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Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

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Arguments are presented that the $T=0$ conductance G of a disordered electronic system depends on its length scale L in a universal manner. Asymptotic forms are obtained for the scaling function $\beta(G)=d\ln G/d\ln L$, valid for both $G \ll G_c \simeq e^2/\hbar$ and $G \gg G_c$. In three dimensions, G_c is an unstable fixed point. In two dimensions, there is no true metallic behavior; the conductance crosses over smoothly from logarithmic or slower to exponential decrease with L.

- All states are localized in 2D

Strongly disordered systems

- Even for a “dirty” sample, the localisation length of a 2DEG is of the order of a kilometer therefore the sample is effectively delocalised at zero magnetic field because of its finite size
- Therefore it is widely accepted that in order to observe insulator-quantum Hall transitions, one needs to deliberately introduce disorder so as to experimentally realise a highly-disordered 2D system

$$\frac{x}{L} = \frac{\theta}{2\pi}$$

$|\leftarrow L \rightarrow|\leftarrow L \rightarrow|$

L : period

1D

2π : period as well

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\int_{-\infty}^{\infty} e^{-x^2}\, d(x^2) = -\left[\frac{1}{e^{x^2}}\right]_{-\infty}^{\infty}$$

$$= -(0 - 0)$$

$$= 0$$

$F = -kx$ Hooke's law

$$-kx = ma = m\ddot{x} = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Simple Harmonic oscillations

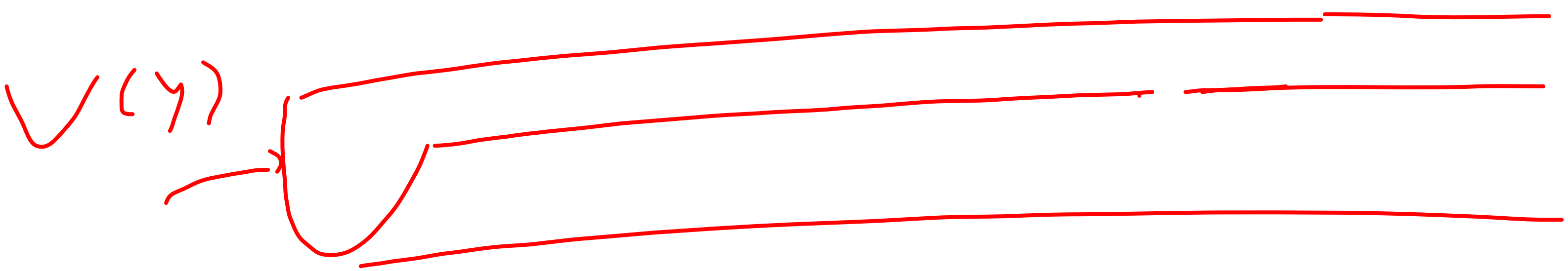
frictionless

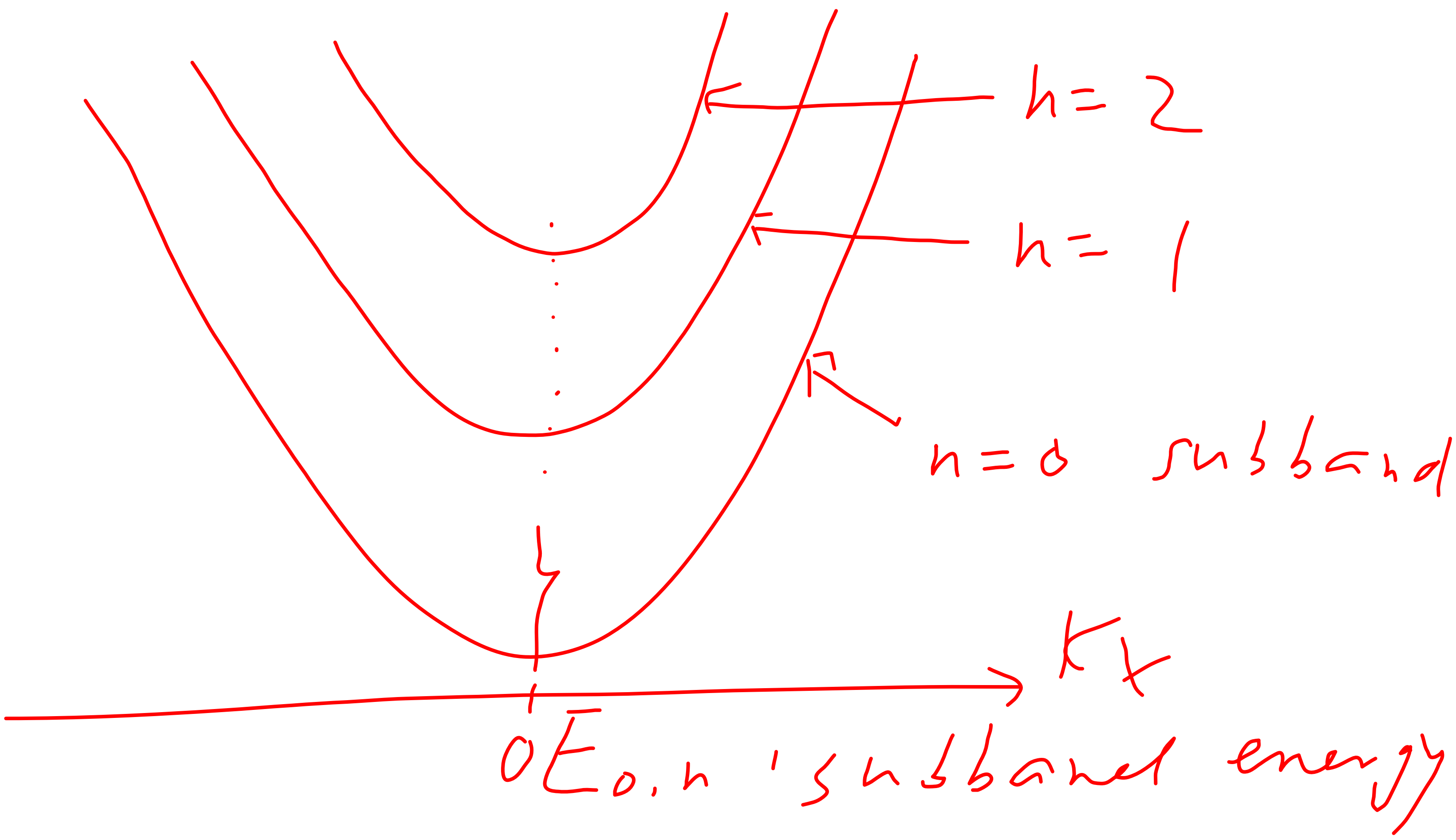
$$E = \frac{\hbar^2 k^2}{2m^*}$$

free-electron like

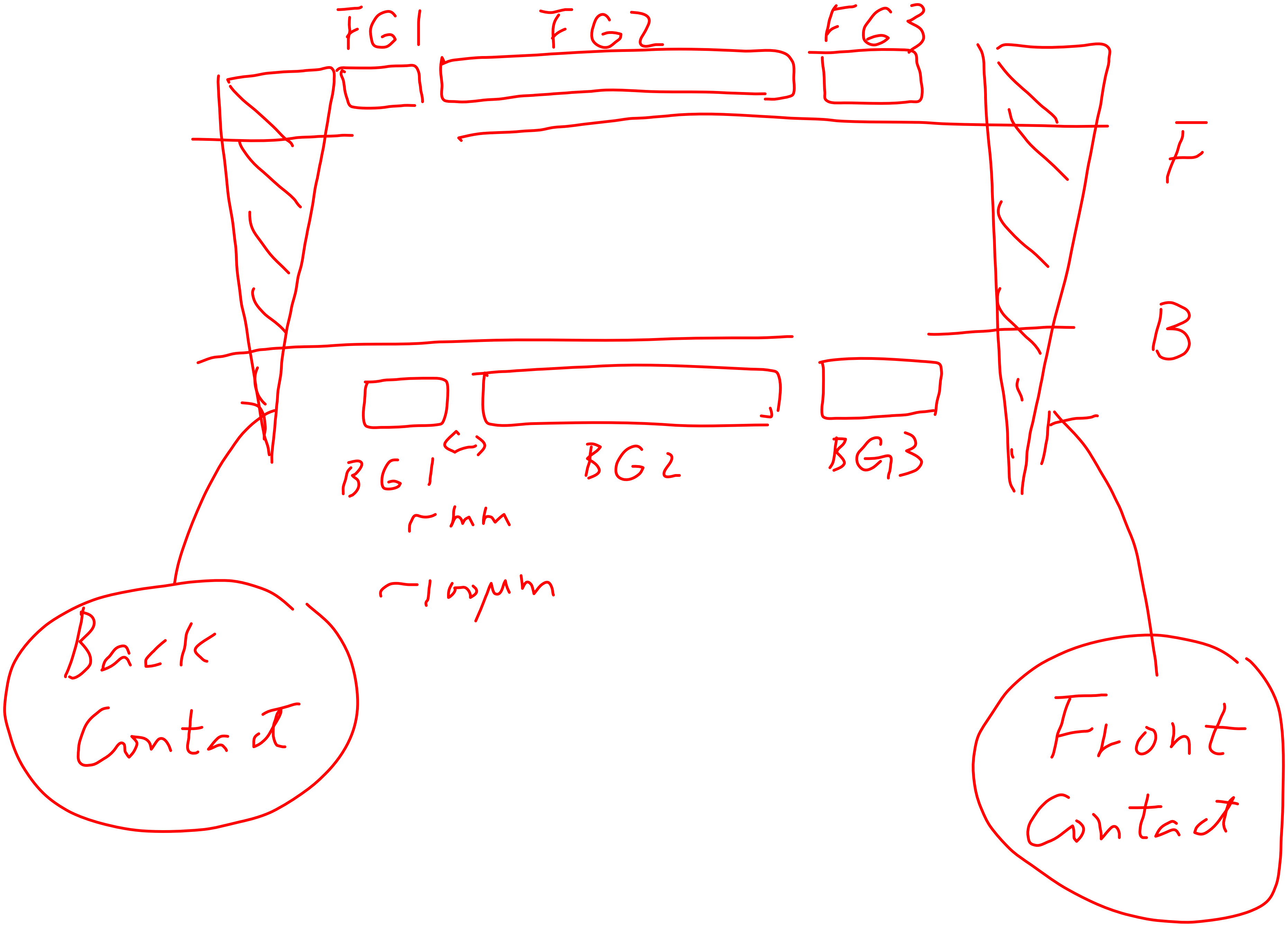
x: translational invariance

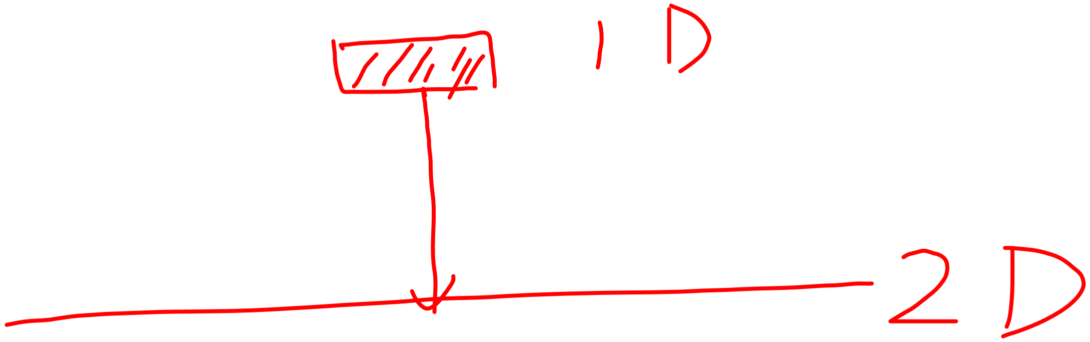
$$: \frac{\hbar^2 k_x^2}{2m^*}$$ infinite



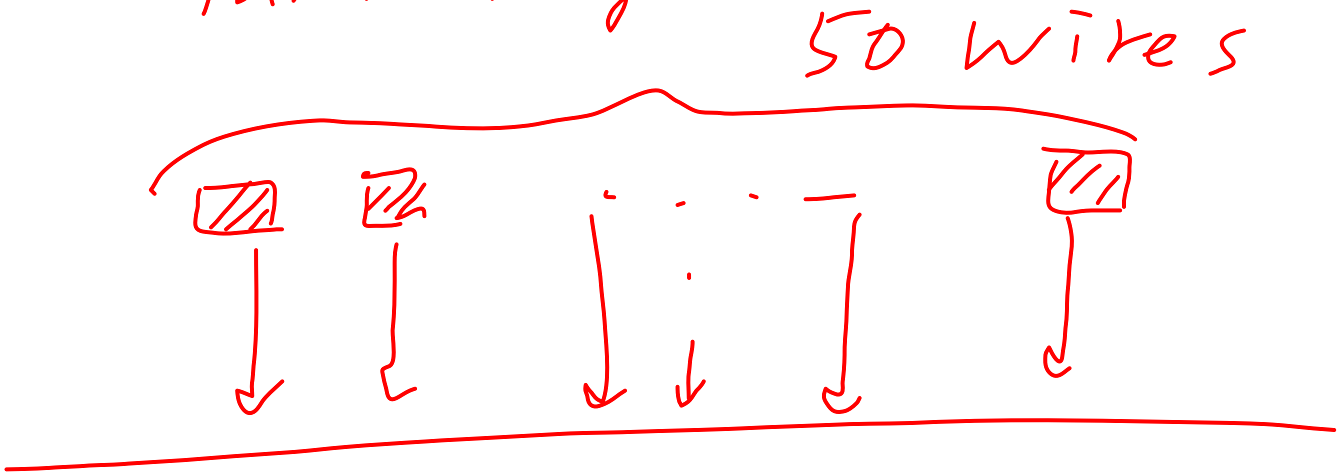


n = 2
n = 1
n = 0 subband
k_x
0
$E_{0,n}$'s subband energy


FG1
FG2
FG3
F
B
BG1
BG2
BG3
~mm
~100μm
Back Contact
Front Contact



Tunnelling area is too small



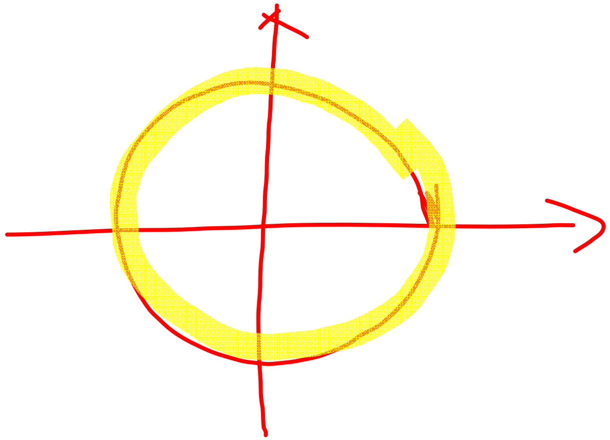
50 Nominally identical wires

Equilibrium tunneling:

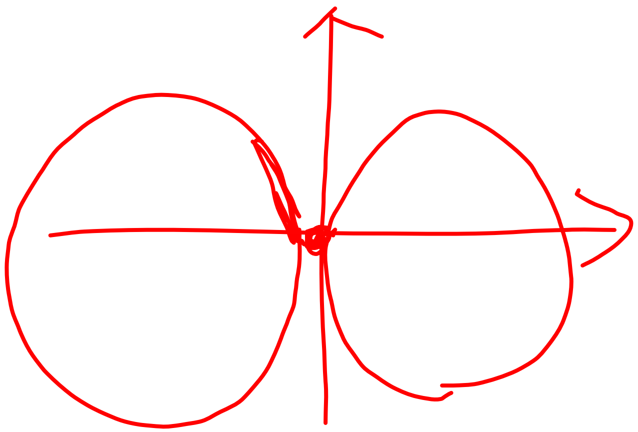
Small applied bias

$$G = \frac{dI}{dV}(ac)$$

2D-2D Tunnelling



Maximum Overlapping



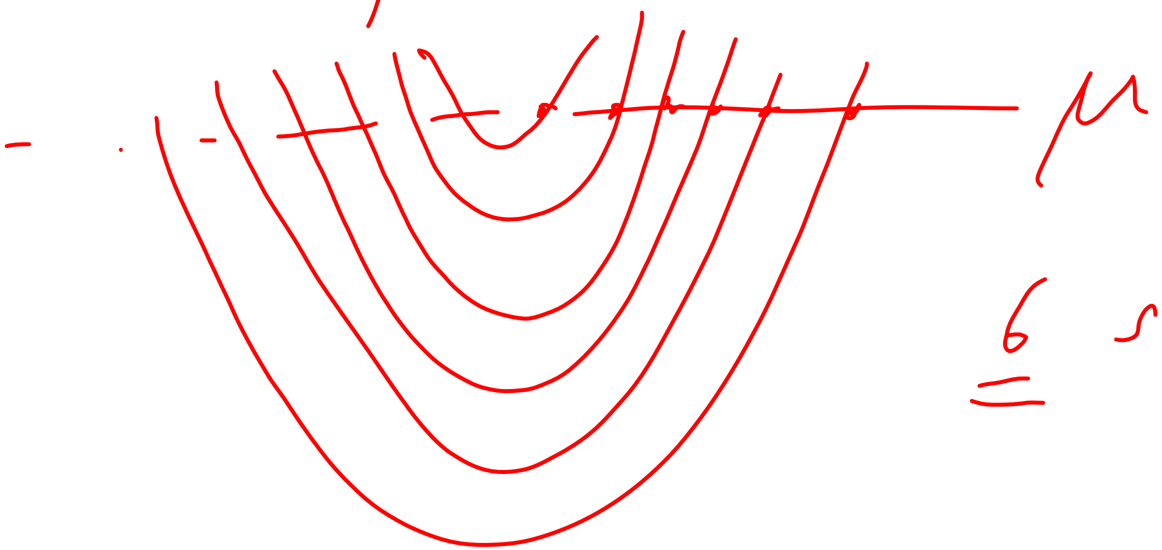
Minimum Overlapping

1D: $E_F = E_{0,n} + \frac{\hbar^2 k_x^2}{2m^*}$

Quasi-1D

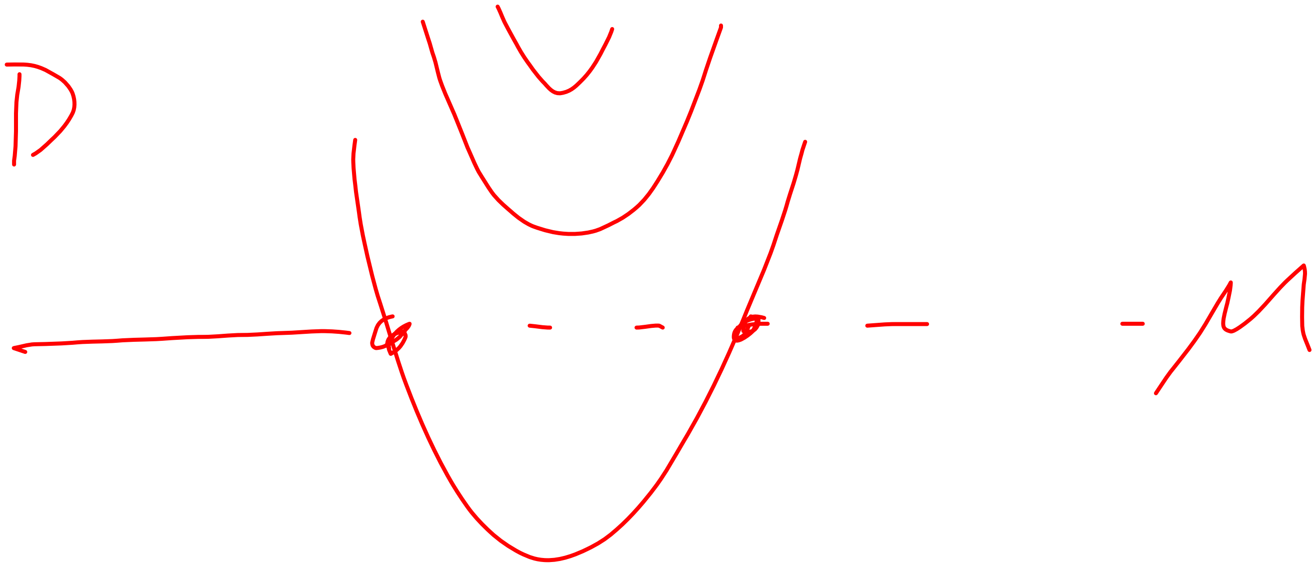
3, 4 . . 6 subbands

occupied



$\underline{\underline{6}}$ subbands

1D



1 subband only

$n <$ 1D, quasi-1D

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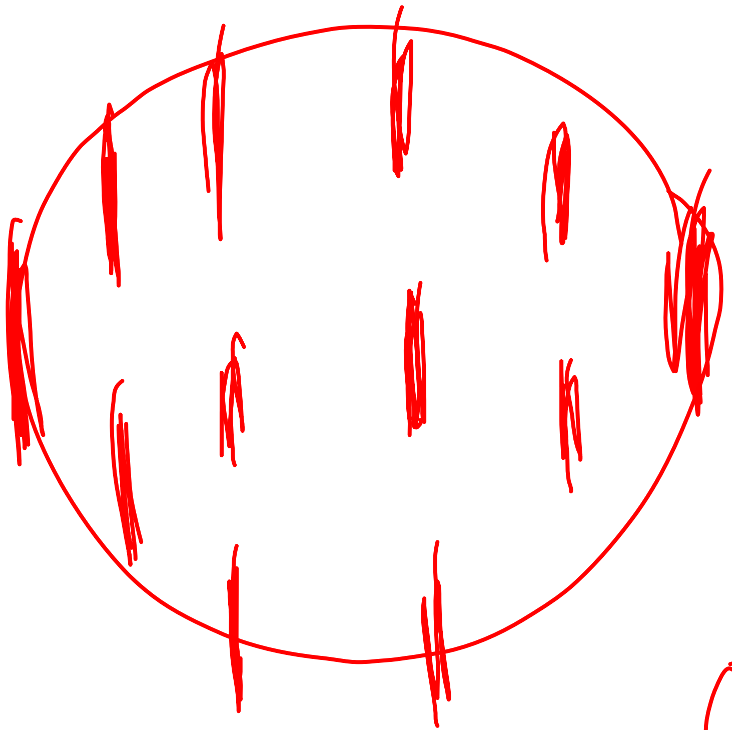


Overlapping

Integral

2D – 1D tunnelling

$$\vec{B} = \nabla \times \vec{A}$$

$$\lambda = \frac{h}{p} = \frac{2\pi}{k}$$

$$\frac{h}{2\pi} k = p \Rightarrow \hbar k = p$$

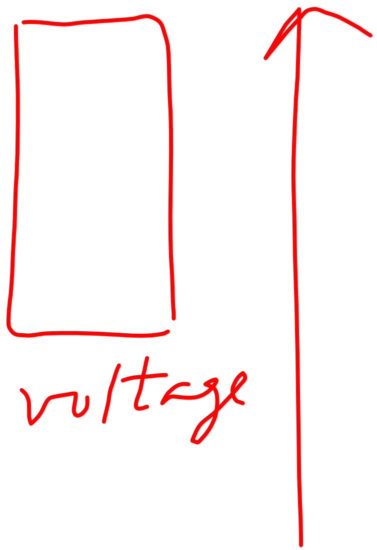
$$\frac{p}{\hbar} = k \quad (\text{wave vector})$$

$$G = \frac{dI}{dV} \cong \frac{1}{R}$$

$$1\,\mu S = 1\,M\Omega$$

$\boxed{\text{No offset}}$

G



fixed ac voltage by 90°

Bx By

Two cooldowns are required