Lecture 12

6.976 Flat Panel Display Devices

Physics of Liquid Crystals II

Outline

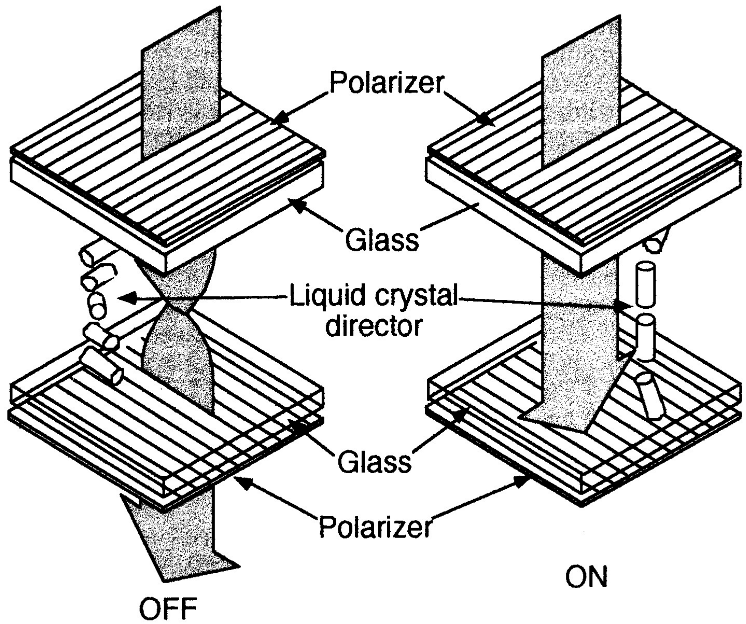
- **Properties of Liquid Crystals**
- **Elastic and Electrostatic Energy Density**
- **Field Effect Liquid Crystals**
- **Optical Properties of TN-LC Cell**

References

- Optics of Liquid Crystal Displays, Pouchi Yeh and Claire Gu, John Wiley & Sons, 1999.
- B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, John Wiley & Sons, New York
- E. Hecht, Optics, Addison-Wesley Publishing
- Peter J. Collings and Michael Hird, Introduction to Liquid Crystals-Chemistry and Physics, Taylor and Francis, 1997
- D. J. Channin and A. Sussman, Liquid Crystal Displays, LCD, Chapter 4 in Display Devices, Ed. Jacques I. Pankove, Spriger-Verlag, 1980.
- T. Scheffer, SID Seminar Notes 2000, Super Twisted Nematic LCDs

Summary of Today's Lecture

- Properties of LCs are in general anisotropic because of of their rod-like structure
- Static behavior of LCs is determined by
  - Balance between the electrostatic and elastic torques
  - Boundary conditions
- The application of an electric field leads to the re-orientation of director and hence change in optical properties
- TN-LC cell behaves as a polarization rotator



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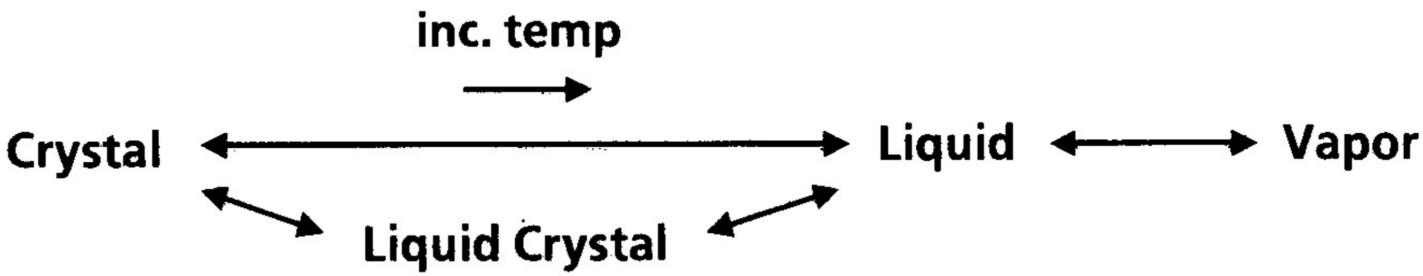
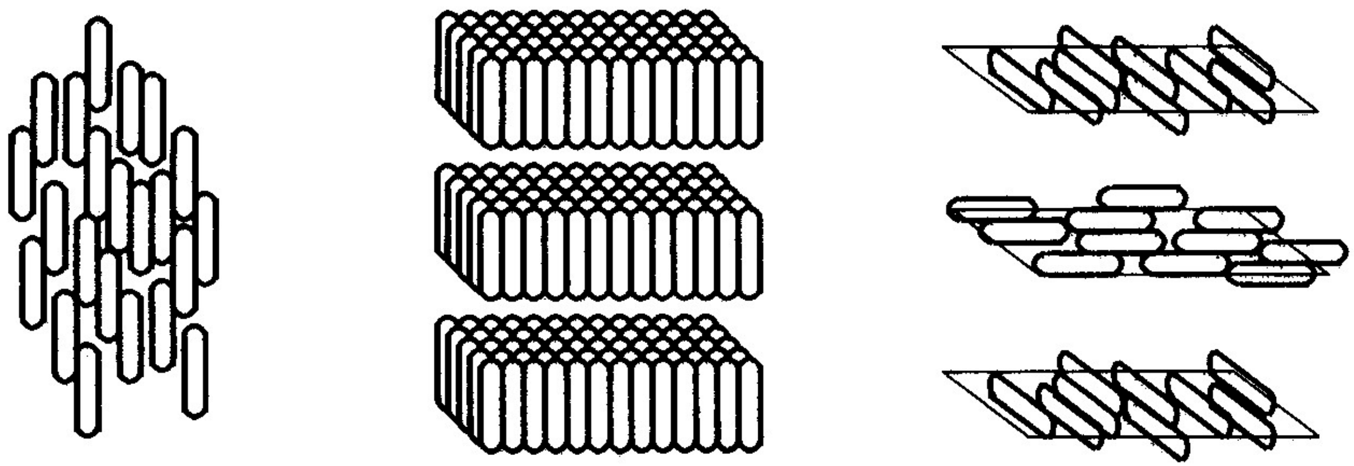
Liquid Crystal Structure

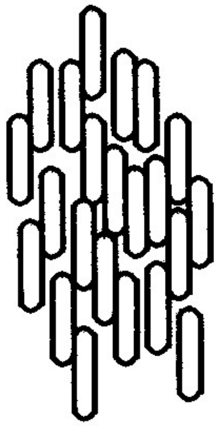




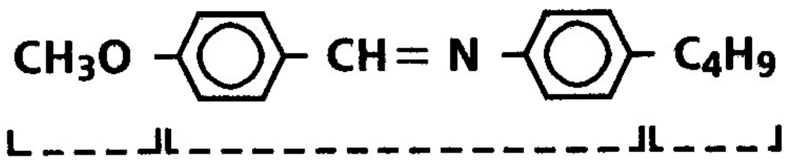
Nematic Liquid Crystal | Smectic Liquid Crystal | Cholesteric Liquid Crystal

Steemers, SID Seminar Notes 1994

Liquid Crystal Molecular Structure



Nematic
liquid Crystal



terminal group | core molecule | terminal group

Data Sheet :

Smectic - Nematic Phase Transition	S→N	- 30C
Nematic - Isotropic Phase Transition	Clearing Point	+ 91C
Viscosity	v	18 mm^2s^{-1}
Dielectric Anisotropy	Δε	+ 6.6
Optical Anisotropy	Δn	+ 0.085

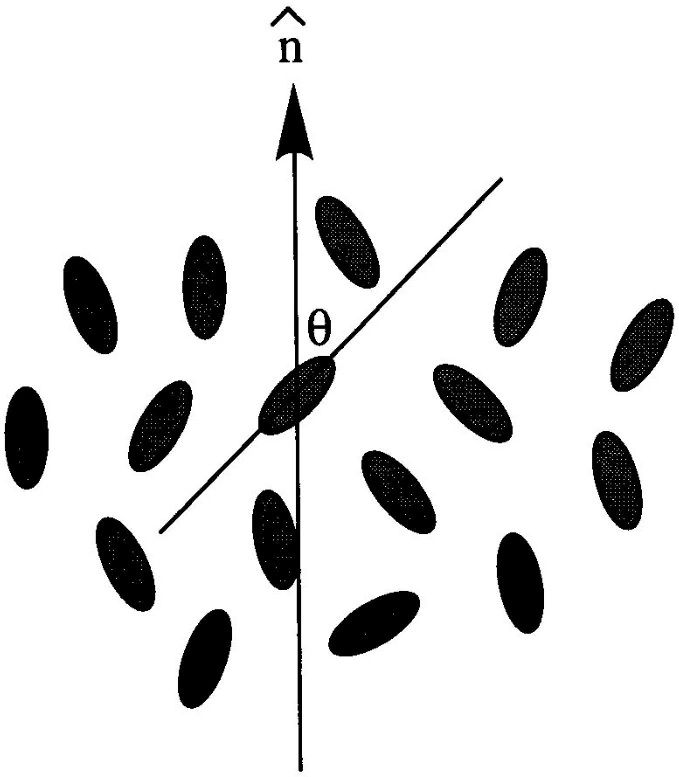
Steemers, SID Seminar Notes 1994

Orientational Order of LCs

- Director **n** at any point is the prefered orientation in the immediate neighborhood
  - In homogeneous LC, it is constant throughout medium
  - In in-homogeneous medium, **n**=**n**(x,y,z)
- Order parameter of an LC is given by

$$S = \langle P_2(\cos\theta) \rangle = \frac{1}{2}\langle 3\cos^2\theta - 1 \rangle$$

  - θ is the angle between the long axis and director **n**



Molecular order in a nematic liquid crystal.

Collings

Dielectric Constants

- Nematic and smetic LCs are uniaxially symmetric with axis of symmetry parallel to the director n.
  - Dielectric constants differ in value along the preferred axis ($\varepsilon_{||}$) and perpendicular to the preferred axis ($\varepsilon_{\perp}$)
- Dielectric anisotropy is $\Delta\varepsilon = \varepsilon_{||} - \varepsilon_{\perp}$ $\quad -2\varepsilon_0 \le \Delta\varepsilon \le 15\varepsilon_0$
- The macroscopic energy is $W_{em} = \frac{1}{2}\overline{D} \bullet \overline{E}$
- If θ is the angle between the director and z-axis

$$D_z = \left(\varepsilon_{||}\cos^2\theta + \varepsilon_{\perp}\sin^2\theta\right)E$$

$$W_{em} = \frac{1}{2}\frac{D_z^2}{\varepsilon_{||}\cos^2\theta + \varepsilon_{\perp}\sin^2\theta}$$

Refractive Index

- As a result of the uniaxial symmetry, LCs have two principal refractive indices n_o and n_e
  - Ordinary refractive index no is for light with e-field polarization perpendicular to director
  - Extra-ordinary refractive index is for light with E-field polarization parallel to the director.
- Birefringence (or optical anisotropy) is

$$\Delta n = n_e - n_o$$

- Macroscopic refractive index is related to molecular polarizability at optical frequencies
  - Optical anisotopicity mainly due to presence of delocalized electrons not participating in chemical bonds —π electrons

Elastic Constants

- In LCs electric field often applied to cause the re-orientation of molecule
- Elastic constants of LCs determine torques that arise when the system is perturbed from its equilibrium configuration
  - Weak torque compared to solids
- Three deformations characterize LC static deformation pattern
  - Splay k_1
  - Twist k_2
  - Bend k_3

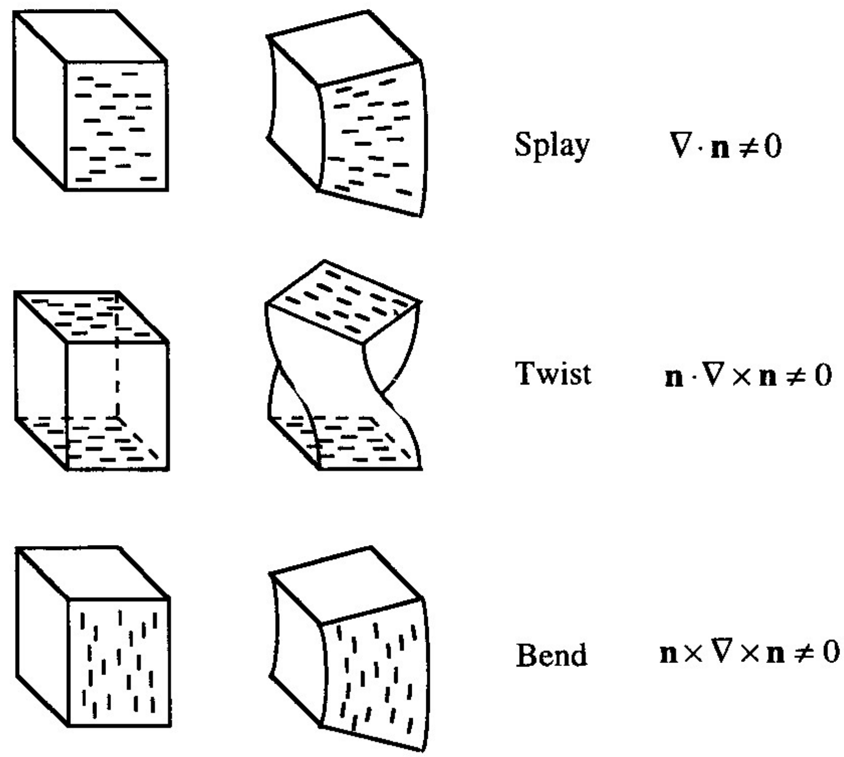


Figure 1.6. Schematic drawing of splay, twist, and bend in LC

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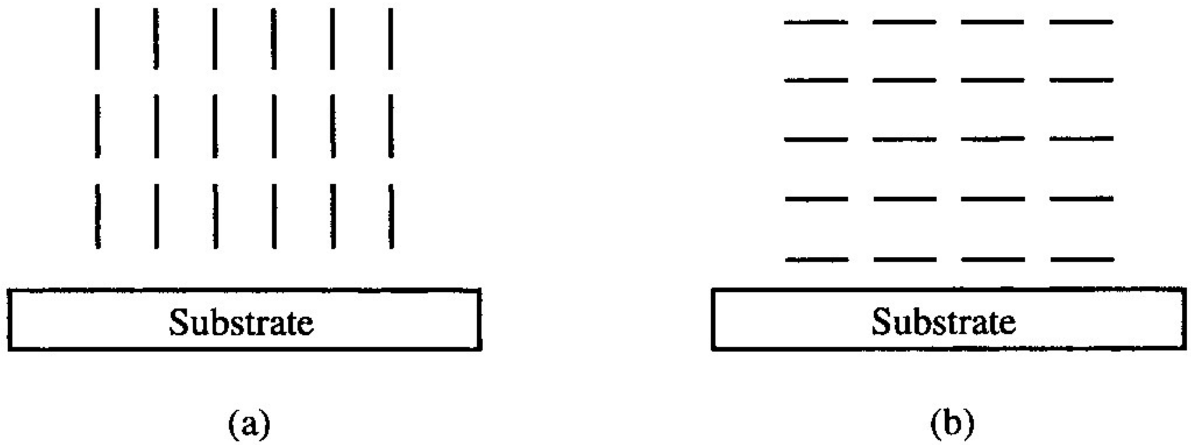
Rotational Viscocity

- Viscosity is internal resistance to fluid flow
  - Arises from intermolecular forces within fluid
  - Ratio of shearing stresses to rate of shear
- Viscosity of LC affects dynamical behavior
  - Increases at low T and decreases at high T
- Rotational viscosity provides resistance to rotation of LC molecules
  - Rotational viscous coefficient γ

Surface Alignment



Yeh & Gu

Figure 1.7. (*a*) Homeotropic (vertical) alignment; (*b*) parallel homogeneous alignment.

- Boundary conditions of an LC cell important for orientation
  - Determine optical properties
- Surface ensures that LC cell is a single domain
- Homeotropic alignment —LC director normal to surface
- Homogeneous alignment —LC director paralle to surface

Rubbing

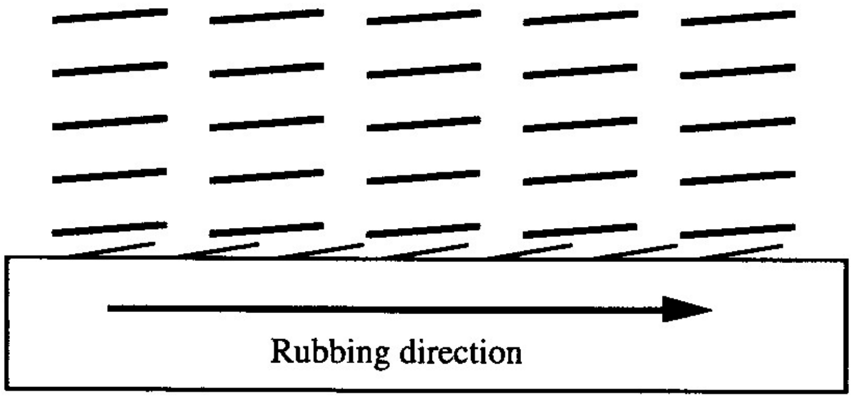


Figure 1.8. Schematic drawing of the anchoring of rodlike LC molecules near the surface of a rubbed substrate.

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- Rubbing surface with linen cloth or lens paper leads to a preferred orientation
  - Determine optical properties
- Rubbing surface also produces a uniform unidirectional tilt

Pre-Tilt Angle

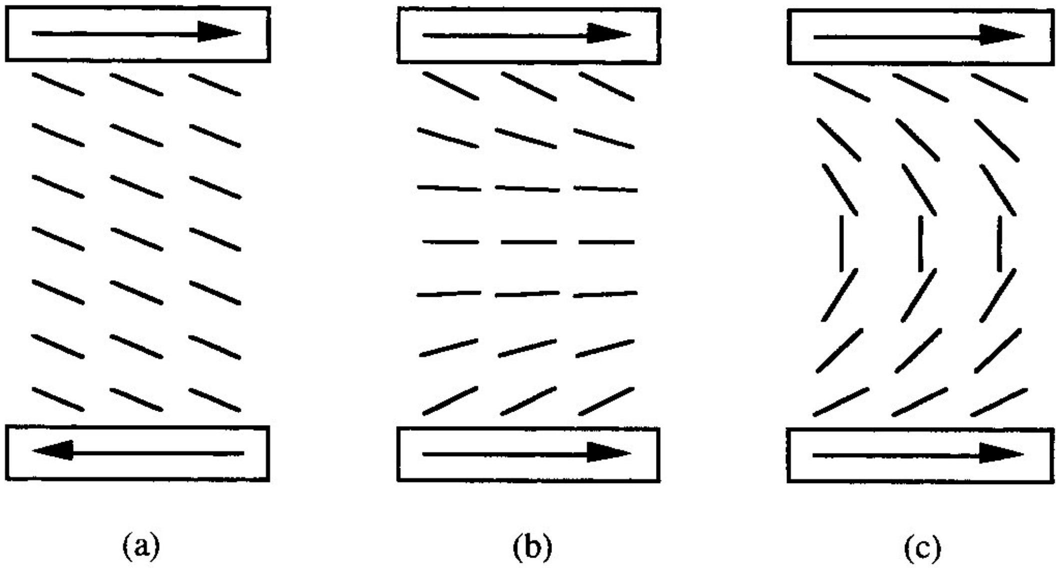

Figure 1.9. Tilt of LC molecular axes due to rubbing of inner surfaces of the LC cell. The arrows indicate the direction of rubbing. (*a*) Parallel alignment occurs when the rubbings are in opposite directions. Splay cell (*b*) or bend cell (*c*) can occur when the rubbings are in the same direction.

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Liquid Crystal Properties

Table 1.2. Properties of Liquid Crystals

	T (°C)	λ (nm)	n_e	n_o	$\varepsilon_\parallel$	$\varepsilon_\perp$	k_1 (10^{-12} N)	k_2 (10^{-12} N)	k_3 (10^{-12} N)	Nematic	Range (°C)	Reference
MBBA[a]	22	589	1.769	1.549	4.7	5.4	6.2	3.8	8.6	20	47	4
PCH-5[b]	30.3	589	1.604	1.4875	17.1	5	8.5	5.1	16.2	30	55	5,12
		633	1.600	1.4851						30	55	
	38.5	589	1.5956	1.4863	16.6	15.3	7.3	4.5	13.2			
		633	1.5919	1.4840								
	46.7	589	1.5849	1.4860	15.9	5.7	5.9	3.9	9.9			
		633	1.5812	1.4836								
K15 (5CB)[c]	25	515	1.736	1.5442	19.7	6.7	6.4	3	10	24	35.3	18
K21 (7CB)[d]	37	577	1.6815	1.5248	15.7	6	5.95	4	6.6	30	42.8	8,15,16,17
M15 (5OCB)[e]	50	589	1.7187	1.5259	17.9	6.7	6.1	3.74	8.4	48	68	9,11,15,16
M21 (7OCB)[f]	60	589	1.6846	1.5139	16.3	6.5	6.9		7.7	54	74	15,16
M24 (8OCB)[g]	70	589	1.6639	1.5078	14.7	6.2	7.3		9.0	67	80	15,16
E5[h]	20	577	1.736	1.5228	19	5.9				−8	50.5	12,15
E7[i]	20	577	1.75	1.5231	19.6	5.1	12	9	19.5	−10	60.5	15
ZLI-1646[j]	20	589	1.558	1.478	10.6	4.6	7.7	4.0	12.2	−20	60	12,15
ZLI-4792[k]	20	589	1.573	1.479	8.3	3.1	13.2	6.5	18.3	−40	92	12

[a] *p*-Methoxybenzylidene-*p*′-*n*-butylaniline
[b] 4-(*trans*-4-Pentylcyclohexyl)benzonitrile
[c] Pentylcyanobiphenyl.
[d] Heptylcyanobiphenyl.
[e] Pentyloxycyanobiphenyl.
[f] Heptyloxycyanobiphenyl.
[g] Octyloxycyanobiphenyl.
[h] 45%K15 + 24%K21 + 10%M15 + 9%M21+12%M24.
[i] 47%K15 + 25%K21 + 18%M24 + 10%T15.
[j] Phenylcyclohexane and biphenylcyclohexane mixtures.
[k] SFM-TFT mixture.

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Elastic Energy Density

$$W_{el} = \frac{1}{2}\left[k_1(\nabla \bullet \bar{n})^2 + k_2(\bar{n} \bullet \nabla \times \bar{n})^2 + k_3(\bar{n} \times \nabla \times \bar{n})^2\right]$$

- Splay Elastic Constant k_1
- Twist Elastic Constant k_2
- Bend Elastic Constant k_3

$$3 < k < 25 \text{ pN}$$

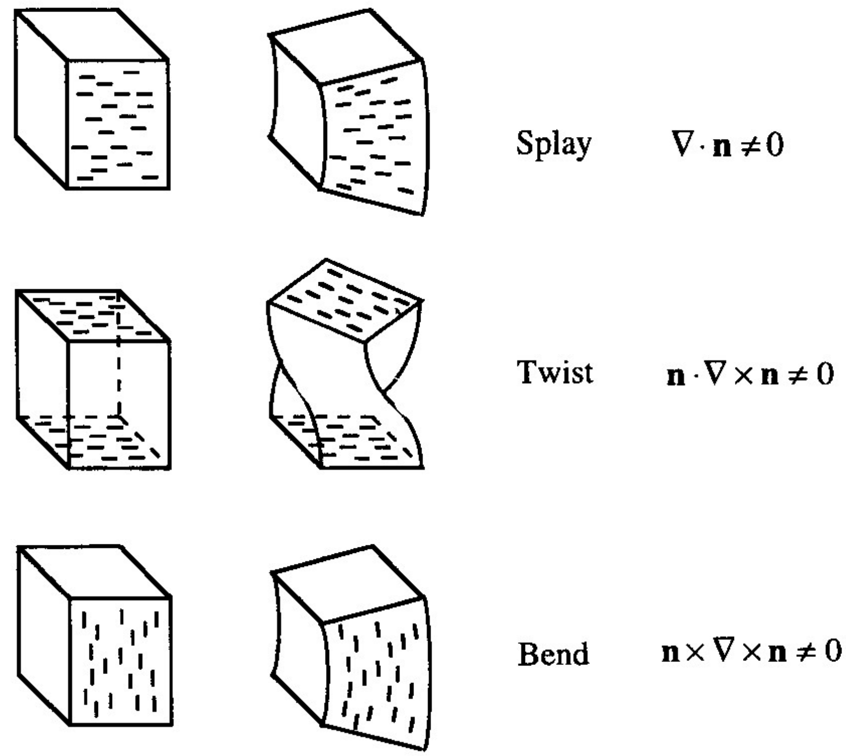

Figure 1.6. Schematic drawing of splay, twist, and bend in LC

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Tilt Mode

- Consider a nematic cell with initial director distribution n(z) parallel to the y-axis
- Electric field applied along z-axis
- n(z) tilted towards z-axis but with B.C. n(0) and n(d) parallel to y-axis
- $\theta(z)$ is the tilt angle at z

$$\overline{n}(z) = (0, \cos\theta, \sin\theta)$$

$$\nabla \bullet \overline{n} = \cos\theta \frac{d\theta}{dz}$$

$$\nabla \times \overline{n} = \left(\sin\theta \frac{d\theta}{dz}, 0, 0 \right)$$

$$W_{el} = \frac{1}{2}\left[k_1 \cos^2\theta + k_3 \sin^2\theta\right]\left(\frac{d\theta}{dz}\right)^2$$

Twist Mode

- Consider a nematic cell with initial director distribution n(z) parallel to the x-axis
- Electric field applied along y-axis
- n(z) tilted towards y-axis but with B.C. n(0) and n(d) parallel to y-axis
- $\phi(z)$ is the twist angle at z

$$\bar{n}(z) = (\cos\phi, \sin\phi, 0)$$

$$\nabla \bullet \bar{n} = 0$$

$$\nabla \times \bar{n} = (-\cos\phi, \sin\phi, 0)\frac{d\phi}{dz}$$

$$W_{el} = \frac{1}{2} k_2 \left(\frac{d\phi}{dz}\right)^2$$

Twist & Tilt Mode

- Consider a nematic cell with initial director distribution n(z) parallel to the xy-plane
- Electric field applied along z-axis tilts the directors
- n(z) tilted towards y-axis but with B.C. n(0) and n(d) parallel to ~~y-axis~~
- $\phi(z)$ is the twist angle at z and $\theta(z)$ is the tilt angle at z

$$\bar{n}(z) = (\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta)$$

$$\nabla \bullet \bar{n} = \cos\theta \frac{d\theta}{dz}$$

$$\nabla \times \bar{n} = \left(\sin\theta\sin\phi\frac{d\theta}{dz} - \cos\theta\cos\phi\frac{d\phi}{dz}, -\sin\theta\cos\phi\frac{d\theta}{dz} - \cos\theta\sin\phi\frac{d\phi}{dz}, 0 \right)$$

$$W_{el} = \frac{1}{2}\left[k_1\cos^2\theta + k_3\sin^2\theta\right]\left(\frac{d\theta}{dz}\right)^2 + \frac{1}{2}\left[k_2\cos^2\theta + k_3\sin^2\theta\right]\cos^2\theta\left(\frac{d\phi}{dz}\right)^2$$

Electromagnetic Energy

$$W_{em} = \frac{1}{2}\overline{D} \bullet \overline{E} = \frac{1}{2}\varepsilon\overline{E} \bullet \overline{E}$$

$$W_{em} = \frac{1}{2}\varepsilon_{\perp}E^2 - \frac{1}{2}\left(\varepsilon_{\perp} - \varepsilon_{\parallel}\right)\left(\overline{n} \bullet \overline{E}\right)^2$$

Constant charge on electrode

$$\Delta W_{em} = \frac{1}{2}\frac{D^2}{\varepsilon_f} - \frac{1}{2}\frac{D^2}{\varepsilon_i}$$

Constant voltage on electrode

$$\Delta W_{em} = \frac{1}{2}\varepsilon_i E^2 - \frac{1}{2}\varepsilon_f E^2$$

Rotational Viscous Torque

- When the director configuration n(x,y,z) is not the equilibrium one, internal torques cause rotation of n towards the equilibrium configuration
- Viscous forces oppose rotation of LC
- In the absence of LC material flow

$$\Gamma_{vis} = -\gamma_1 \bar{v} \times \frac{\partial \bar{n}}{\partial t} = -\gamma_1 \bar{n} \times \left(\bar{\Omega} \times \bar{n} \right)$$

where

$\bar{\Omega}(x, y, z)$ is director angular velocity

$\bar{v}$ is flow velocity

γ_1 is the viscous coefficient

Total Free Energy

Total Free Energy W

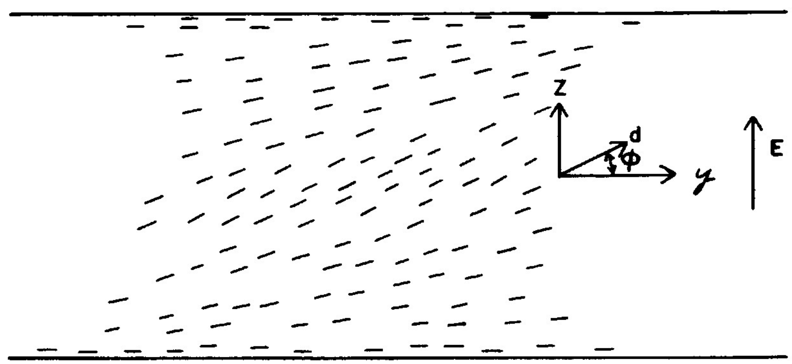
$$W = W_{el} + W_{em}$$

Equilibrium configuration of LC determined

$$\delta \left| \int_V W(x, y, z) dxdydz \right| = 0$$

Surface alignment provides the boundary conditions that director must satisfy along with above equation

Forces acting on LC



Channin

$$\overline{n}(z) = (0, \cos\theta, \sin\theta)$$

$$W = \frac{1}{2}\left[k_1 \cos^2\theta + k_3 \sin^2\theta\right]\left(\frac{d\theta}{dz}\right)^2 - \frac{1}{2}\varepsilon_\perp E^2 + \frac{1}{2}\left(\varepsilon_\perp - \varepsilon_\parallel\right)E^2 \sin^2\theta$$

Equilibrium between elastic and electromagnetic forces

$$\frac{1}{2}\left[k_1 \cos^2\theta + k_3 \sin^2\theta\right]\left(\frac{d\theta}{dz}\right)^2 +$$

$$\left[\cancel{\left[k_1 \cos^2\theta + k_3 \sin^2\theta\right]}\left(\frac{d\theta}{dz}\right)^2 + \frac{1}{2}\Delta\varepsilon E^2\right]\cancel{\cos}\ \sin^2\theta = \text{constant}$$

$$\theta(0,t) = \theta(d,t) = 0$$

Forces acting on LC

(Add Viscous Torque)

Assume $k_1 \approx k_3 \Rightarrow k_1 - k_3 = 0$

$$k_1 \frac{\partial^2 \theta}{\partial z^2} + \Delta\varepsilon E^2 \theta - \gamma_1 \frac{\partial \theta}{\partial t} = 0$$

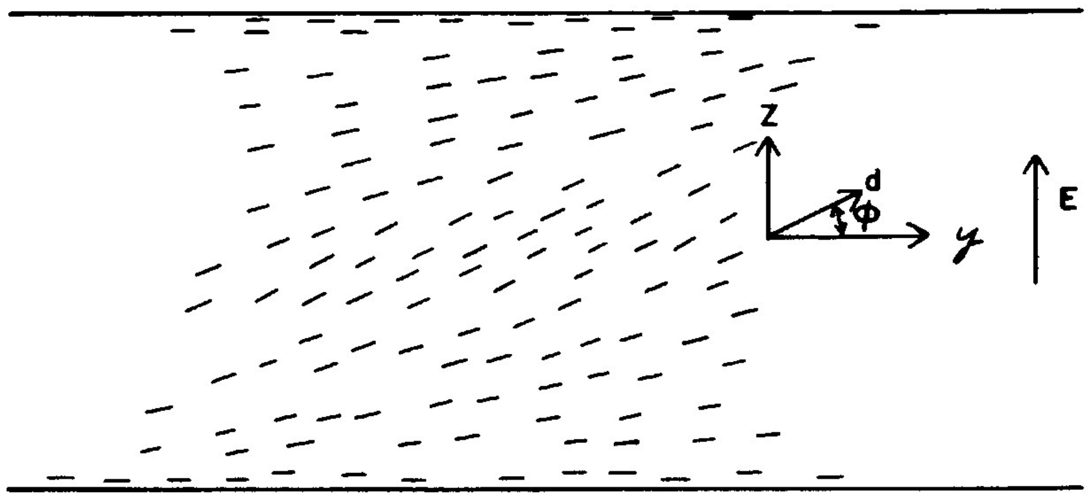
$$\theta(0, t) = \theta(d, t) = 0$$

$$\theta(z, t) = \theta_0 e^{-t/\tau} \cos\left(\frac{\pi}{d}(d - 2z)\right)$$

$$\theta_0 = \theta(z = 0, t = 0)$$

$$\tau = \gamma_1 \left[k_1 \left(\frac{2\pi}{d}\right)^2 - \Delta\varepsilon E^2 \right]$$

Freedericksz Transistion



Channin

$$E = E_c = \frac{\pi}{d}\sqrt{k_1/\Delta\varepsilon}; \quad \tau = 0 \qquad V_c = \pi\sqrt{k_1/\Delta\varepsilon}$$

$$E < E_c; \qquad \tau > 0$$

Disturbance decays with time

$$E > E_c; \qquad \tau < 0$$

Disturbance grows with time

Freedericksz Transistion

With zero field the decay is exponential

$$\tau_{relaxation} = \gamma_1 k_1 \left(\frac{2\pi}{d} \right)^2$$

When E>E$_c$, elastic torques are negligible compared to electromagnetic and viscous torques

$$\frac{1}{\tau_{response}} = \frac{1}{\theta} \frac{\partial\theta}{\partial t} = \frac{\Delta\varepsilon E^2}{\gamma_1} = \frac{\Delta\varepsilon V^2}{\gamma_1 d^2}$$

Twisted Nematic Transmission

Consider propagation of light along the axis of twist of the TN-LC which is linear with z

$$\psi(z) = \alpha z \qquad \Gamma = \frac{2\pi}{\lambda}(n_e - n_o)d$$

Total twist angle $\phi = \psi\,(d) = \alpha z$

Divide d into N incremental layers of equal width $\Delta z = d/N$

For the mth layer $\qquad z = z_m = m\,\Delta z\,, \quad 1 \le m \le N$

$$\phi_m = m\,\Delta\phi = m\,\alpha\,\Delta z$$

where ϕ_m is the angle made by the mth layer with the x-axis

Twisted Nematic Transmission

The Jones matrix of the wave retarder whose slow axis (optic axis) makes an angle $\phi_m = m\,\Delta\,\phi$ is given by

$$T_m = R(-\phi_m)T_r R(\phi_m)$$

R(ϕ) is the coordinate rotation matrix

$$\sout{T_r = \begin{pmatrix} \mathbf{exp}(-jn_e k_o \Delta z) & 0 \\ 0 & \mathbf{exp}(jn_e k_o \Delta z) \end{pmatrix}}$$

$$T_r = \mathbf{exp}(-j\phi\Delta z)\begin{pmatrix} \mathbf{exp}\left(-j\beta\frac{\Delta z}{2}\right) & 0 \\ 0 & \mathbf{exp}\left(j\beta\frac{\Delta z}{2}\right) \end{pmatrix}$$

$$\phi = \frac{n_e + n_o}{2}k_o \qquad \text{and} \qquad \beta = (n_e - n_o)k_o$$

Twisted Nematic Transmission

Overall transmission can be expressed as a product of individual transmissions

$$T = \prod_{m=1}^{N} T_m = \prod_{m=1}^{N} R(-\phi_m) T_r R(\phi_m)$$

Note that $\quad R(\phi_m)R(\phi_{m-1}) = R(\phi_m - \phi_{m-1}) = R(\Delta\phi)$

$$T_r R(\Delta\phi) = \begin{pmatrix} \exp\left(-j\beta\frac{\Delta z}{2}\right) & 0 \\ 0 & \exp\left(j\beta\frac{\Delta z}{2}\right) \end{pmatrix} \begin{pmatrix} \cos\alpha\Delta z & \sin\alpha\Delta z \\ -\sin\alpha\Delta z & \cos\alpha\Delta z \end{pmatrix} = \begin{pmatrix} \cos\frac{\phi}{N} e^{-j\frac{\Gamma}{2N}} & \sin\frac{\phi}{N} e^{-j\frac{\Gamma}{2N}} \\ -\sin\frac{\phi}{N} e^{j\frac{\Gamma}{2N}} & \cos\frac{\phi}{N} e^{j\frac{\Gamma}{2N}} \end{pmatrix}$$

$$T = R(-\phi)[T_r R(\Delta\phi)]^N = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{bmatrix} \cos\frac{\phi}{N} e^{-j\frac{\Gamma}{2N}} & \sin\frac{\phi}{N} e^{-j\frac{\Gamma}{2N}} \\ -\sin\frac{\phi}{N} e^{j\frac{\Gamma}{2N}} & \cos\frac{\phi}{N} e^{j\frac{\Gamma}{2N}} \end{bmatrix}^N$$

Adiabatic Following

(Waveguiding in TN-LC)

When $\alpha << \beta$ $\quad R(\Delta\phi) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$T = R(-\phi_N)[T_r]^N R(\phi_1) = R(-\alpha N\Delta z)\begin{pmatrix} \mathbf{exp}\left(-j\beta\frac{\Delta z}{2}\right) & 0 \\ 0 & \mathbf{exp}\left(j\beta\frac{\Delta z}{2}\right) \end{pmatrix}^N = R(-\alpha N\Delta z)\begin{pmatrix} \mathbf{exp}\left(-j\beta N\frac{\Delta z}{2}\right) & 0 \\ 0 & \mathbf{exp}\left(j\beta N\frac{\Delta z}{2}\right) \end{pmatrix}$$

In the limit as $N \to \infty$; $\Delta z \to 0$; $N\Delta z \to d$

$$T = R(-\alpha d)\begin{pmatrix} \mathbf{exp}\left(-j\beta\frac{d}{2}\right) & 0 \\ 0 & \mathbf{exp}\left(j\beta\frac{d}{2}\right) \end{pmatrix}$$

T≡wave retarder βd with slow axis-along the x-axis followed by polarization rotator αd.

Adiabatic Following
(Waveguiding in TN-LC)

Consider an input beam of light polarized parallel to the LC director (c-axis)

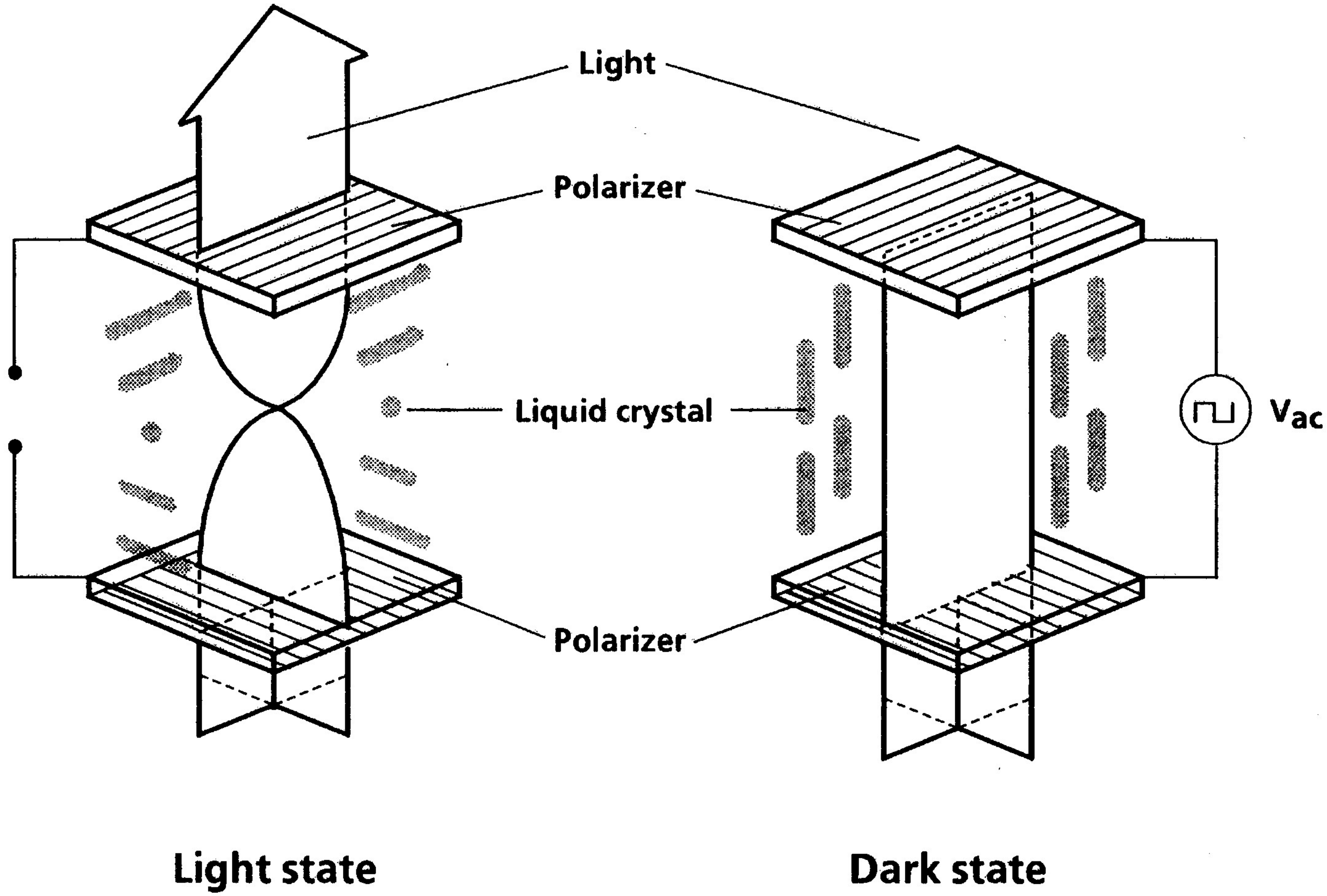
$$\begin{pmatrix} V_e \\ V_o \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} V_e' \\ V_o' \end{pmatrix} = R\cancel{(\alpha d)} \begin{pmatrix} e^{-j\frac{\Gamma}{2}} & 0 \\ 0 & e^{+j\frac{\Gamma}{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} V_e' \\ V_o' \end{pmatrix} = e^{-j\frac{\Gamma}{2}} R\cancel{(\alpha d)} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Net effect is to provide a phase shift and rotate the polarization by an angle of $\alpha d = \phi$

Twisted Nematic LC Cell



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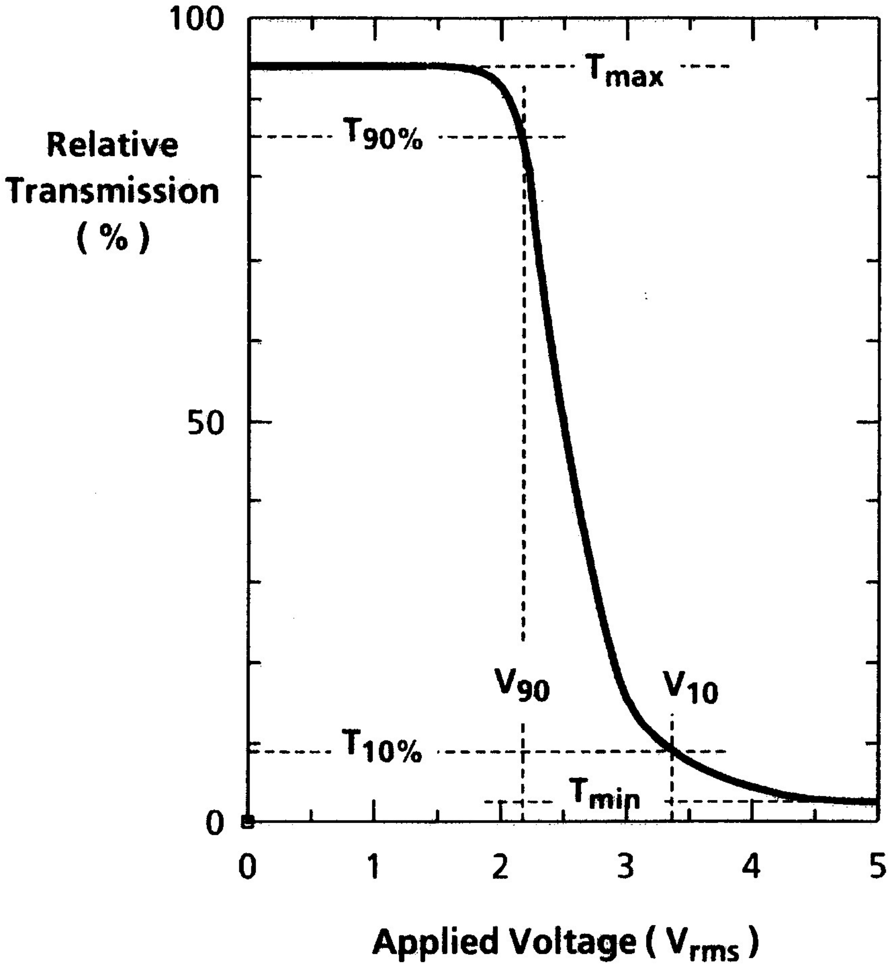
Electro-optic Response of TN-LC Cell



- V_{10}, V_{90}
- Contrast ratio = T_{max} / T_{min}
- Gray scale

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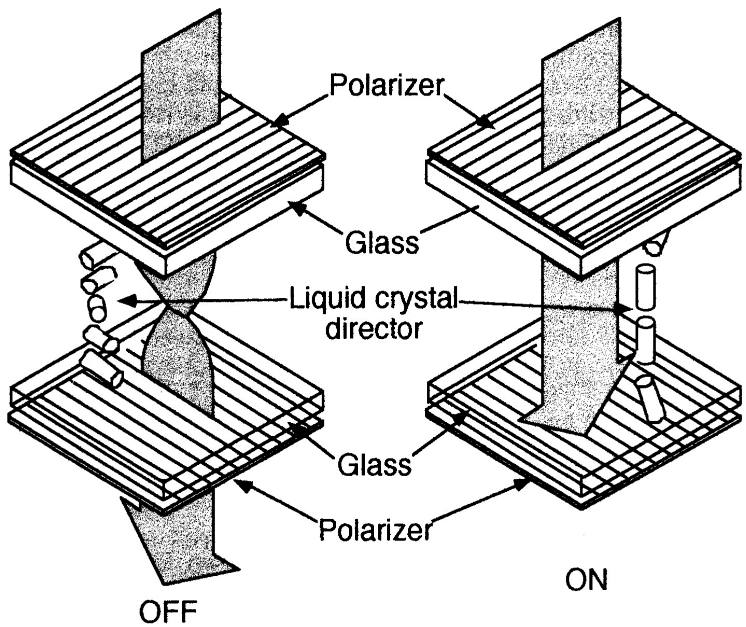
Summary of Today's Lecture

- Properties of LCs are in general anisotropic because of of their rod-like structure
- Static behavior of LCs is determined by
  - Balance between the electrostatic and elastic torques
  - Boundary conditions
- The application of an electric field leads to the re-orientation of director and hence change in optical properties
- **TN-LC cell behaves as a polarization rotator**



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