium generating functional for $\rho^{\text{Gauss}}(t_0)$

$$Z[J,R] = e^{iW[J,R]} = \int \mathcal{D}\varphi \; e^{iS[\varphi] + iJ\cdot\varphi + \frac{i}{2}\varphi\cdot R\cdot\varphi}$$

with t_0 … Re(t), $\to \infty$

double Legendre transf.

add fermions

2PI effective action
$\Gamma[\phi, G]$ $= S[\phi] +$ 1-loop quantum corrections + 2PI diagrams

large N expansion

$\underbrace{\text{LO diagrams}}_{\sim N^1} + \underbrace{\text{NLO diagrams}}_{\sim N^0} + \ldots$ + fermion-boson-loop

stationary conditions

add e.o.m. for 4 fermion propagator components

$$\left[\partial_t^2 + M^2(x;\phi)\right]\phi(t) = \text{fermion backreaction} + \text{2PI corrections}$$

$$\left[\Box_x + M^2(x;\phi)\right] G(x,y) = -i \int_z \left[\Sigma(x,z;\phi,G)\right] G(z,y) - i\delta(x-y)$$

effective mass … self-energy

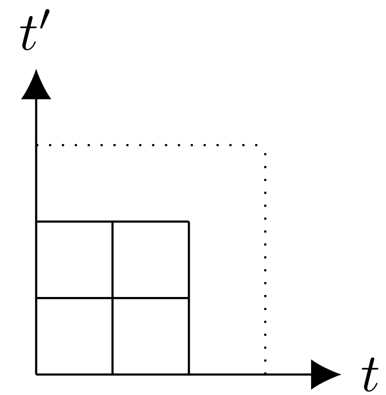
Numerical solution of the equations of motion

- Symmetries: spatial homogeneity & isotropy

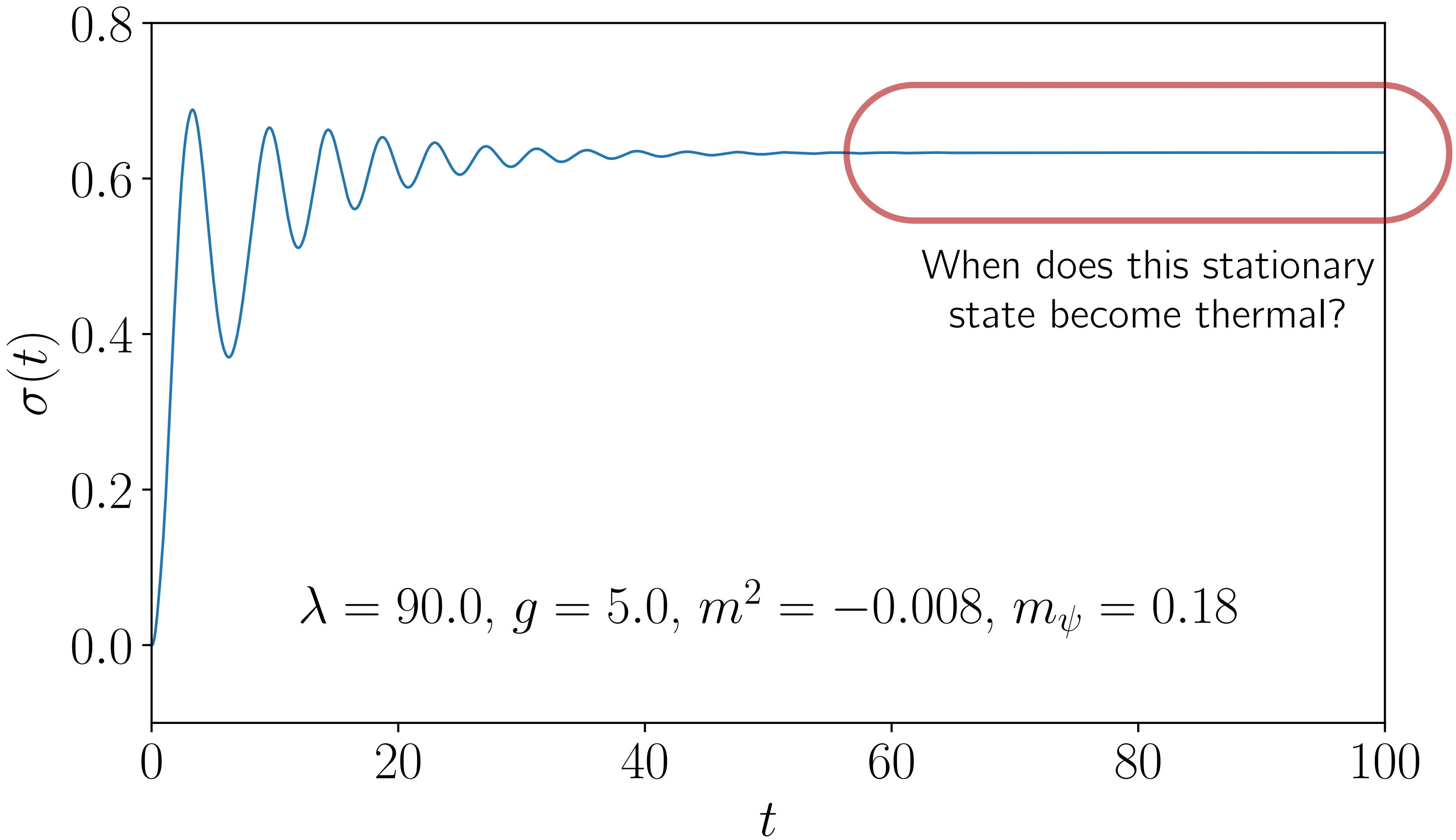
- Propagator decomposition: $G(x,y) = F(x,y) + \frac{i}{2}\rho(x,y)\ \mathsf{sgn}(x^0 - y^0)$

  statistical function (F) — spectral function (ρ)

- real-time evolution:
  - specify initial conditions as free-field propagators + vanishing field
  - iterative numerical computation of the time-evolution in two temporal directions



Real-time evolution of the macroscopic field



(1) Thermal equilibrium is a time-translation invariant state.

- time-translation invariance implies

$$G(t, t', |\mathbf{p}|) \rightsquigarrow G(\omega, |\mathbf{p}|)$$

in general depending on $t + t'$ and $t - t'$

independent of $t + t'$
here is something

- temporal Wigner transformation:

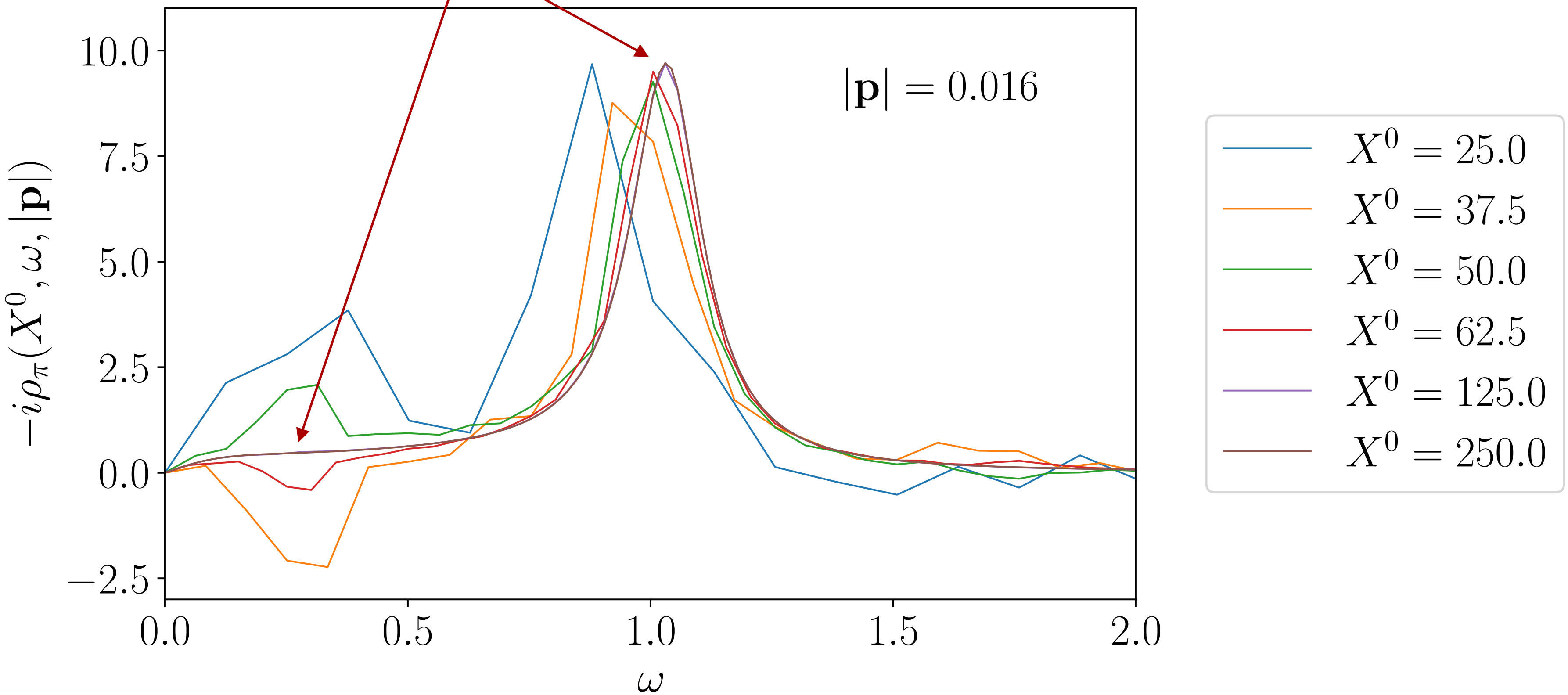
$$\rho(t, t', |\mathbf{p}|) \ \rightarrow \ \rho(X^0, \omega, |\mathbf{p}|)$$

center-of-mass time $X^0 = \frac{t+t'}{2}$

frequency
$X^0 = \frac{t+t'}{2}$

The two-point functions become time-translation invariant.

constant in $X^0 \sim t + t'$ for $X^0 \gtrsim 150$



(2) Thermal eq. as state with thermal particle distributions.

- thermal initial density matrix implies fluctuation-dissipation relation:

$$F_{\text{eq}}(\omega, |\mathbf{p}|) = -i \left(\frac{1}{2} + n_{\text{th}}(\omega) \right) \rho_{\text{eq}}(\omega, |\mathbf{p}|)$$

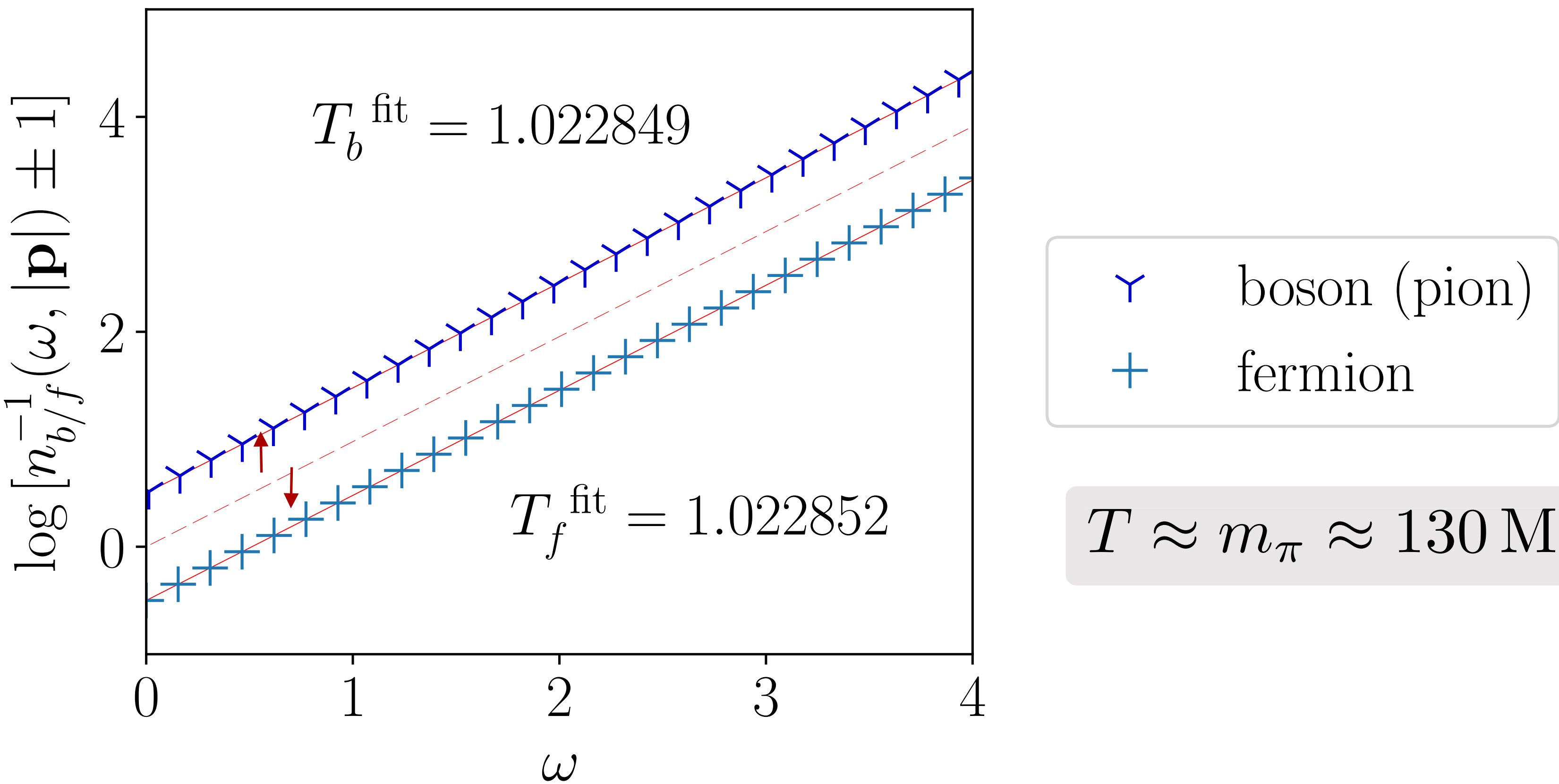
- effective particle number:

$$n(\omega, |\mathbf{p}|) = i \frac{F(\omega, |\mathbf{p}|)}{\rho(\omega, |\mathbf{p}|)} - \frac{1}{2}$$

- in thermal equilibrium:

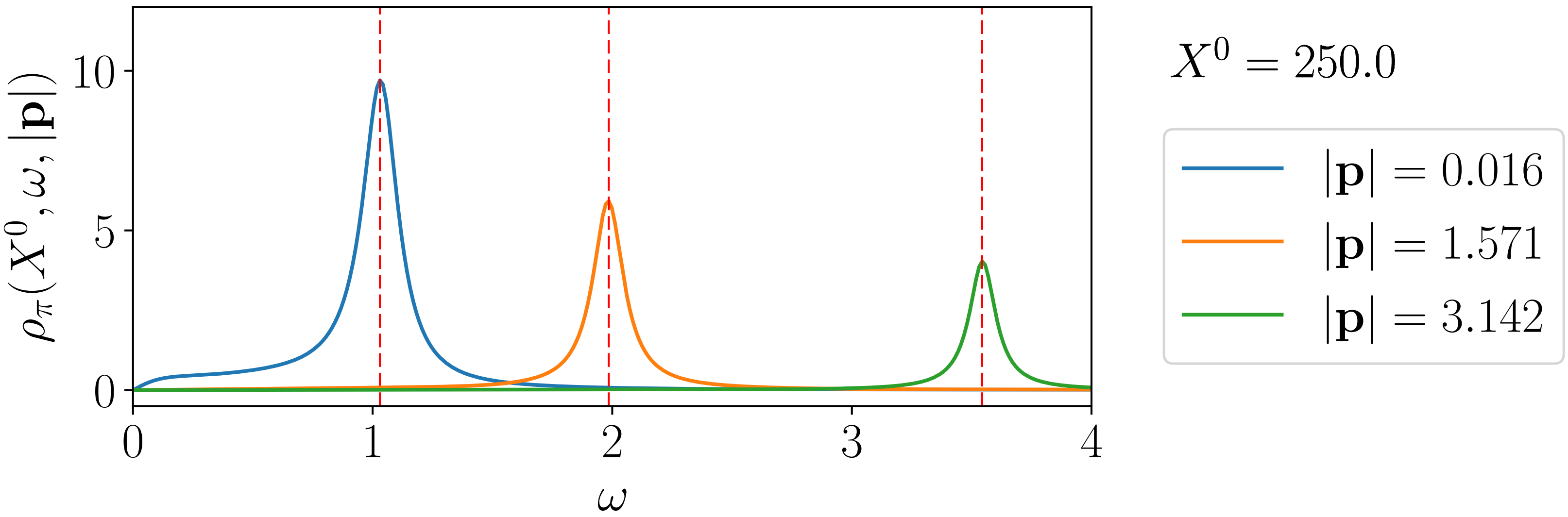
$$n(\omega, |\mathbf{p}|) \to n_{\mathrm{BE/FD}}(\omega) = \frac{1}{e^{\beta\omega} \mp 1} \text{ with } \beta = 1/T$$

Determination of the thermalization temperature using the Bose-Einstein and Fermi-Dirac distribution



$T \approx m_\pi \approx 130\,\text{MeV}$

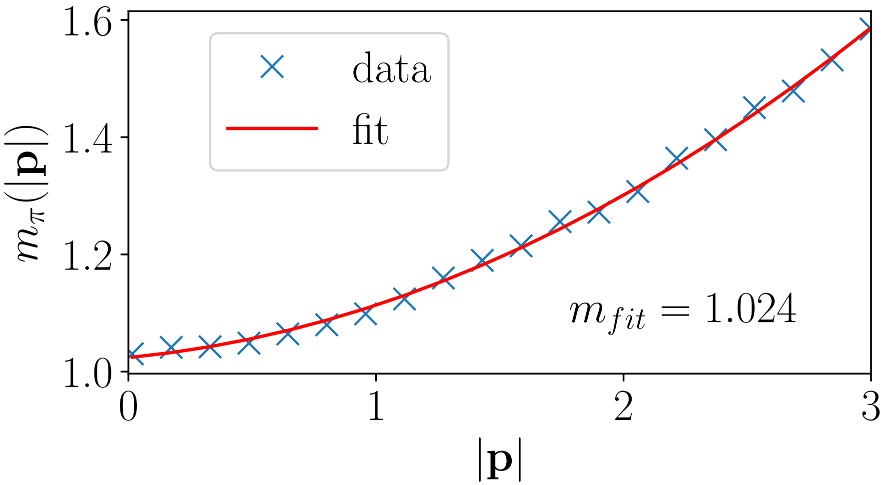
Particle masses from spectral functions by dispersion relation



- particle mass from peak position:

$$m(|\mathbf{p}|) = \sqrt{\omega^2_{\text{peak}}(|\mathbf{p}|^2) - |\mathbf{p}|^2}$$

- physical mass at zero momentum



A step forward
in describing the thermalizing of the QGP in a heavy ion collision

We were able to

- include non-equilibrium dynamics
- observe the approach of thermal equilibrium
- determine the physical mass spectrum

Next steps:

- non-zero baryon-chemical potential
- expanding box size
- scaling behavior around critical point