Lecture 10
6.976 Flat Panel Display Devices

Light Valves

Outline

- **Monochromatic Plane Waves**
- **Electromagnetic Propagation in Anisotropic Media**
- **Spatial Light Modulators**

Electromagnetic Propagation in Anisotropic Media

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

Constituitve Equations

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

$$\mathbf{B} = \mu \mathbf{H} = \mu_o \mathbf{H} + \mathbf{M}$$

E=Electric field vector
H=Magnetic field vector
D=Electric displacement
B=Magnetic induction
ρ=Electric charge density
J= Current density
P=electric polarization
M=Magnetic polarization
ε=permitivity tensor
ε_o=permitivity of vacuum
μ=permeability tensor
μ_o=permeability of vacuum

$$\mathbf{D}_i = \varepsilon_{ij} \mathbf{E}_j$$

$$\varepsilon = \varepsilon_o \begin{pmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{pmatrix} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$$

Plane Wave in Homogeneous Media

Electric Field Vector

$$\bar{\mathbf{E}}\,\mathbf{exp}\left[\mathbf{i}\left(\mathbf{wt} - \bar{\mathbf{k}} \cdot \bar{\mathbf{r}}\right)\right]$$

Magnetic Field Vector

$$\bar{\mathbf{H}}\,\mathbf{exp}\left[\mathbf{i}\left(\mathbf{wt} - \bar{\mathbf{k}} \cdot \bar{\mathbf{r}}\right)\right]$$

$$\bar{\mathbf{k}} = \frac{\mathbf{w}}{\mathbf{c}}\mathbf{n}\bar{\mathbf{s}} \qquad \bar{\mathrm{s}} = \text{unit vector in propagation direction}$$

Plugging these in Maxwell's equation reduces to

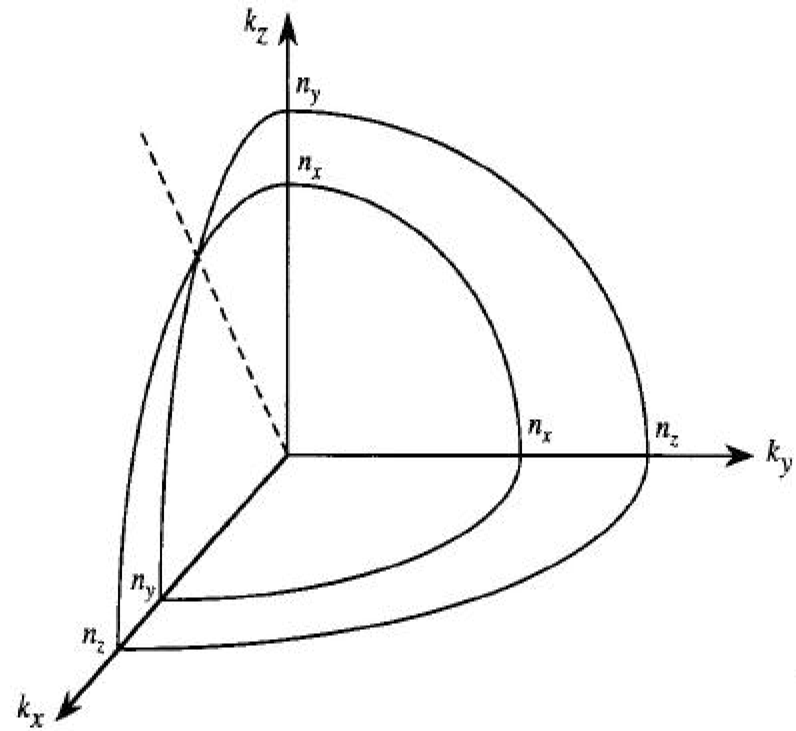
$$\bar{\mathbf{k}}\ ´\left(\bar{\mathbf{k}}\ ´\ \bar{\mathbf{E}}\right) + \mathbf{w}^2\mathbf{me}\bar{\mathbf{E}} = 0$$

This leads to a relation between ω and **k**

$$\mathbf{det}\begin{vmatrix} \mathbf{w}^2\mathbf{me}_x - \mathbf{k}_y^2 - \mathbf{k}_z^2 & \mathbf{k}_x\mathbf{k}_y & \mathbf{k}_x\mathbf{k}_z \\ \mathbf{k}_y\mathbf{k}_x & \mathbf{w}^2\mathbf{me}_y - \mathbf{k}_x^2 - \mathbf{k}_z^2 & \mathbf{k}_y\mathbf{k}_z \\ \mathbf{k}_z\mathbf{k}_x & \mathbf{k}_z\mathbf{k}_y & \mathbf{w}^2\mathbf{me}_z - \mathbf{k}_x^2 - \mathbf{k}_y^2 \end{vmatrix} = 0$$

Normal Surface

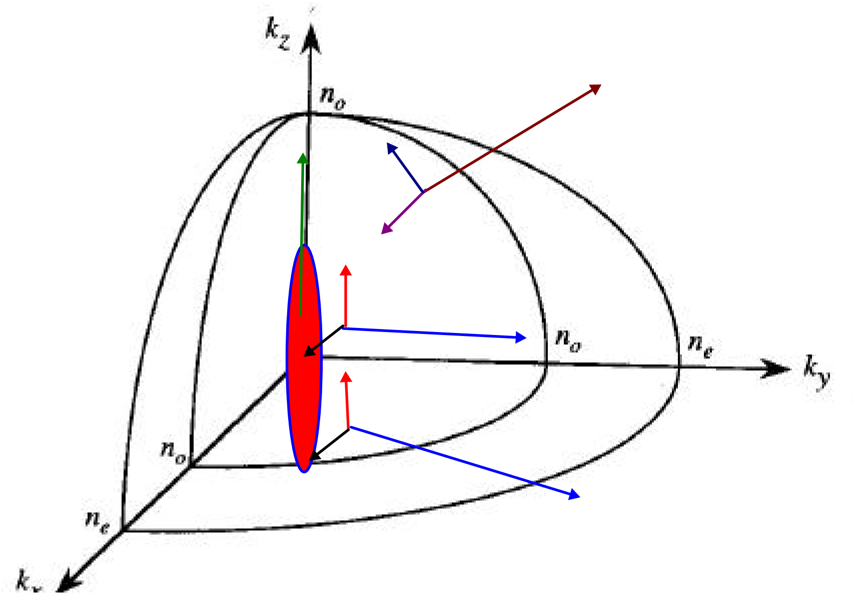
- Solution for $n_x(\varepsilon_x)$ at a given frequency represents a 3D surface in **k** space known as normal surface
  - consists of two shells having four points in common
  - optic axis = two lines that go through the origin and these four points
- Given a direction of propagation, there are two **k** values that are intersections of propagation direction and normal surface
  - **k** values ⇒ different phase velocities (ω/k) of waves propagating along the direction
- Along an arbitrary direction of propagation, **s**, there can exist two independent plane waves linearly polarized propagating with phase velocities $(\pm c/n_1$ and $(\pm c/n_2)$



Yeh & Gu

Classification of Media

- Normal surface is determined by the principal indices of refraction, n_x, n_y, n_z
- $n_x \neq n_y \neq n_z \Rightarrow$ biaxial material
  - Two optical axes
- $n_o^2 = \varepsilon_x/\varepsilon_o = \varepsilon_y/\varepsilon_o$ and $n_e^2 = \varepsilon_z/\varepsilon_o$
  - $\Rightarrow$ uniaxial material (z-axis)
  - Normal surface consists of a sphere and ellipsoid of revolution
  - n_o is ordinary index and n_e extraordinary index
  - $n_e - n_o$ either +ve or -ve
- $n_x = n_y = n_z \Rightarrow$ isotropic material
  - Normal surface degenerate in a single sphere



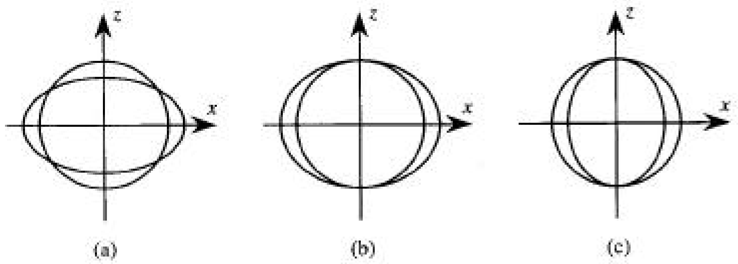


Figure 3.2. Intersection of the normal surface with the *xz* plane for (*a*) biaxial media, (*b*) positive uniaxial media, and (*c*) negative uniaxial media.

Other Method

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Table 3.2a. Refractive Indices of Some Typical Solid Crystals

Isotropic	Fluorite, CaF_2		1.392	
	Sodium chloride, NaCl		1.544	
	Diamond, C		2.417	
	CdTe		2.69	
	GaAs		3.40	
	Ge		3.40	
	InP		3.61	
	GaP		3.73	
Uniaxial		n_o		n_e
Positive	Ice, H_2O	1.309		1.310
	MgF_2	1.378		1.390
	Quartz, SiO_2	1.544		1.553
	Beryllium oxide, BeO	1.717		1.732
	Zircon, $ZrSiO_4$	1.923		1.968
	SnO_2	2.01		2.10
	ZnS	2.354		2.358
	CdS	2.483		2.511
	Rutile, TiO_2	2.616		2.903
Negative	KDP, KH_2PO_4	1.507		1.467
	ADP, $(NH_4)H_2PO_4$	1.522		1.478
	Beryl, $Be_3Al_2(SiO_3)_6$	1.598		1.590
	Sodium nitrate, $NaNO_3$	1.587		1.366
	Calcite, $CaCO_3$	1.658		1.486
	Tourmaline	1.638		1.618
	Sapphire, Al_2O_3	1.768		1.760
	Lithium niobate, $LiNbO_3$	2.300		2.208
	Barium titanate, $BaTiO_3$	2.416		2.364
	Proustite, Ag_3AsS_3	3.019		2.739
Biaxial		n_x	n_y	n_z
	Gypsum, $CaSO_4 \cdot 2H_2O$	1.520	1.523	1.530
	Feldspar	1.522	1.526	1.530
	Mica	1.552	1.582	1.588
	Topaz, $Al_2(SiO_4)(OH,F)_2$	1.619	1.620	1.627
	Sodium nitrite, $NaNO_2$	1.344	1.411	1.651
	$YAlO_3$	1.923	1.938	1.947
	SbSI	2.7	3.2	3.8

Note: The refractive indices of most materials depend on the wavelength (dispersion). The listed numbers are typical values.

Table 3.2b. Refractive Indices of Some Typical Nematic Liquid Crystals [1]

	T(°C)	Wavelength (nm)	n_e	n_o
MBBA (Schiff base)	25	467.8	1.837	1.575
		480	1.825	1.57
		508.6	1.802	1.563
		589	1.764	1.549
		643.8	1.749	1.544
RO-TN-601	25	467.8	1.718	1.515
		480	1.7116	1.5131
		508.6	1.7041	1.5098
		546.1	1.6937	1.506
Phase 4 Licristal (EM-Merck), azoxy	25	546.1	1.856	1.5606
		589.3	1.8291	1.553
K15 (5CB) (BDH, Ltd.), Cyanoalkylbiphenyl	25	436		
		509	1.7411	1.5443
		577	1.7201	1.5353
		644	1.7072	1.5292
	30	436	1.7648	1.5624
		509	1.725	1.5481
		577	1.7044	1.539
		644	1.6926	1.5323
K21 (7CB) (BDH, Ltd.), cyanoalkylbiphenyl	37	436	1.736	1.5443
		509	1.6998	1.5329
		577	1.6815	1.5248
		644	1.6702	1.5186
	41	436	1.714	1.5517
		509	1.6805	1.5389
		577	1.6632	1.5305
		644	1.6526	1.5236
M15(5OCB) (BDH, Ltd.), cyanoalkoxybiphenyl	50	589	1.7187	1.5259
M21(7OCB) (BDH, Ltd.), cyanoalkoxybiphenyl	60	589	1.6846	1.5139
M24(8OCB) (BDH, Ltd.), cyanoalkoxybiphenyl	70	589	1.6639	1.5078

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Light Propagation in Uniaxial Media

$$\varepsilon_x = \varepsilon_y = \varepsilon_o n_o^2; \quad \varepsilon_z = \varepsilon_o n_e^2$$

Normal surface

$$\left(\frac{k_x^2 + k_y^2}{n_e^2} + \frac{k_z^2}{n_o^2} - \frac{\omega^2}{c^2}\right)\left(\frac{k^2}{n_o^2} - \frac{\omega^2}{c^2}\right) = 0$$

- Sphere gives the relationship between ω and **k** for the ordinary (**O**) wave
- Ellipsoid of revolution gives the relationship between ω and **k** for the extraordinary (**E**) wave
- The two surfaces touch at two points on z-axis

Eigen-refractive indices are

O - wave $\quad n = n_o$

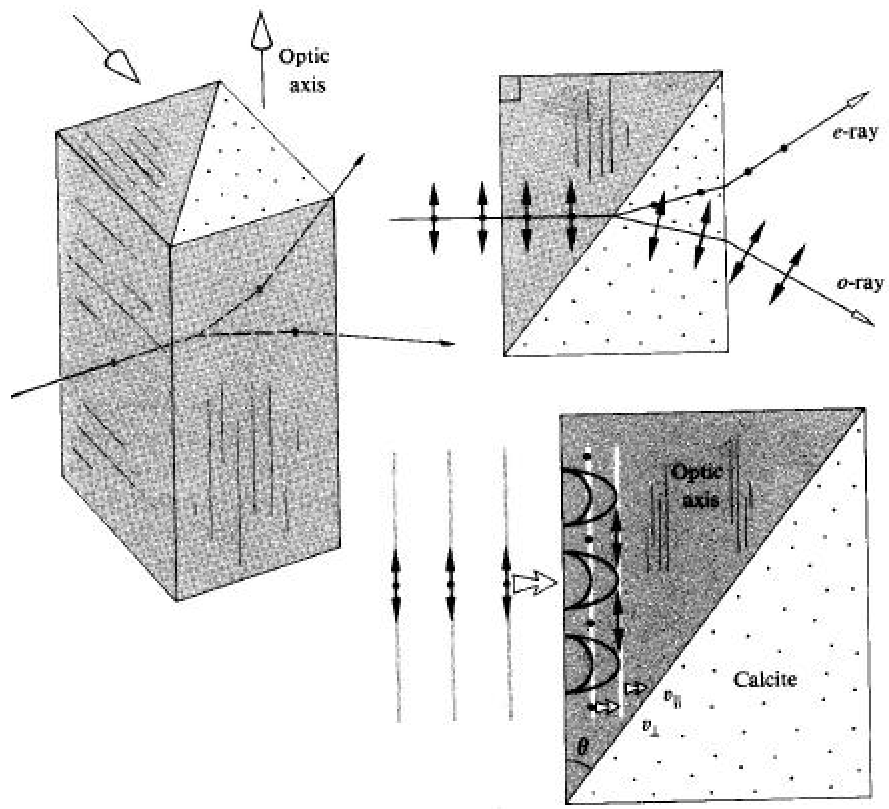
E - wave $\quad \dfrac{1}{n^2} = \dfrac{\cos^2\theta}{n_o^2} + \dfrac{\sin^2\theta}{n_e^2}$

θ is the angle between propagation direction and optic axis

Phase Retardation

$$\mathbf{D} = \mathbf{C_o D_o} \exp\left[-i\bar{\mathbf{k}}_o \cdot \bar{\mathbf{r}}\right] + \mathbf{C_e D_e} \exp\left[-i\bar{\mathbf{k}}_e \cdot \bar{\mathbf{r}}\right]$$

- Inside a uniaxial medium, a phase retardation develops between O-wave and E-wave
  - Due to diff. in phase velocity
- Phase retardation leads to a new polarization state
  - Þ **Birefringent** plates can be used to alter polarization state of light



Hecht

Phase Retardation

Wave propagating in uniaxial medium perpendicular to z(c-) axis

$$\mathbf{E} = \mathbf{C_o \bar{x} \exp\left[-i\bar{k}_o \cdot \bar{r}\right] + C_e \bar{z} \exp\left[-i\bar{k}_e \cdot \bar{r}\right]}$$

Assuming $C_o = C_e = 1$

$$\text{At } y = 0$$

$$\mathbf{E = x + z}$$

Linearly polarized

$$\text{At } y = d_{\mathbf{l}/4} = \frac{\mathbf{p}/2}{\mathbf{w}/\mathbf{c}(\mathbf{n_e} - \mathbf{n_o})}$$

$$\mathbf{E} = \mathbf{\exp(ik_e} d_{\mathbf{l}/4}\mathbf{)[ix + z]}$$

Circularly polarized

$$\text{At } y = d_{\mathbf{l}/2} = 2d_{\mathbf{l}/4}$$

$$\mathbf{E} = \mathbf{\exp(ik_e} d_{\mathbf{l}/2}\mathbf{)[-x + z]}$$

Linearly polarized but ± to original

- Birefringent plate with thickness $d_{\lambda/4}$ is known as **quarter-wave plate** and it is used to convert a linear polarization to circular polarization
- Birefringent plate with $d_{\lambda/4}$ is known as **half-wave plate** and it is used to change direction of linear polarization

Polarization by Selective Reflection

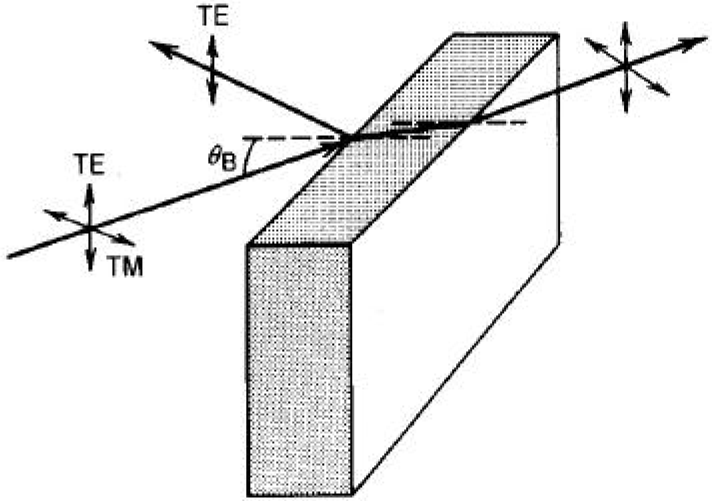


Figure 6.6-2 Brewster-angle polarizer.

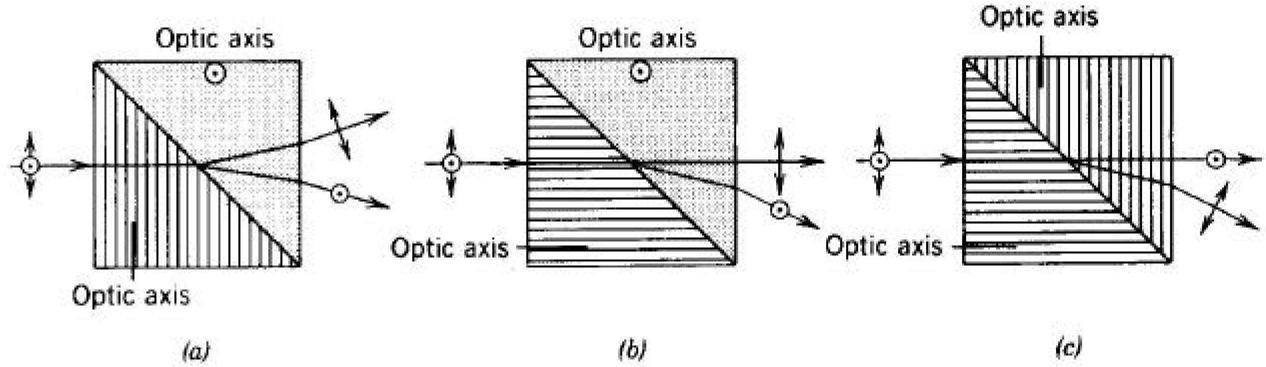
Saleh & Teich

- Reflection of light from the boundary between two dielectric materials is polarization dependent
- At the Brewsters angle of incidence
  - Light of TM polarization is totally refracted
  - Only TE component is reflected

$$\mathbf{n_i \sin\theta_B = n_t \sin\theta_t} \qquad \mathbf{\theta_t = 90^\circ - \theta_B}$$

$$\mathbf{n_i \sin\theta_B = n_t \cos\theta_B} \quad \Rightarrow \quad \mathbf{\tan\theta_B = n_i / n_t}$$

Polarization by Selective Refraction



Saleh & Teich

Figure 6.6-3 Polarizing prisms: (*a*) Wollaston prism; (*b*) Rochon prism; (*c*) Sénarmont prism. The directions and polarizations of the exiting waves differ in the three cases. In this illustration, the crystals are negative uniaxial (e.g., calcite).

- In an anisotropic crystal, two polarizations of light refract at different angles
  - Spatially separation
- Devices are usually two cemented prisms of uniaxial crystals in different orientations

Wave Retarders
(Wave Plates)

- Retarders change the polarization of an incident wave
- One of the two constituent polarization state is caused to lag behind the other
  - Fast wave advanced
  - Slow wave retarded
- Relative phase of the two components are different at exit
- Converts polarization state into another
  - Linear to circular/elliptical
  - Circular/elliptical to linear

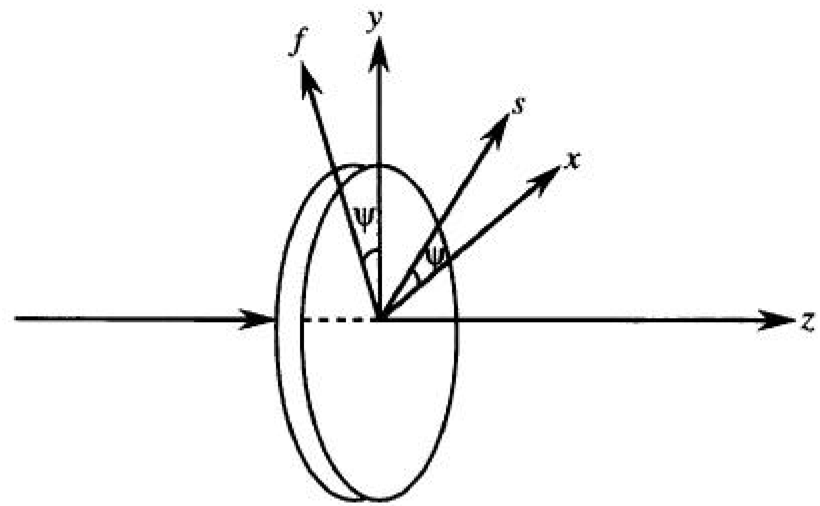


Figure 4.1. A retardation plate with azimuth angle ψ.

Yeh & Gu

$$G = \frac{2p}{l}\left(n_s - n_f\right)d$$

Wave Retarders
(Wave Plates)

- **Wave retarders** are often made of anisotropic materials
  - uniaxial
- When light wave travels along a principal axis, the normal modes are linearly polarized pointing along the other two principal axes (x, y)
  - Travel with principal refractive indices n_f, n_s
- **Intensity** modulated by relative **phase retardation**

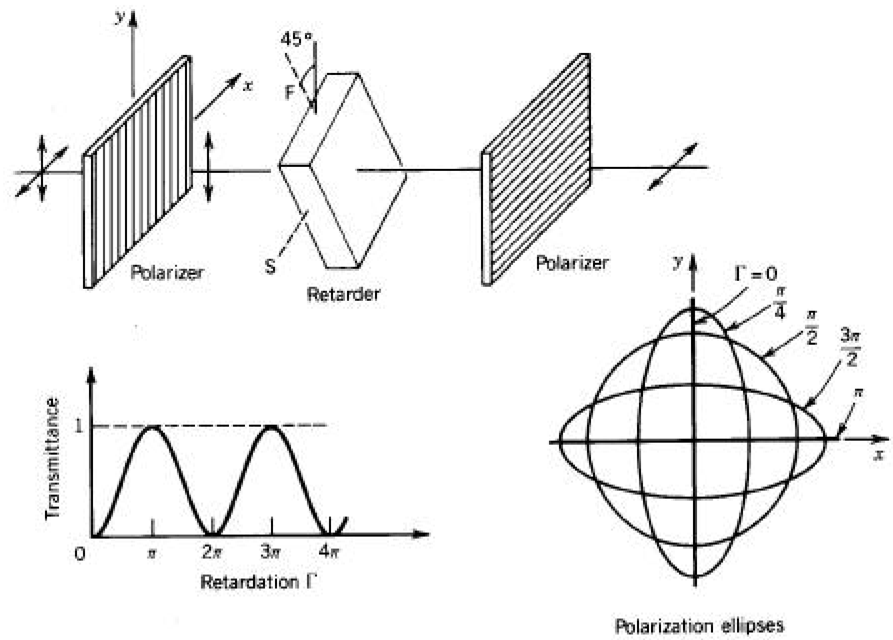


Figure 6.6-4 Controlling light intensity by use of a wave retarder with variable retardation Γ between two crossed polarizers.

Saleh & Teich

$$G = \frac{2p}{l}(n_s - n_f)d = k_o(n_s - n_f)d$$

Anisotropic Absorbtion and Polarizers

To take care of absorbtion, generalize the refractive index to complex number

$$\hat{\mathbf{n}}_o = \mathbf{n}_o - i\mathbf{k}_o$$

$$\hat{\mathbf{n}}_e = \mathbf{n}_e - i\mathbf{k}_e$$

$\mathbf{k}_o, \mathbf{k}_e$ are extinction coefficient

O-type polarizer transmits ordinary waves and attenuates extraordinary wave i.e. $\kappa_o=0$
E-type polarizer transmits extraordinary waves and attenuates ordinary wave i.e. $\kappa_e=0$

Define

T_1=transmission with polarization // to the transmission axis
T_2=transmission with polarization ± to the transmission axis

$$\text{Extinction Ratio} = \frac{T_2}{T_1}$$

$$T_o = \frac{\mathbf{T}_1 + \mathbf{T}_2}{2}$$

Transmittance of unpolarized light through polarizer

$$\mathbf{T}_p = \frac{\mathbf{T}_1^2 + \mathbf{T}_2^2}{2}$$

Transmittance of unpolarized light through pair of // polarizers

$$\mathbf{T}_x = \frac{\mathbf{T}_1\mathbf{T}_2}{2}$$

Transmittance of unpolarized light through pair of ± polarizers

Optical Activity

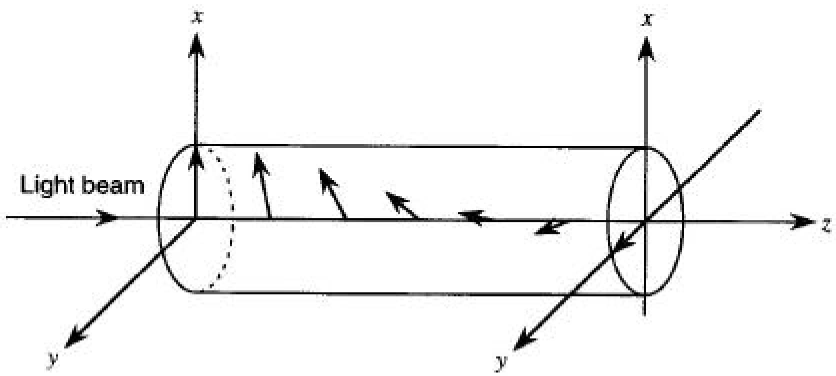


Figure 3.6. Rotation of the plane of polarization by an optically active medium.

Yeh & Gu

- Optically active materials are substances that rotate a beam of light traversing through them in the direction of the optical axis.
  - Usually given in °/mm

Could be induced by external signal such as

- Electric field (electro-optic effect)
- Optical signal (photo-refractive effect)

Spatial Light Modulators

- Spatial light modulators (or light valves) are the buiding blocks of optical information processors and display systems.
- Consider a plane monochromatic wave of the form:

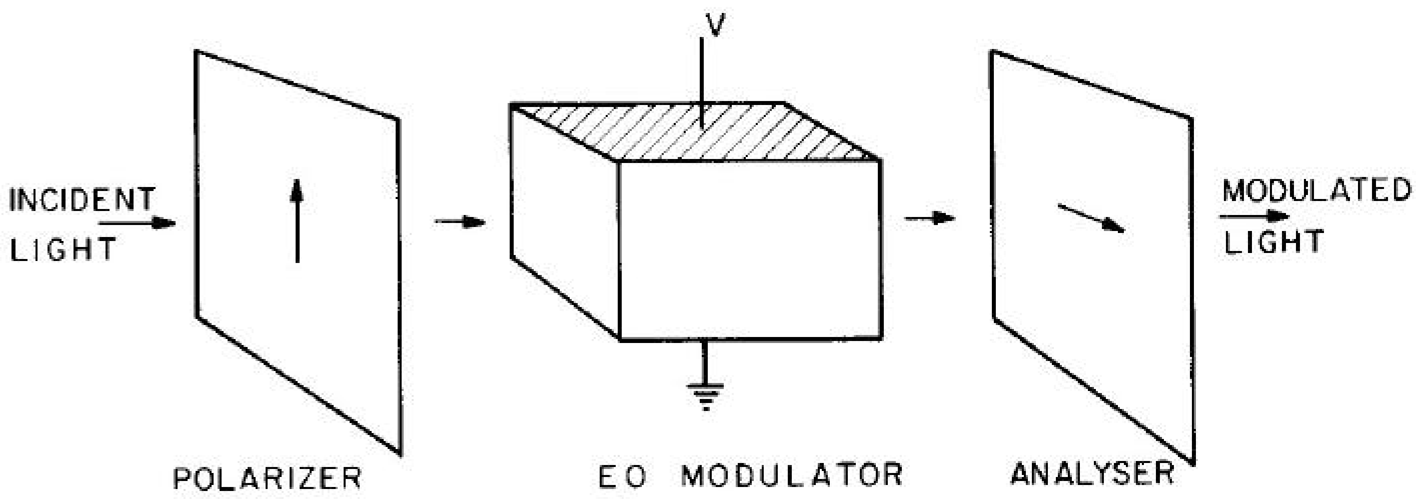
$$\overline{\mathbf{E}}(\mathbf{r},\mathbf{t}) = \hat{\mathbf{e}}\,\mathbf{Re}\left\{\mathbf{E}_o \exp\left[\mathbf{j}\left(\mathbf{wt} - \overline{\mathbf{k}} \cdot \overline{\mathbf{r}} + \mathbf{f}\right)\right]\right\}$$

- A spatial light modulator (SLM) is a device that can modify the phase, polarization and/or amplitude of a 2D light beam as a function of either:
  - A time varying electrical drive signal (electrically addressed SLM)
  - The intensity distribution of another time-varying optical signal (optically addressed SLM)

Examples of Light Modulation Schemes

- Electro-optic effect
  - Pockels effect $\Delta n \propto E$ ($LiNbO_3$)
  - Kerr effect $\Delta n \propto E^2$ (PLZT)
- Photorefractive effect
  - $\Delta n \propto$ Exposure ($LiNbO_3$,$BaTiO_3$)
- Molecular alignment by electric field
  - Torque=PXE (Liquid Crystals)
- Micromechanical
  - Electrostatic deformation (membranes, gels, oil films)
- Thermal
  - Thermoplastics, smectic A & nematic liquid crystals
- Electrophoresis
  - motion of charged particles in an electric field

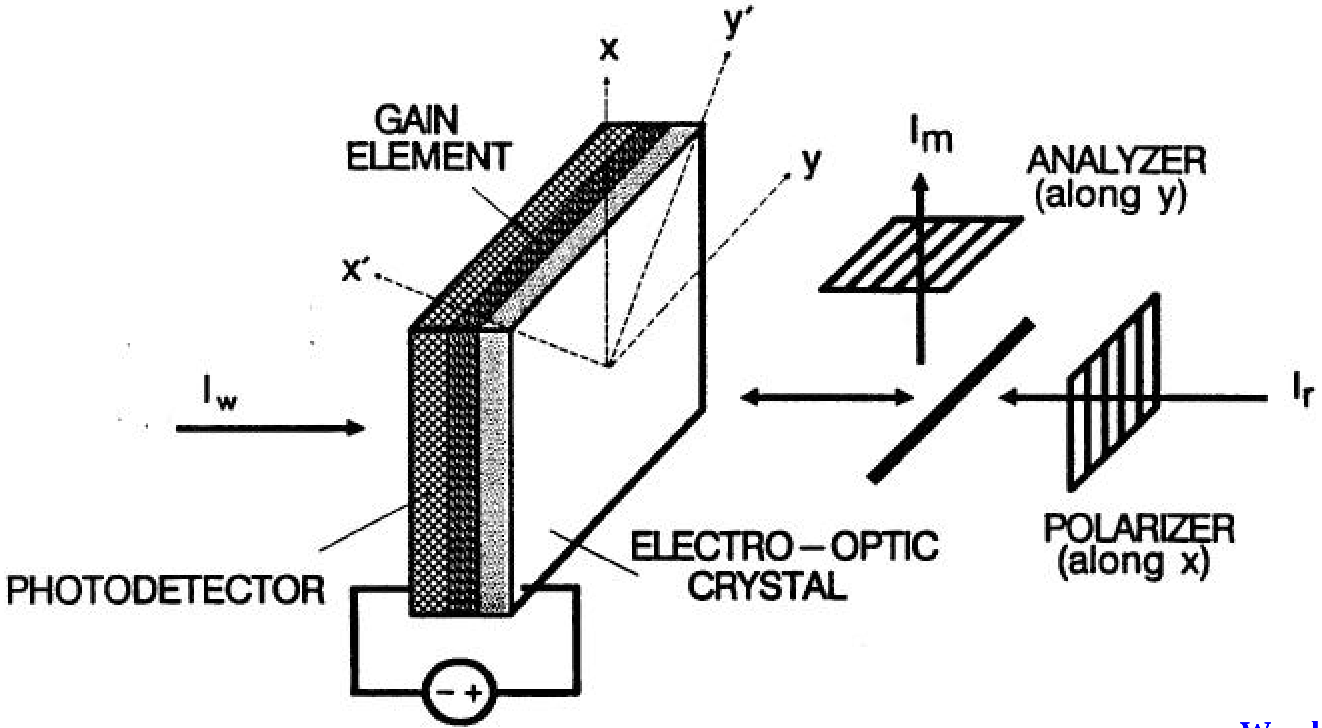
Electro-optic Modulation of Light



Warde

- Electric field induced birefringence rotates plane of polarization
- Field applied transversely to direction of propagation of light
- Analyzer transmits an amplitude proportional to cosine of polarization angle wrt analyzer

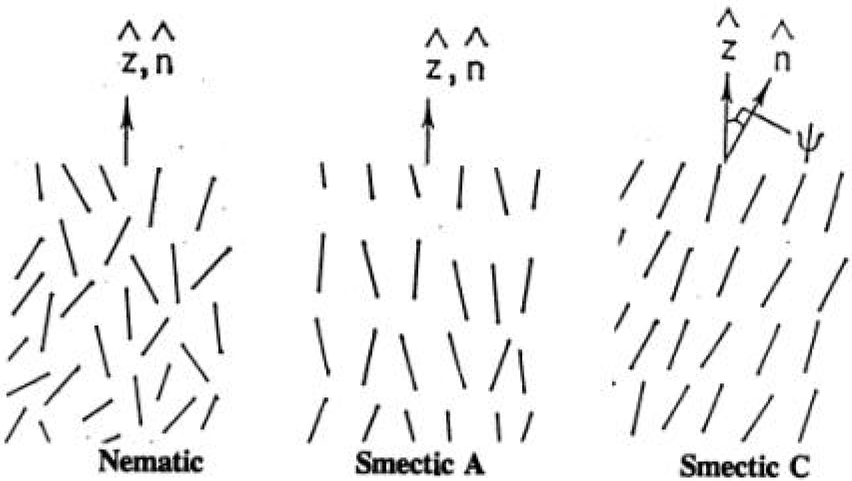
Electro-Optic SLMs



Warde

- Write light illuminates photodetector & accumulates charge
- Voltage built up across electro-optic crystal
- Electric field induced birefringence
- Phase retardation
- Reflects intensity modulated signal

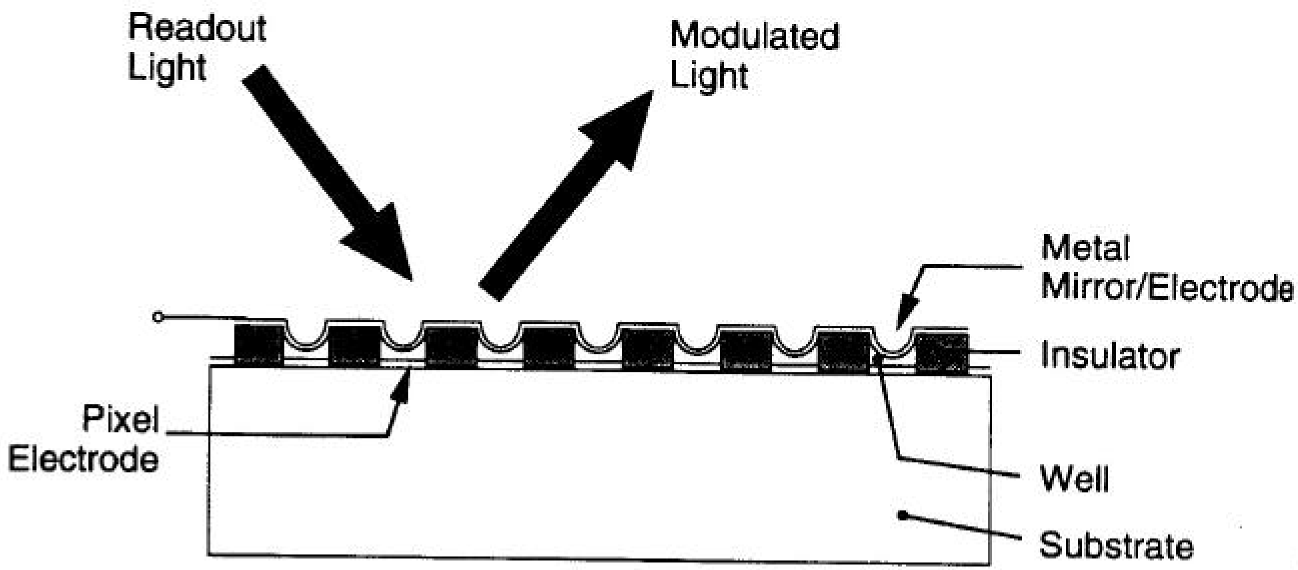
Liquid Crystal Devices



Warde

- Nematic liquid crystal
  - The re-orientation of molecules with the electric field alters the birefringence of the material.
- Electroclinic Smetic liquid crystal
  - The tilt angle of molecules is linear with applie delectric field
- Surface stabilized Smetic Ferro-electric liquid crystal
  - Molecules switch between two surface stabilized states because of the torque resulting from coupling of ferro-electric polarization to the applied E

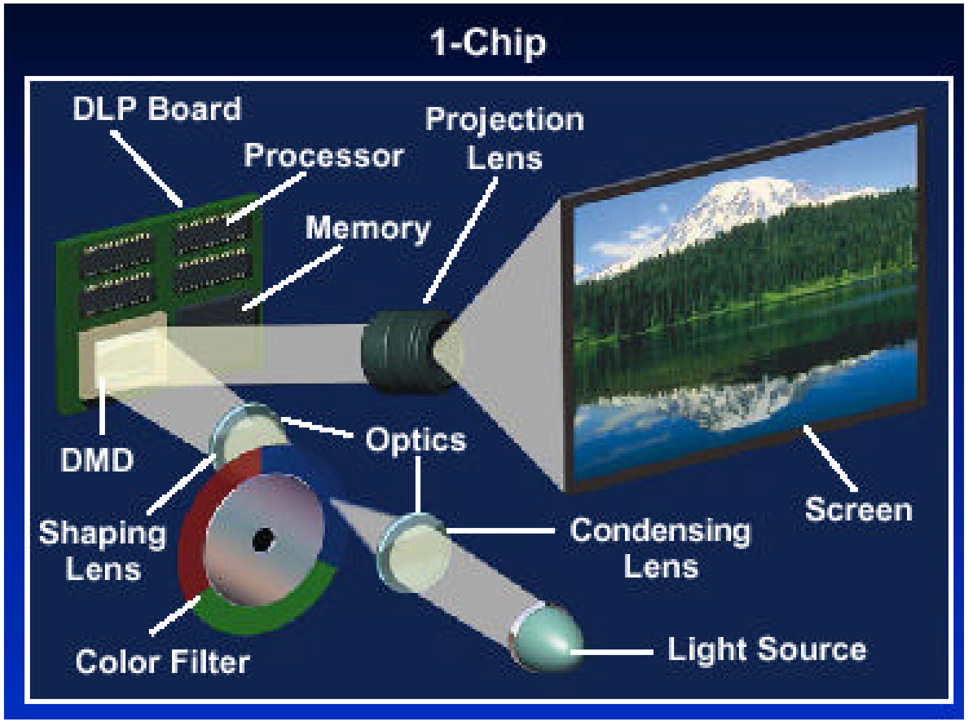
Membrane Mirror Light Modulator

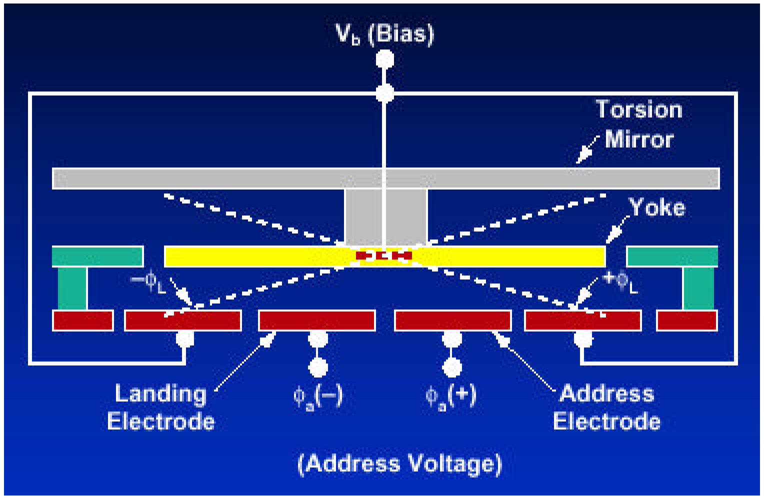


Warde

- Electrostatic forces deform mirror into wells etched in supporting surface.
- Readout light diffracts from membrane mirror

Deformable Mirror Device



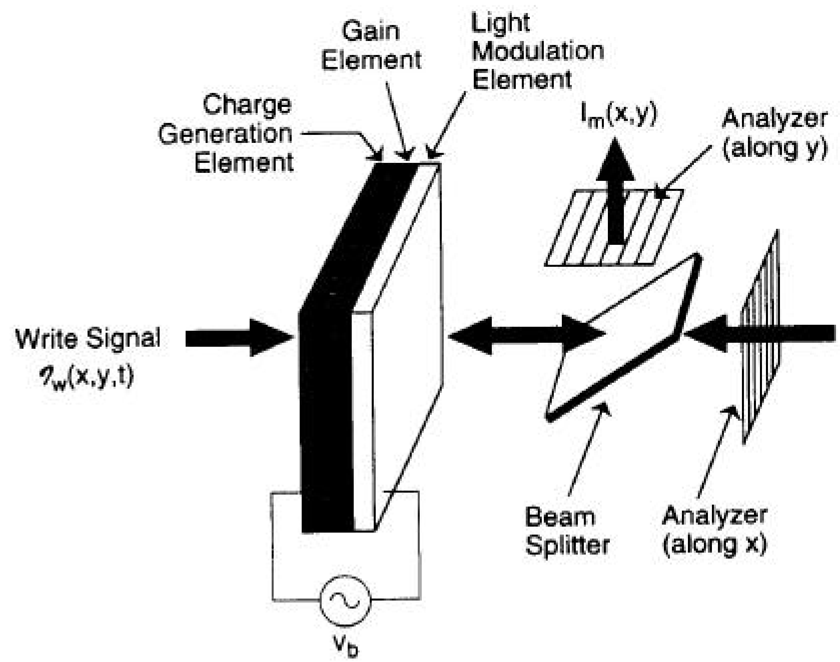


Courtesy Texas Instruments

- Array of micro-mirrors integrated with SRAM array
- Signal stored in each SRAM cell applies voltage to each mirror
- Applied voltage deflects Mirror and hence direct light

Summary of Today's Lecture

- Light valves modulate light coming from an independent light source of high intensity
- Modulation derived from phenomena causing
  - Reflection
  - Diffraction
  - Scattering
  - Polarization change
- Modulation of
  - Amplitude
  - Phase
  - Polarization
  - Intensity



Warde