# Lecture 11

## 6.976 Flat Panel Display Devices

# Physics of Liquid Crystals I

## Outline

- **Polarization Devices**
- **Jones Matrix Method**
- **Liquid Crystals**

# References


- Jin Au Kong, Electromagnetic Wave Theory, EMW Publishing, Cambridge, MA, USA
- B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, John Wiley & Sons, New York
- E. Hecht, Optics, Addison-Wesley Publishing
- Peter J. Collings and Michael Hird, Introduction to Liquid Crystals-Chemistry and Physics, Taylor and Francis, 1997
- D. J. Channin and A. Sussman, Liquid Crystal Displays, LCD, Chapter 4 in Display Devices, Ed. Jacques I. Pankove, Spriger-Verlag, 1980.
- P. Yeh and C. Gu, Optics of Liquid Crystal Displays, John Wiley & Sons, New York, 1999.

# Jones Vectors

$$\mathbf{J} = \begin{bmatrix} \mathbf{A_x e^{j\delta_x}} \\ \mathbf{A_y e^{j\delta_y}} \end{bmatrix}$$

From this we can determine intensity

$$\mathbf{I} = \left|\mathbf{A_x}\right|^2 + \left|\mathbf{A_y}\right|^2$$

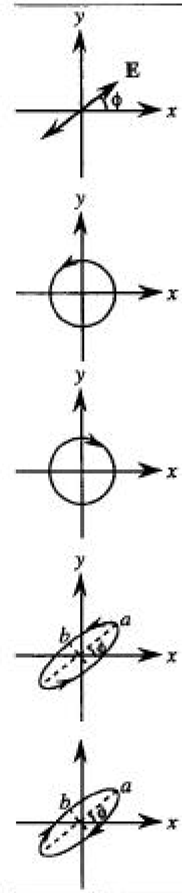


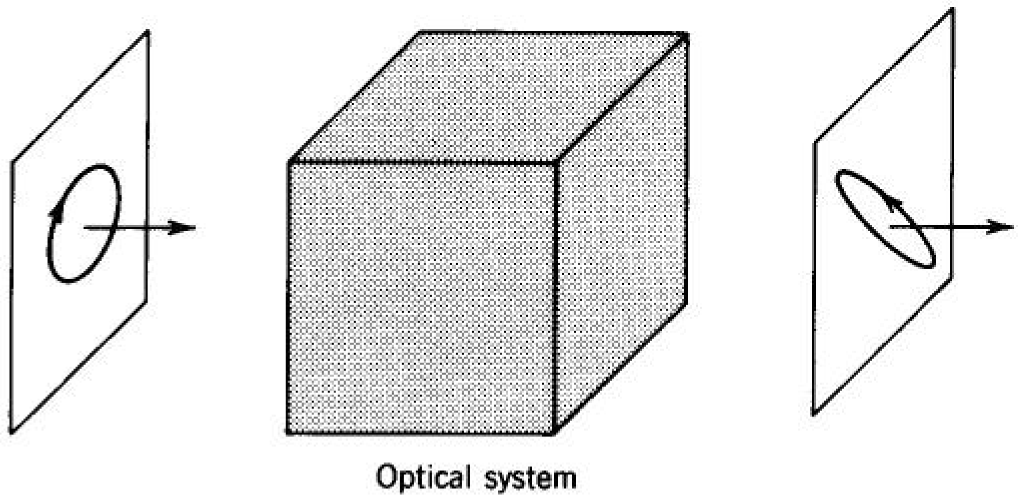
**Table 4.1. Jones Vectors**

| Polarization State | Jones Vector |
|---|---|
| (linear, angle $\phi$) | $\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$ |
| (circular) | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$ |
| (circular) | $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ |
| (elliptical) | $\begin{pmatrix} a\cos\phi + ib\sin\phi \\ a\sin\phi - ib\cos\phi \end{pmatrix}$ |
| (elliptical) | $\begin{pmatrix} a\cos\phi - ib\sin\phi \\ a\sin\phi + ib\cos\phi \end{pmatrix}$ |

*Note:* $a$ and $b$ are the principal axes of the ellipse.

**Yeh & Gu**

# Jones Matrix Formulation




Saleh & Teich

**Figure 6.1-4** An optical system that alters the polarization of a plane wave.

$$\begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix}$$

$$\mathbf{J}_2 = \mathbf{T}\mathbf{J}_1$$

# Linear Polarizer

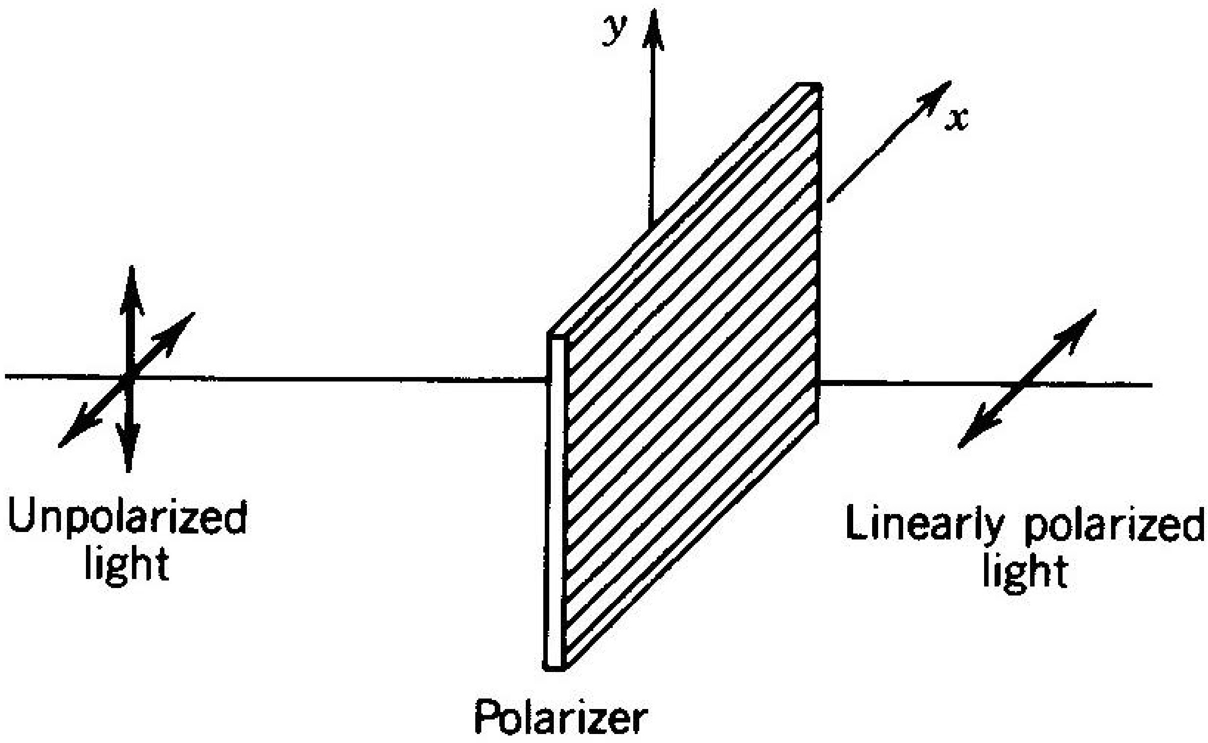



**Figure 6.1-5** The linear polarizer.

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

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# Wave Retarder

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & \mathbf{e}^{-j\Gamma} \end{bmatrix}$$

$$\Gamma = \frac{2\pi}{\lambda}\left(\mathbf{n}_s - \mathbf{n}_f\right)\mathbf{d}$$

$$\mathbf{T} = \mathbf{e}^{-j\phi} \begin{bmatrix} \mathbf{e}^{-j\Gamma/2} & 0 \\ 0 & \mathbf{e}^{+j\Gamma/2} \end{bmatrix}$$

$\phi$ = absolute phase change

$\Gamma$ = relative phase change

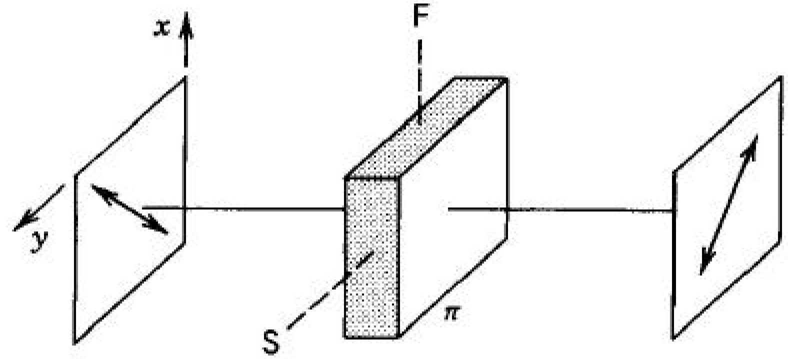
# Half-Wave Retarder Plate

$$\mathbf{T} = \begin{bmatrix} -j & 0 \\ 0 & j \end{bmatrix}$$

$$\Gamma = \frac{2\pi}{\lambda}\left(\mathbf{n}_s - \mathbf{n}_f\right)\mathbf{d} = \pi$$

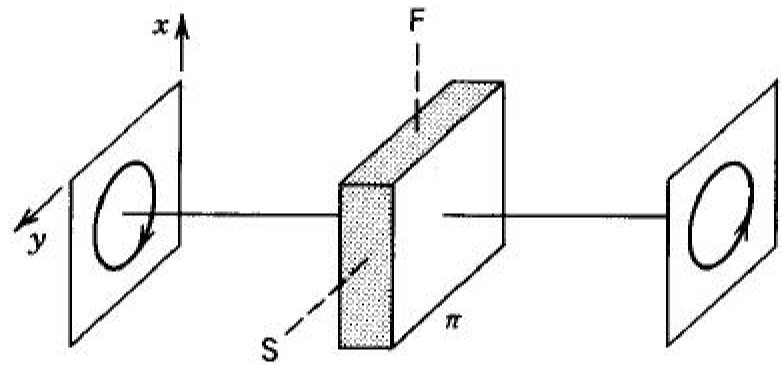
$$\begin{bmatrix} -j & 0 \\ 0 & j \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -j \\ j \end{bmatrix} = -j\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Polarization rotated by 90º




$$\begin{bmatrix} -j & 0 \\ 0 & j \end{bmatrix}\begin{bmatrix} 1 \\ j \end{bmatrix} = \begin{bmatrix} -j \\ -1 \end{bmatrix} = -j\begin{bmatrix} 1 \\ -j \end{bmatrix}$$

R-circularly polarized ⇒ L-circularly polarized

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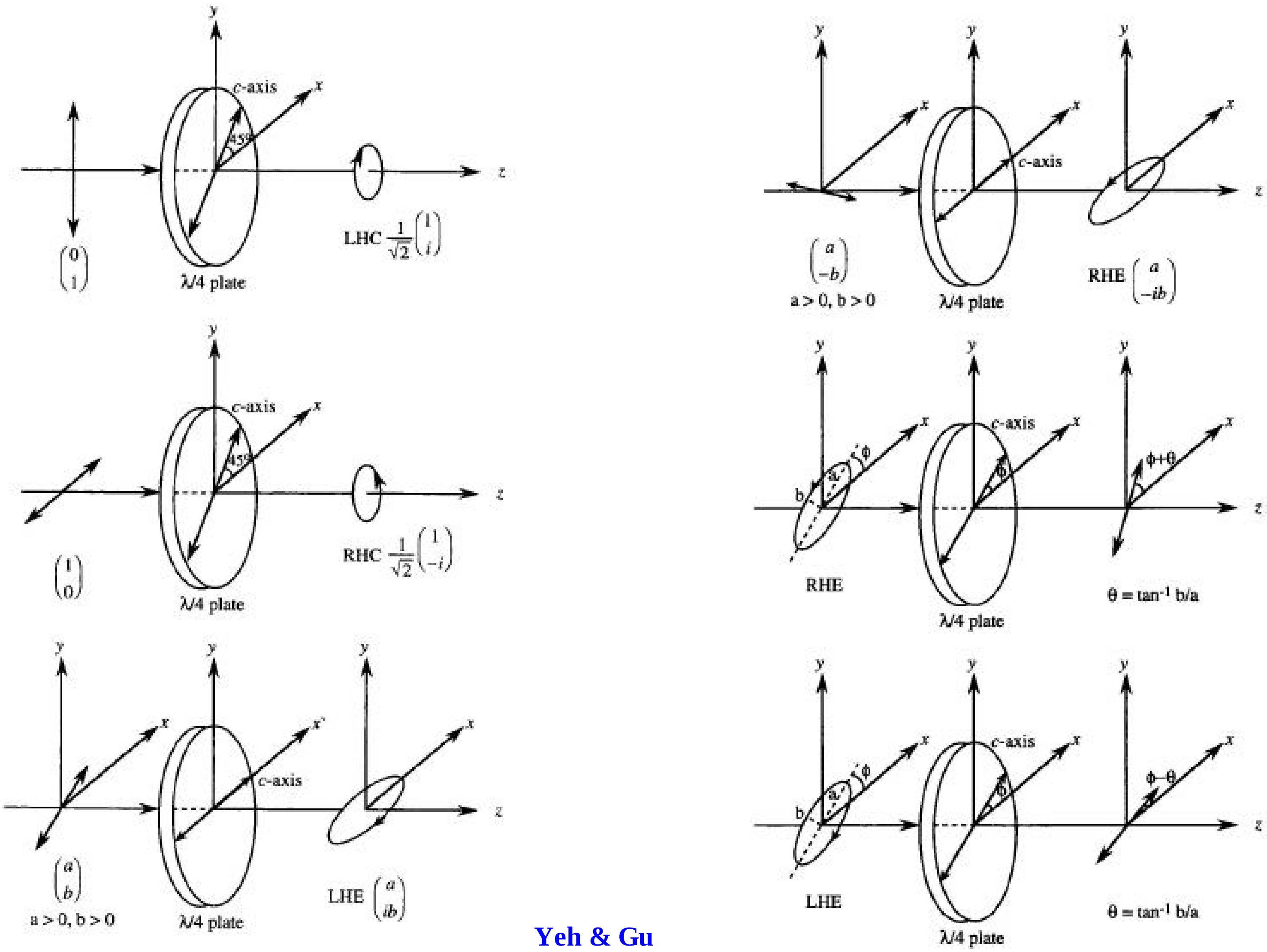
# Effect of Quarter Wave Plate

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# Polarization Rotators

$$T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Takes linearly polarized wave

$$\begin{bmatrix} \cos\theta_1 \\ \sin\theta_1 \end{bmatrix}$$

converts to

$$\begin{bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{bmatrix}$$

**where** $\theta_2 = \theta_1 + \theta$

# Coordinate Transformation

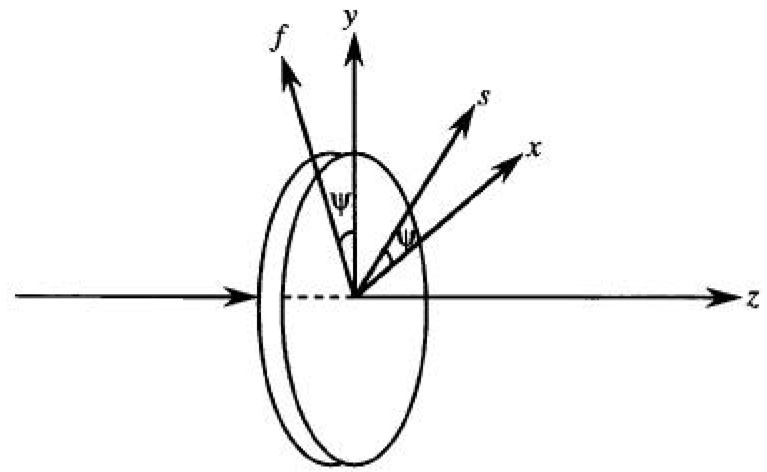


**Figure 4.1.** A retardation plate with azimuth angle ψ.

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$$\begin{bmatrix} J_s \\ J_f \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} J_x \\ J_y \end{bmatrix}$$

$$R(\psi) = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}$$

If ψ=45°

$$R(45°) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$T_{xy} = R(-\psi)\, T_{sf}\, R(\psi)$$

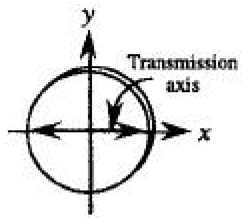
For half - wave plate

$$T_{xy} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -j & 0 \\ 0 & j \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

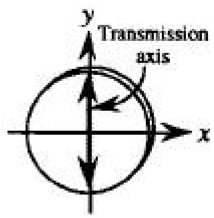
# Jones Matrices

## (Polarizers)

*Polarizers*




$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$




$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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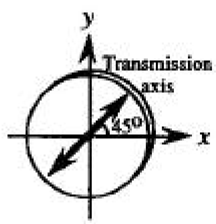
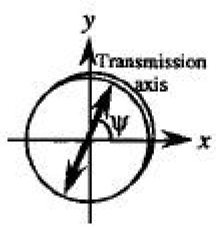



$$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$




$$R(-\psi\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}R(\psi)$$

$$= \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix}\begin{pmatrix} \cos\psi & \sin\psi \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos^2\psi & \cos\psi\sin\psi \\ \sin\psi\cos\psi & \sin^2\psi \end{pmatrix}$$

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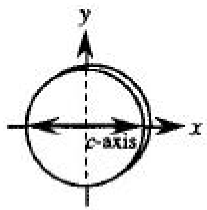
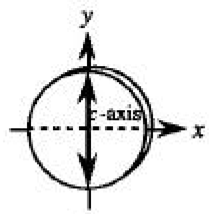
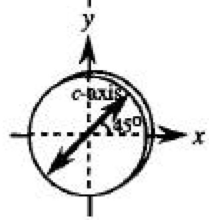
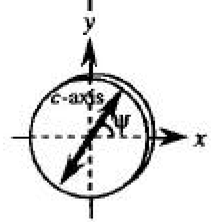
* Here we assume that the $c$ axis is perpendicular to the $z$ axis (i.e., $\theta = 90°$). In general, E (3.3-4) must be used for $n_e$ if $\theta \neq 90°$.

# Jones Matrices

## (Wave Plates)

**Table 4.2. Jones Matrices**

| Optical Element | Jones Matrices |
|---|---|
| | *Wave Plates* |
|  | $\Gamma \equiv \frac{2\pi}{\lambda}(n_e - n_o)d$ <br> $\begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix}$ |
|  | $\begin{pmatrix} e^{i\Gamma/2} & 0 \\ 0 & e^{-i\Gamma/2} \end{pmatrix}$ |
|  | $\begin{pmatrix} \cos\frac{\Gamma}{2} & -i\sin\frac{\Gamma}{2} \\ -i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{pmatrix}$ |
|  | $R(-\psi)\begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix}R(\psi)$ <br> $= \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix}\begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix}\begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix}$ |

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