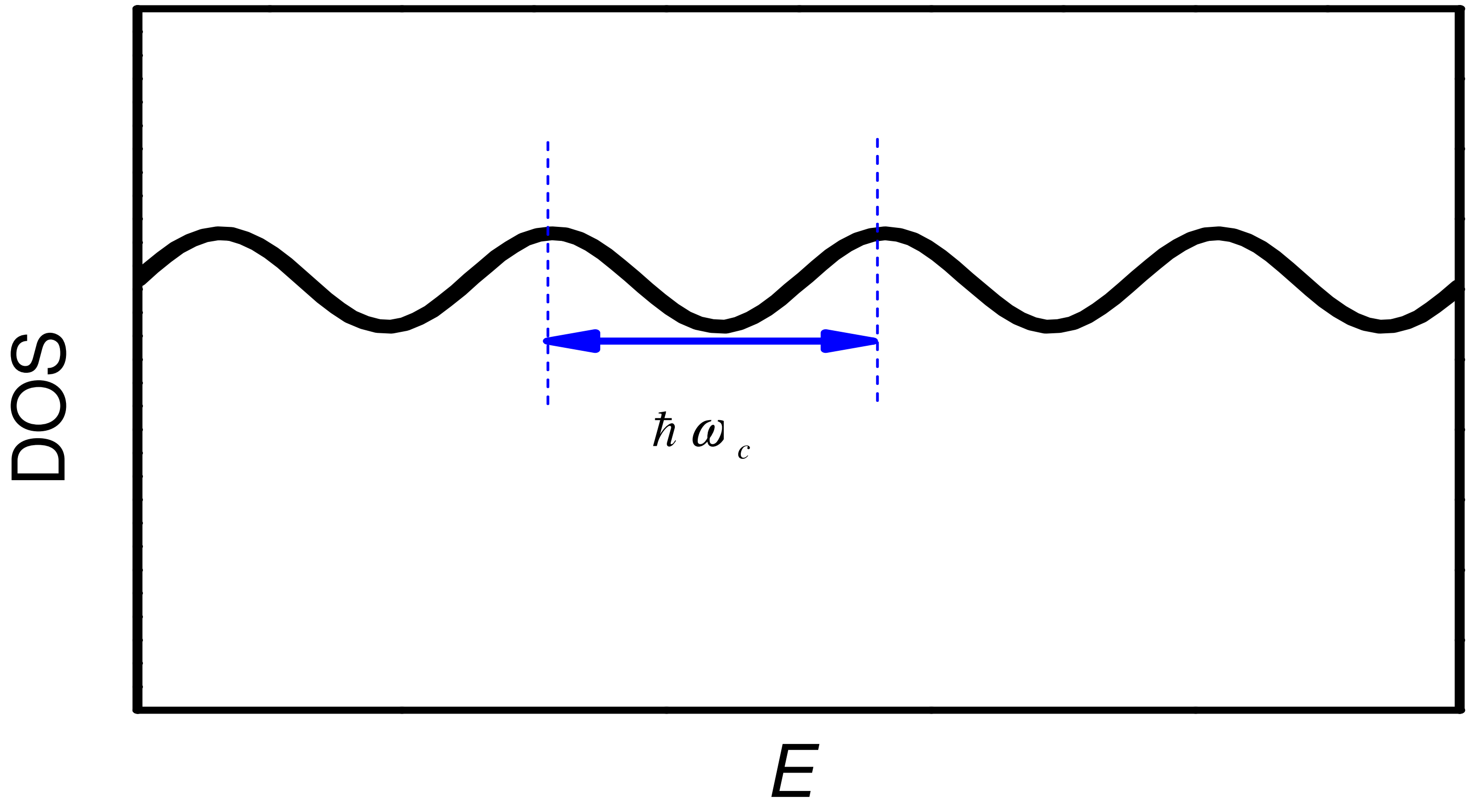
Conventional Shubnikov-de Haas (SdH) theory (Landau quantisation in the metallic regime)

$$\Delta\rho_{SdH}(B,T) = C\exp\left(\frac{-\pi}{\mu B}\right)\frac{\dfrac{2\pi^2 k_B m^* T}{\hbar eB}}{\sinh\left(\dfrac{2\pi^2 k_B m^* T}{\hbar eB}\right)}$$

$$\rho_{xx} = \rho_0 + \Delta\rho_{SdH}\cos[\pi(\nu - 1)] \quad (1)$$

Landau quantisation modulates the DOS

without inducing a QH liquid



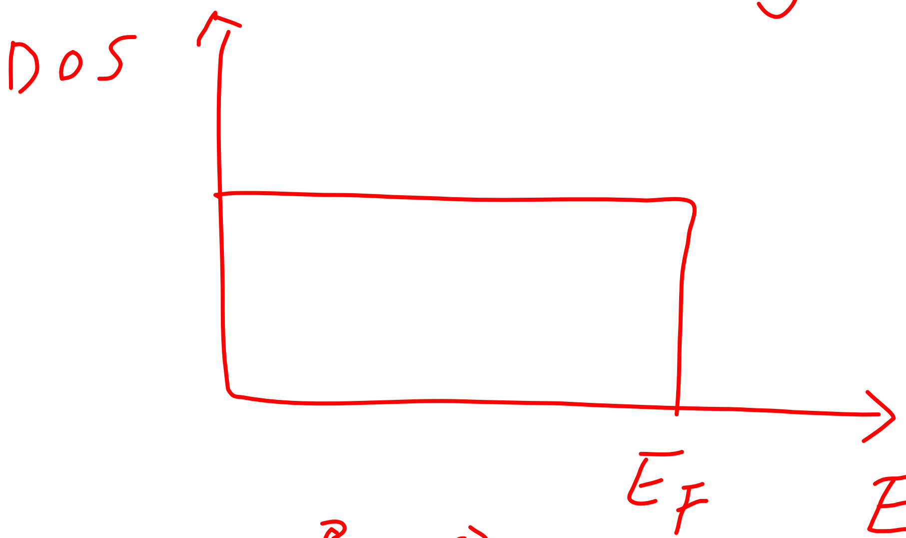
Picture for conventional SdH oscillations

$$\frac{m^*}{2\pi\hbar^2} \times 2 \quad \text{2D DOS}$$

$$= \frac{m^*}{\pi\hbar^2} \quad \text{when considering spin-degeneracy}$$

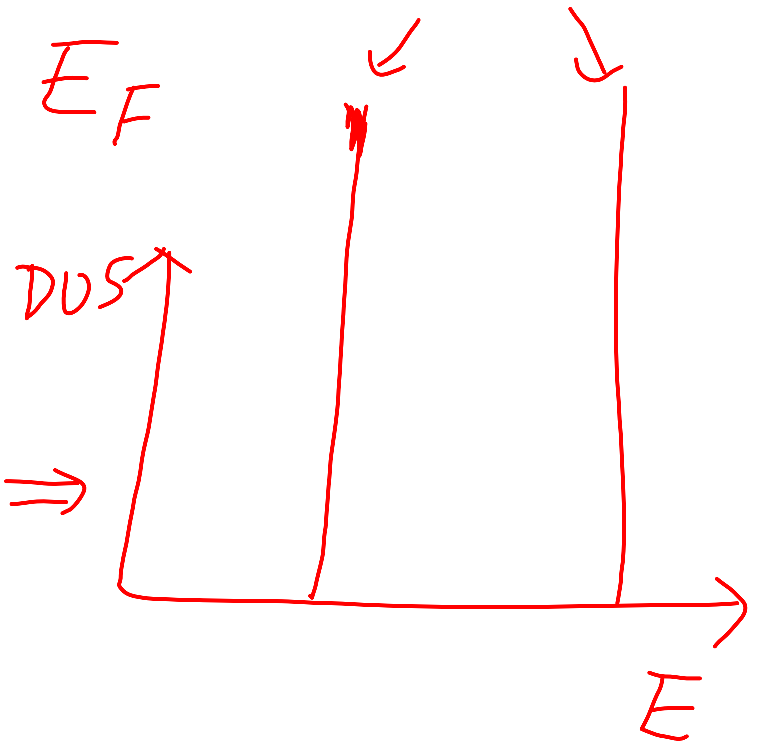
δ-function.

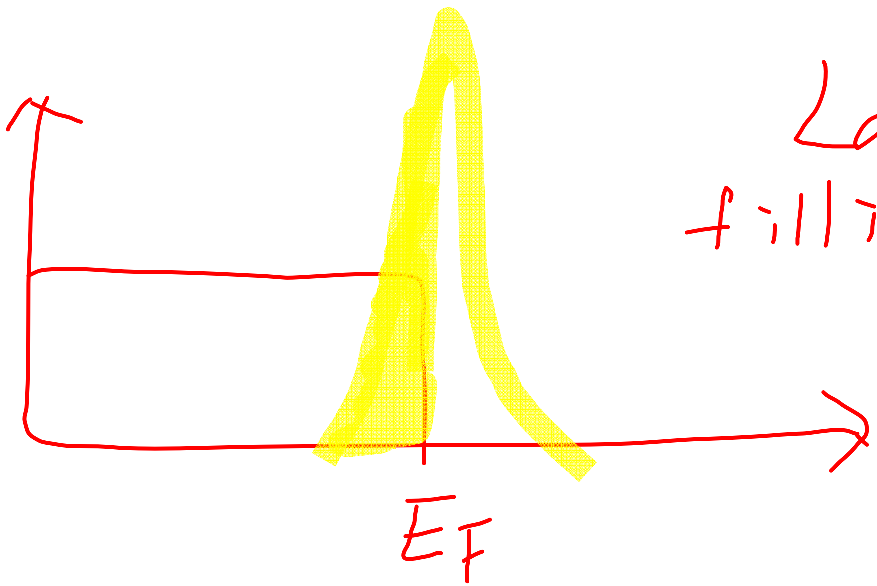
$$n_{2D} = \frac{\#}{m^2 \cdot J} \times E_F$$



$B = 0$

$\Rightarrow$





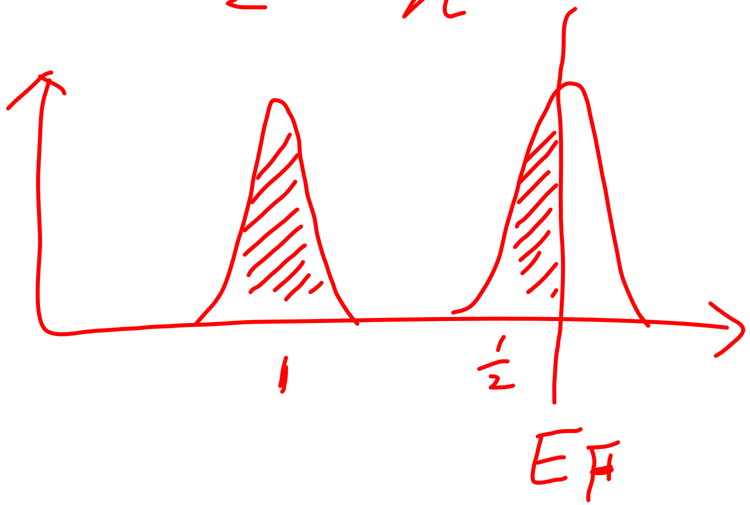
Landau level filling factor

$$\boxed{\frac{\nu e B}{h} = n_{2D}}$$

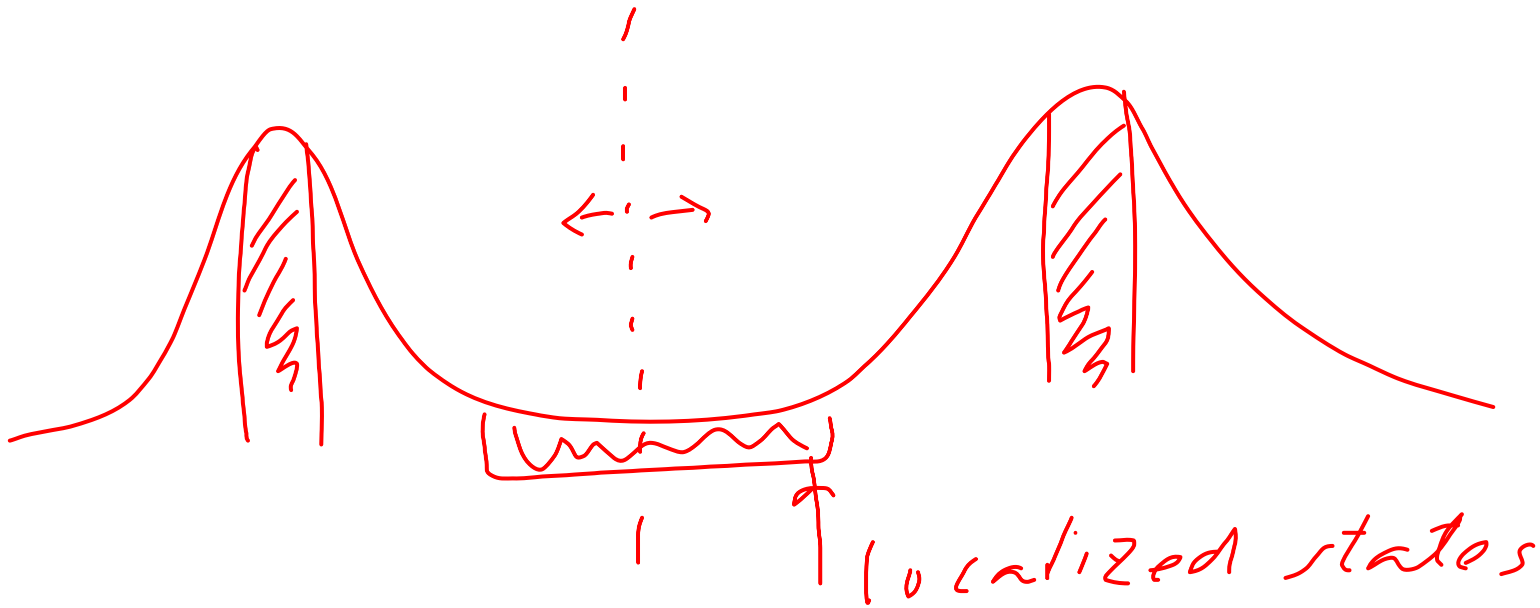
Half-filled

$$\left(\frac{1}{2}\right)\left(\frac{eB}{h}\right) = n_{2D}$$

νB : finite as $B \to 0$



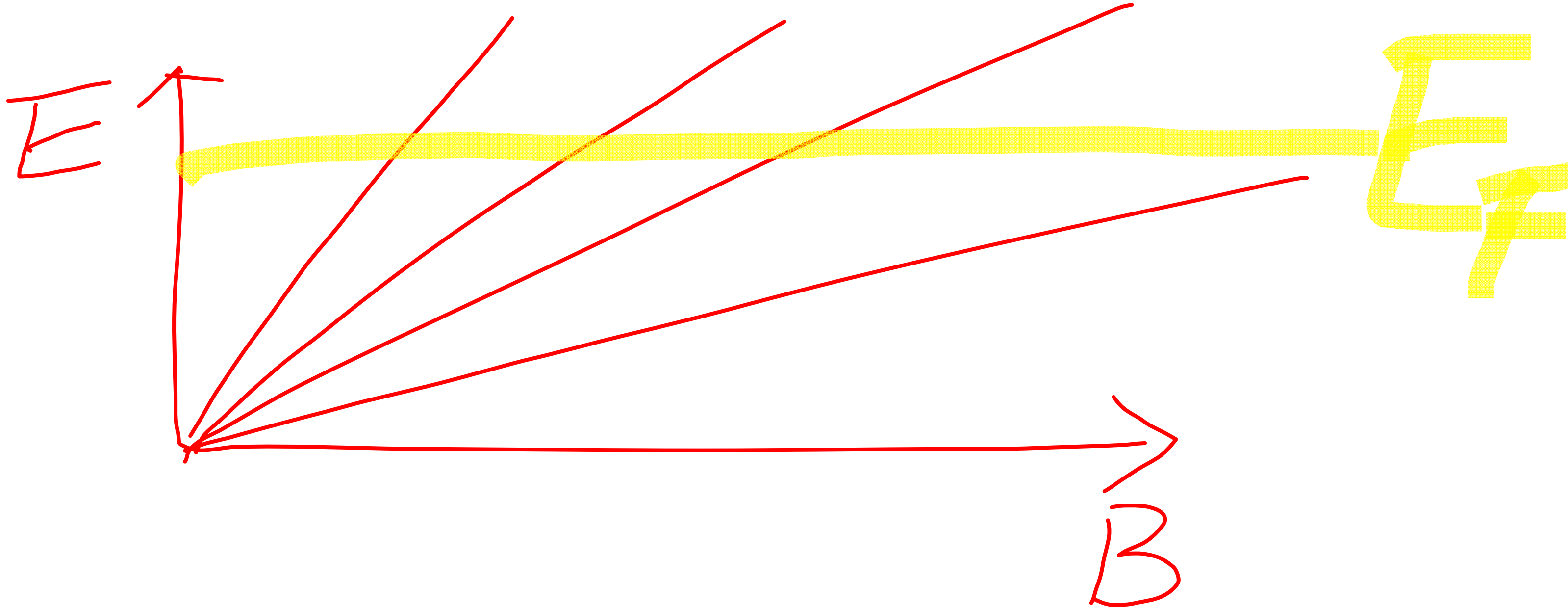
$$\nu = 1 + \frac{1}{2} = \frac{3}{2}$$

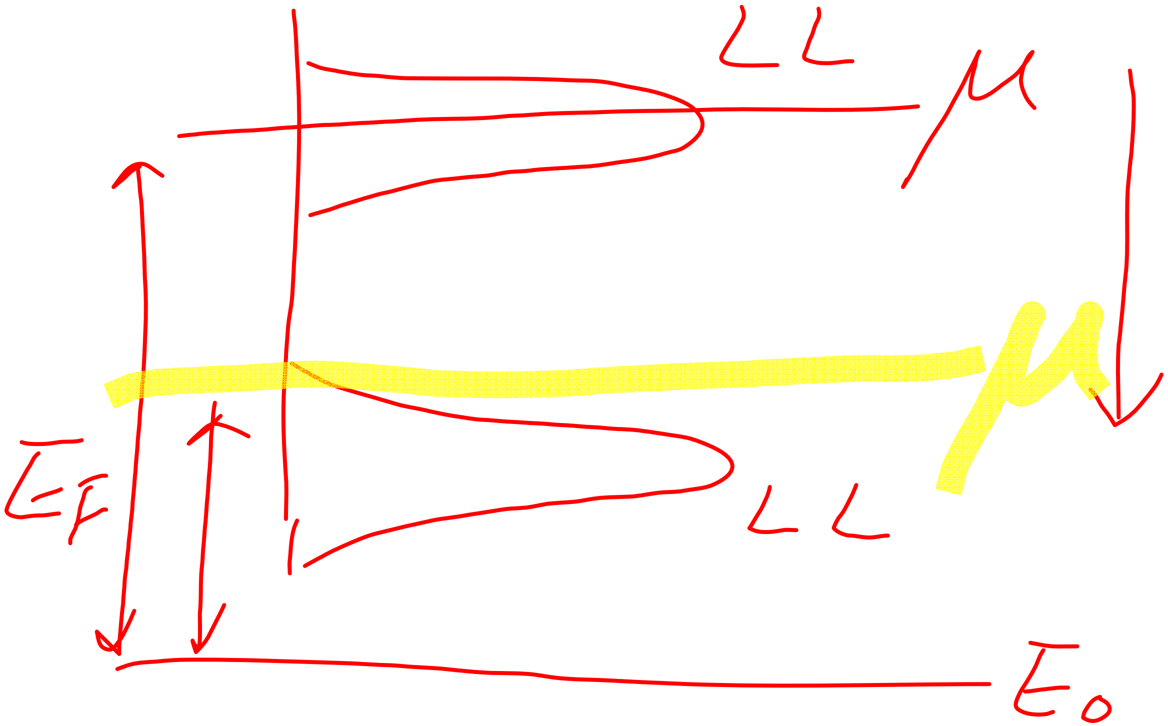


E_F

1. $B \to 0$
2. Integer filling factors

$$\boxed{\frac{\nu e B}{h} = n_{2D}}$$

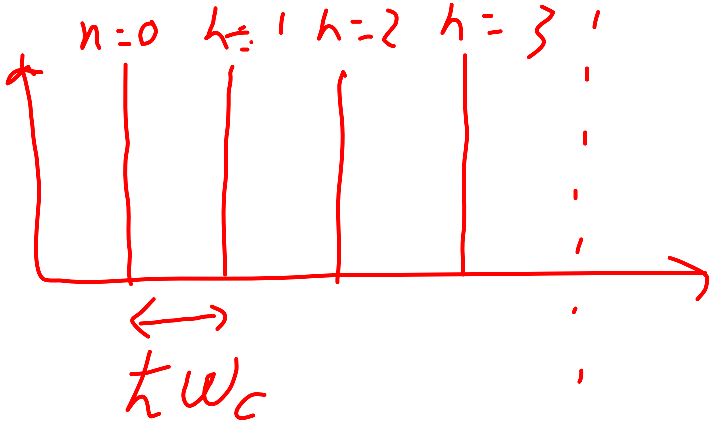




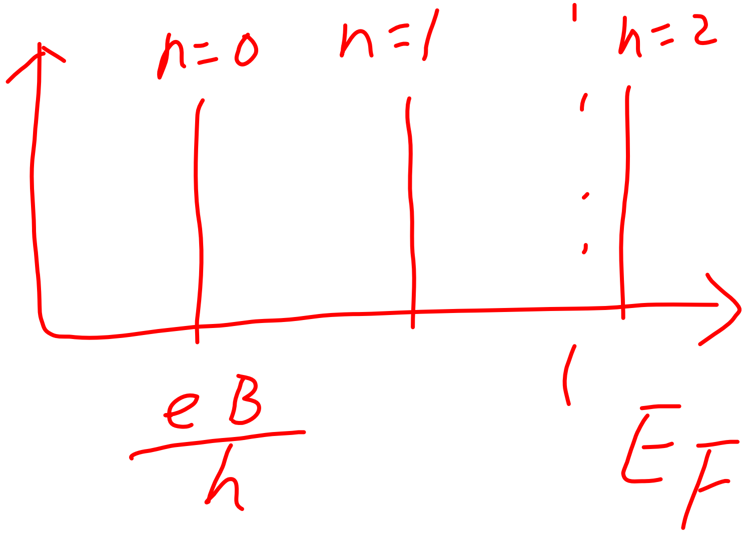
E_F : pinned

E_F shrinks

Fixed n_{2D} varying B

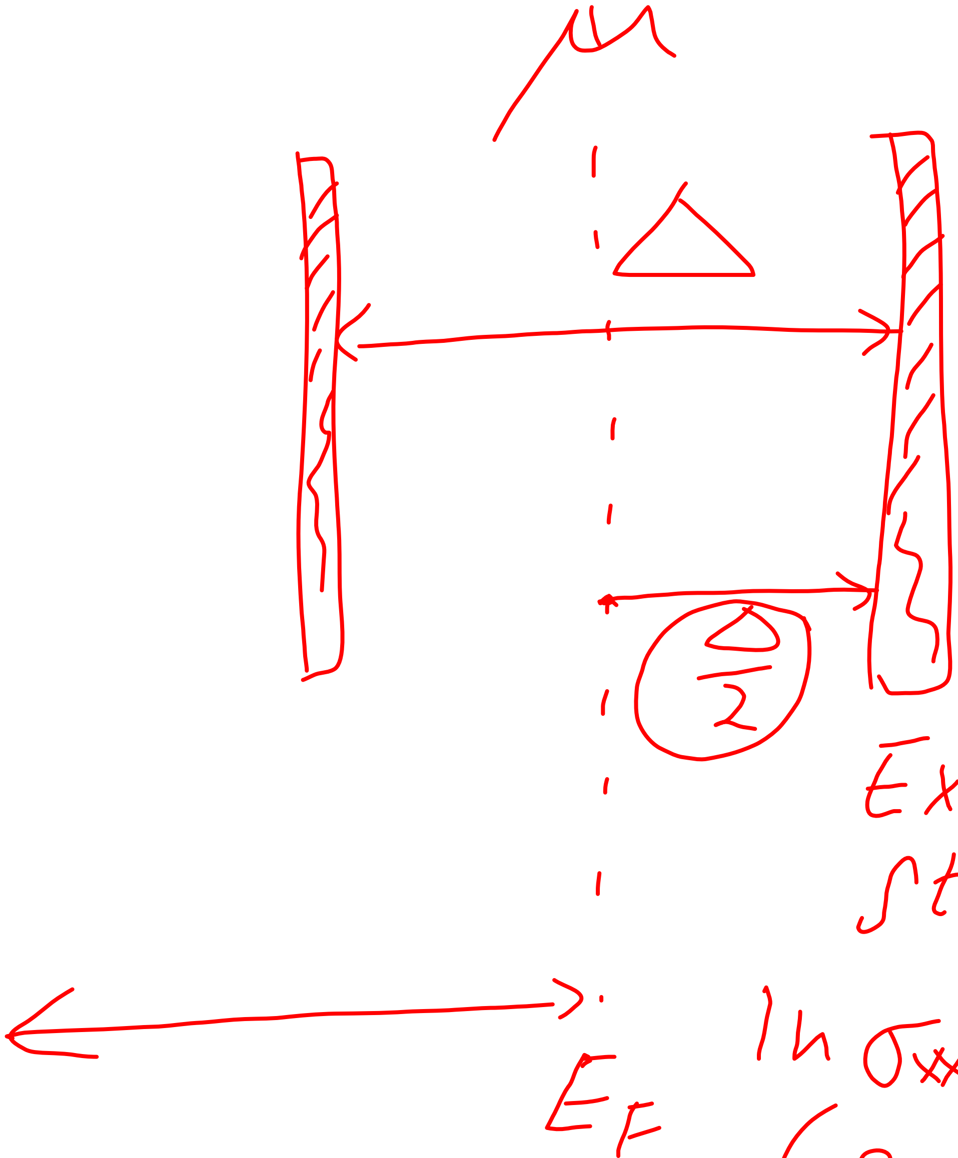


low B



High B

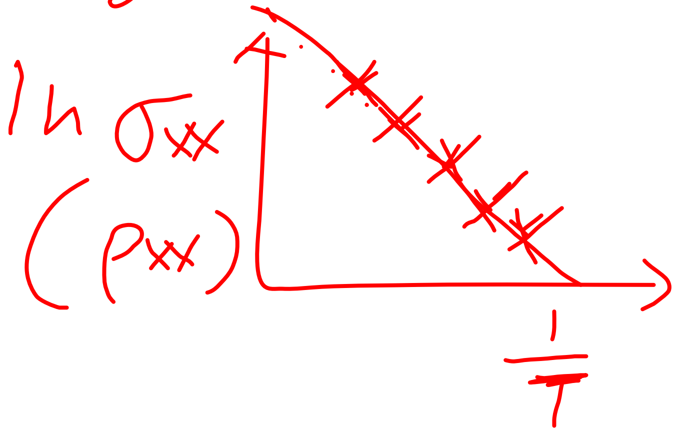
A 2DEG is not a perfect metal. About 10% of the electric field can penetrate a 2DEG!



$$\sigma_{xx} \propto \exp\left(-\frac{\Delta}{2k_BT}\right)$$

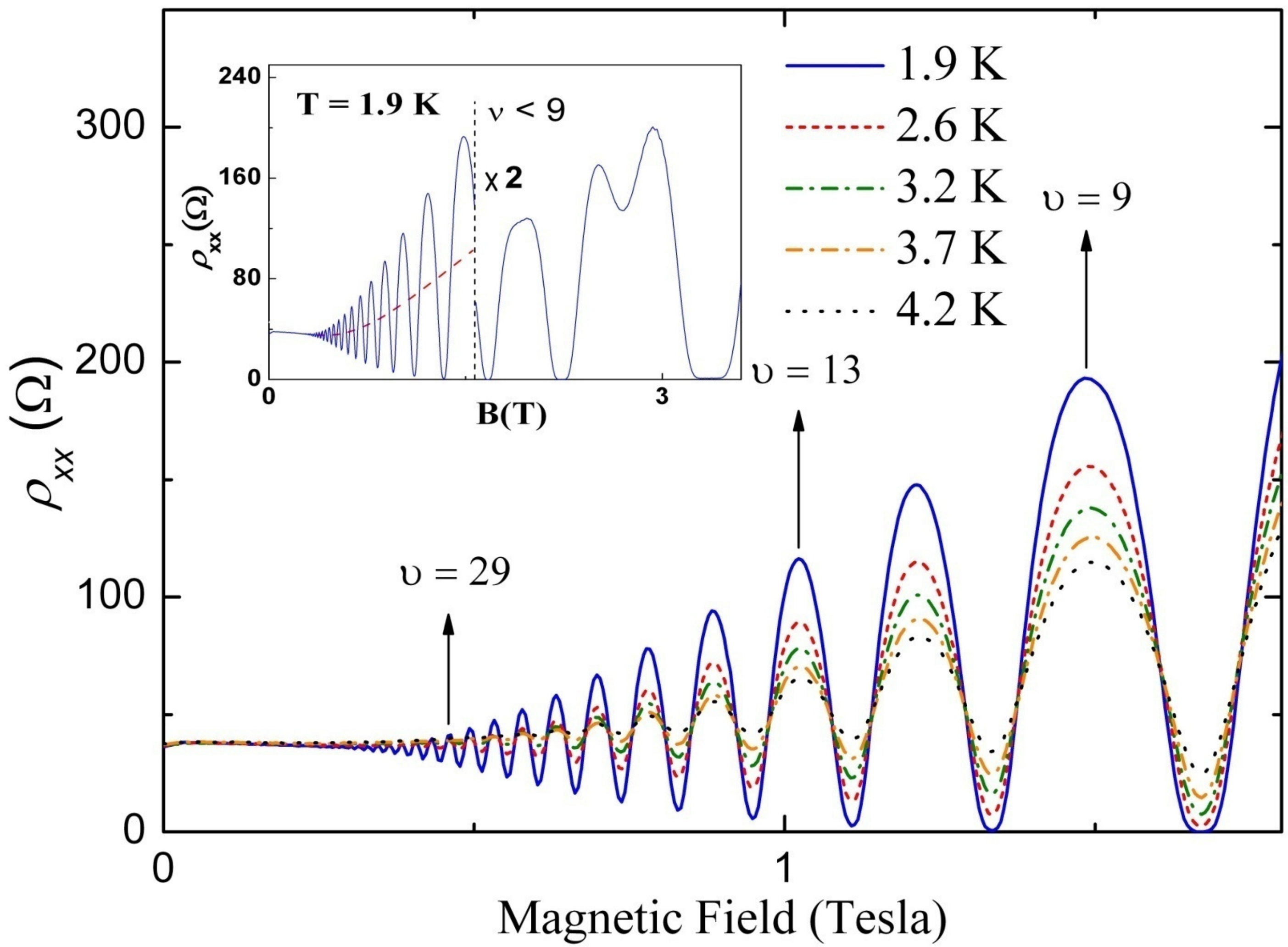
Activation energy Δ

Extended states



$$\ln \sigma_{xx} \propto -\frac{\Delta}{2k_B T} + C$$

1. $\Delta \gg k_B T$
2. σ_{xx} ρ_{xx} varied by a decade (a factor of 10)
3. σ_{xx} and $\rho_{xx} \to 0$



Introducing positive magnetoresistance (PMR) background

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^{\ 2} + \rho_{xy}^{\ 2}}$$

$$\text{both } \sigma_{xx} \to 0 \ \text{ and } \ \rho_{xx} \to 0 \text{ as } T \to 0$$

$$\rho_{xy} = \frac{h}{\nu e^2}$$

Quantum Hall Conductor

- ρ_{xx}, or more precisely, σ_{xx} increases with increasing T

- Activated behaviour: $\sigma_{xx} \propto \exp(-\Delta/2k_B T)$, where Δ is the mobility gap