Singularities in String Theory

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Spacetime singularities

Understanding the physics of spacetime singularities is a major challenge for theoretical physics.

Big Bang / Big Crunch

beginning or end of time, the origin of the universe?

Black holes

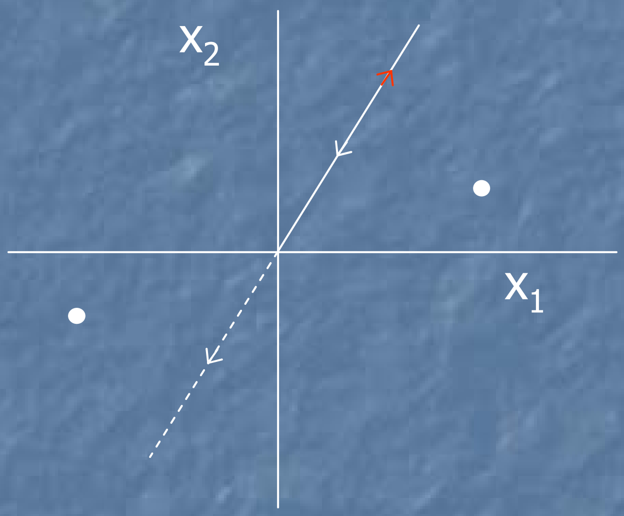
loss of information?

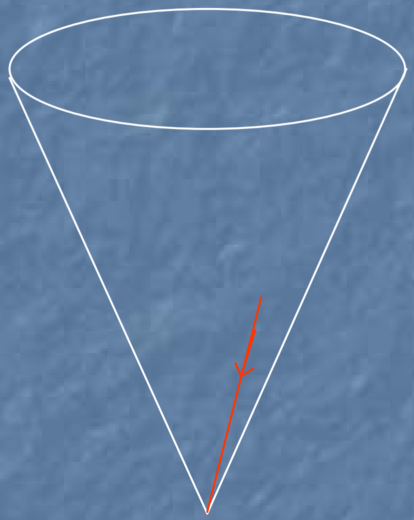
String theory and spacetime singularities

- It is generally believed that understanding spacetime singularities requires a quantum theory of gravity.
- String theory is thus the natural framework to address this problem.
- One hopes that string theory will lead to a detailed theory of the Big Bang which in turns leads to experimental tests of string theory.

Static example 1: Orbifolds

- $(x_1, x_2) \sim (-x_1, -x_2)$ 2d cone





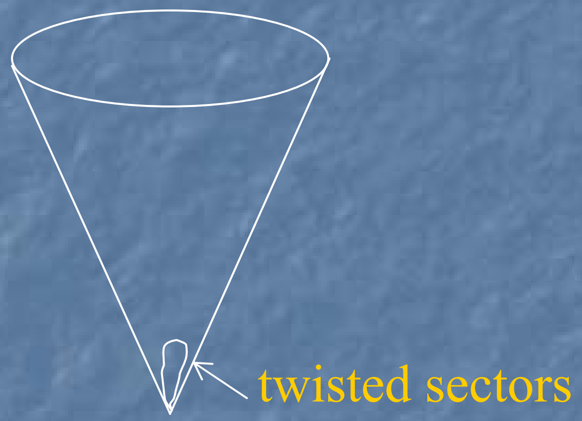
- $(x_1, x_2, x_3, x_4) \sim (-x_1, -x_2, -x_3, -x_4)$ A_1 singularity

Classical general relativity is singular at the tip of the cone.

String theory on orbifolds

Dixon, Harvey, Vafa, Witten

- The extended nature of string theory introduces additional degrees of freedom localized at the tip of the cone: twisted sectors.

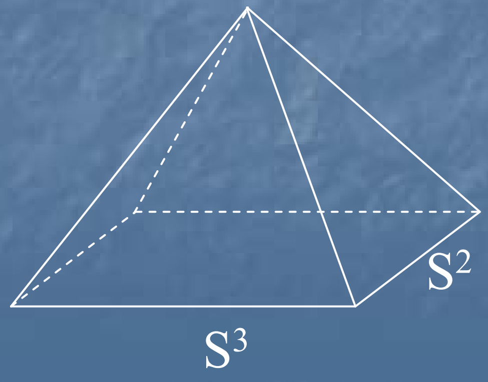


- Including the twisted sectors, string S-matrix is unitary and physics is completely smooth in perturbation theory.

Static example 2: Conifold

Strominger

- General relativity is singular.
- Perturbative string theory is singular.
- By including the non-perturbative degrees of freedom (D-branes wrapping the vanishing three cycle) at the tip of the cone, the string S-matrix is again smooth.



Lessons

String theory introduces new degrees of freedom.

String S-matrix is completely smooth.

Cosmological singularities

- Possibilities:
  - Beginning of time: need initial conditions, wave functions of the Universe etc.
  - Time has no beginning or end: Need to understand how to pass through the singularity.

- New Challenges:
  - What are the right observables?
  - What are the right degrees of freedom?

From Big Crunch to Big Bang: is it possible?

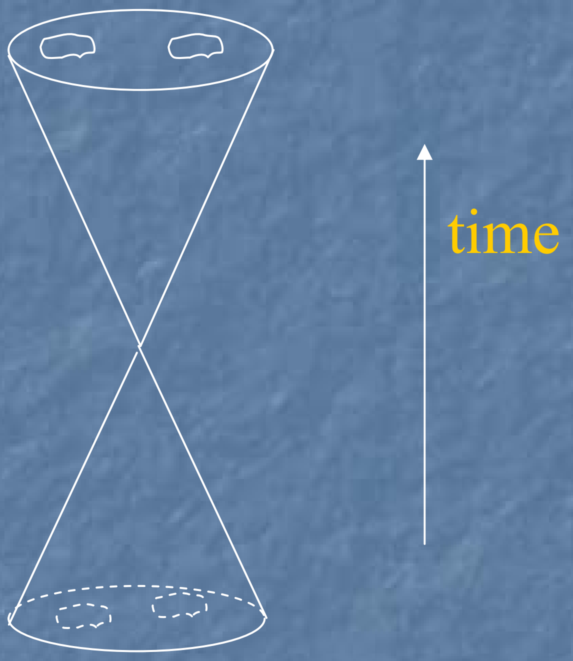
(A lower dimensional Toy Model)

- Exact string background.

- The Universe contracts and expands through a singularity.

- One can compute the S-matrix from one cone to the other.

- Same singularity in certain black holes (a closely related problem).



Results from string perturbation theory

Liu, Moore, Seiberg
Horowitz, Polchinski

- For special kinematics the string amplitudes diverge.
- The energy of an incoming particle is blue shifted to infinity by the contraction at the singularity, which generates infinitely large gravitational field and distorts the geometry.

Results from string perturbation theory

- String perturbative expansion breaks down as a result of large backreaction.
- The same conclusion applies to other singular time-dependent backgrounds.

Nekrasov, Cornalba, Costa; Simon; Lawrence; Fabinger, McGreevy; Martinec and McElgin, Berkooz, Craps, Kutasov, Rajesh; Berkooz, Pioline, Rozali; ………

Lessons and implications

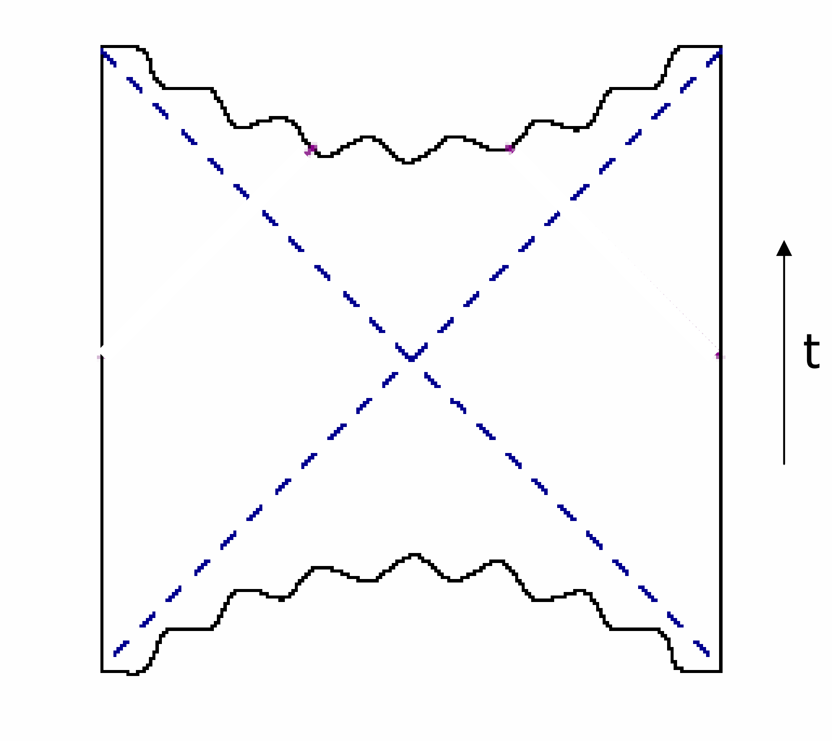
- Perturbative string theory is generically singular at cosmological singularities.
- One needs a full non-perturbative framework to deal with the backreaction.
- No clear evidence from string theory so far a non-singular bounce is possible.

Nonperturbative approaches

- AdS/CFT
- BFSS Matrix Theory

In these formulations, spacetime is no longer fixed from the beginning, rather it is dynamically generated. One only needs to specify the asymptotic geometry.

Schwarzschild black holes in AdS



Maldacena;
Witten

Quantum gravity in this black hole background is described by an SU(N) Super Yang-Mills at finite temperature on S^3.

Classical gravity corresponds to large N and large t'Hooft coupling limit of Yang-Mills theory.

Understanding Black hole singularity from finite temperature Yang-Mills ?

- Find the manifestation of the black hole singularity in the large N and large t' Hooft limit of Yang-Mills theory.
- understand how
  - finite N (quantum gravitational)
  - t' Hooft coupling (stringy)

effects resolve it.

Challenges

- The singularities are hidden behind the horizons.
- Need to decode the black hole geometry from boundary Yang-Mills theory.

Mapping of physical quantities

- Gauge invariant operator $\mathcal{O}$
- Field φ in AdS
- Dimension ν
- Particle mass m
- Finite temperature two-point functions of $\mathcal{O}$
- Free propagator of φ in the Hartle-Hawking vacuum of the AdS black hole background

The boundary theory has a continuous spectrum in the large N limitdespite being on a compact space.

Large dimension limit

Festuccia and Liu

- We consider the following large operator dimension limit of the boundary Wightman function $G_+(\omega)$ in momentum space.

$$\nu \to \infty, \quad \omega = \nu u$$

$$G_+(\omega) \to 2\nu e^{\nu Z(u)}$$

Relation with bulk geodesics

Festuccia and Liu

- Z (u) is given by the Legendre transform of the proper distance of a bulk geodesic with initial velocity

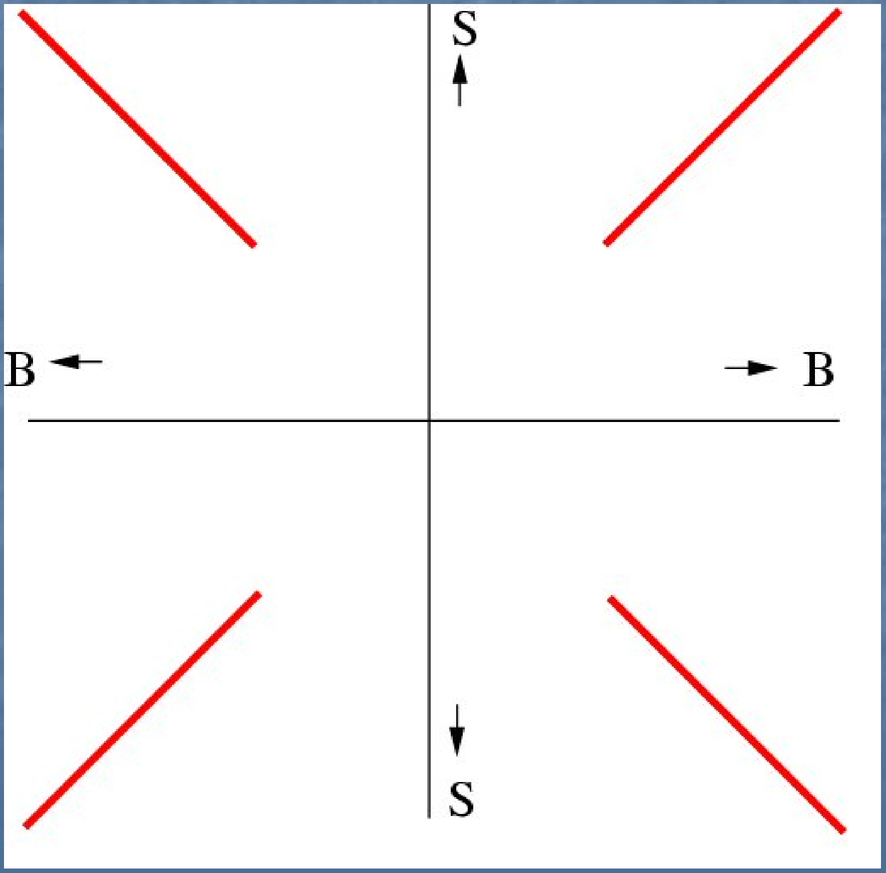
$$E = i\,u$$

- The geodesic starts and ends at the boundary and is specified by a turning point

$$r_c\,(u)$$

Mapping of the boundary momentum space to bulk geometry

r_c (u) maps the boundary momentum space to the black hole spacetime



Yang-Mills theory at finite N

- At finite N, no matter how large, Yang-Mills theory has a discrete spectrum on a compact space.

- This implies that $G_+(\omega)$ has the form

$$G_+(\omega) = \sum \delta(\omega - \omega_i)$$

with ω_i real.

Summary

- Certain static singularities in GR are resolved in perturbative string theory, while others are resolved by invoking non-perturbative degrees of freedom.

- Understanding the cosmological singularities is a big challenge for string theory.

- Non-perturbative framework like the AdS/CFT correspondence gives promising avenue for attacking the problem.