

Coherence of tunneling between one-dimensional Luttinger liquids

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The problem of coherence/incoherence of electron tunneling between one-dimensional Luttinger liquids coupled by a small interchain single-particle tunneling amplitude t_{\perp} is studied in terms of Schwinger-Keldysh double-time or closed-time path formalism. Using an unconventional functional bosonization procedure, we represent the one-dimensional Luttinger Liquid in terms of a “system” of free electrons, and an “environment” of fluctuating bosonic phases. The coupling between the system and environment is introduced by the interchain single-particle tunneling of the amplitude t_{\perp} . We find that the conditions for incoherence and irrelevance (in renormalization-group sense) of t_{\perp} coincide, and thus the relevant but incoherent regime is not possible.

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I. INTRODUCTION

The problem of one-dimensional (1D) Luttinger liquids^{1–10} coupled by a small interchain single-particle tunneling amplitude t_{\perp} attracted a great deal of attention in recent years^{11–30}. It was shown³¹ that, unless intrachain interaction is extraordinarily strong, t_{\perp} is a relevant perturbation in the renormalization-group sense. Then Clarke, Strong, and Anderson^{32,33} put forward the hypothesis that, while being relevant, the single-particle interchain hopping is incoherent. Thus, the transverse electron motion is not quantum, but rather classical, so there is no coherent transverse energy band. The mathematical foundation of their theory is based on the analogy with the so-called two-level model^{35,36}. The latter model consists of a quantum particle that has two degenerate states connected by a matrix element Δ . The particle interacts with an “environment”, a thermal bath of oscillators with distributed frequencies. If initially the particle is in one of the two states, then, in the absence of interaction with the environment, the probability to find the particle in the other state oscillates in time, according to elementary quantum mechanics. When interaction with the environment is sufficiently strong, this probability does not oscillate, but changes in time exponentially, saturating at the probability 1/2. This behavior is called incoherent, because it corresponds to a classical, rather than quantum-mechanical, coupling of the two states. At a stronger interaction with the environment, the particle gets indefinitely localized in the initial state, and does not make a transition to the other state at all. In this regime, Δ is irrelevant^{37,38}. In the two-level model, there is a finite range of the particle-environment interaction strength where Δ is relevant but incoherent. Clarke, Strong, and Anderson^{32,33} conjectured that such a situation may take place also when t_{\perp} (interchain single-particle tunneling amplitude) couples two (or more) 1D Luttinger-liquid chains.

While this is certainly a very interesting suggestion, it raises a number of questions. First, the two-level system

is a zero-dimensional system, where the particle evolves only in time. On the other hand, in a 1D Luttinger liquid, electrons also move in space along the chains. The effect of different dimensionality has to be taken into account and may change the results. Second, in the standard approach to decoherence, it is necessary to identify the “system”, the degree of freedom whose evolution we follow, and the “environment”, the other degrees of freedom that we do not follow. Tracing over the environmental degrees of freedom causes decoherence. The distinction between the “system” and “environment” is clear for the two-level model, but is not obvious in the case of coupled Luttinger liquids. Third, the issue of coherence-decoherence cannot be decided by studying only the second-order term in t_{\perp} . It is necessary to examine the general structure of expansion in t_{\perp} to high orders.

In this paper, we offer a systematic study of these issues. To address the second question, we utilize a somewhat unconventional form of 1D bosonization. In this approach, the noninteracting electrons represent the “system”, the fluctuating bosonic phase fields represent the “environment”, and the coupling between the system and environment is introduced by the interchain single particle tunneling of amplitude t_{\perp} . To address the question of interchain hopping coherence, we use perturbative expansion in powers of t_{\perp} within the double-time functional integral, also known as the Schwinger-Keldysh closed-time-path (CTP) formalism³⁹. Our criterion of coherence-incoherence is formulated in exactly the same way as for the two-level model^{35,36}, except there is no spatial coordinate x in latter case (a discussion for the two-level model is given in Appendix C). We find the result that the conditions for incoherence and irrelevance of t_{\perp} of coupled Luttinger liquids coincide, and thus the *relevant but incoherent regime is not possible*.

The outline of this paper is as follows. In Sec. II, we present our functional bosonization approach for the 1D Luttinger liquid. The anomalous power-law correlation functions of the 1D Luttinger liquid can be reproduced exactly in this approach. In Sec. III, we briefly discuss the basic idea of the Schwinger-Keldysh CTP formalism.

In Sec. IV we carry out the perturbative expansion in powers of t_{\perp} within the CTP and discuss the condition of coherent-incoherent transition for the coupled Luttinger liquids. A conclusion is given in Sec. V. The explicit evaluation of the chiral Jacobian (due to the chiral transformation made in our functional bosonization) and detailed calculations of 1D Luttinger correlation functions in our approach are given in Appendix A and B respectively. We discuss how our criterion of coherence-incoherence applies to the two-level model in Appendix C.

II. FUNCTIONAL BOSONIZATION OF 1D LUTTINGER LIQUID

We now present our functional bosonization approach for the 1D Luttinger liquid. Our functional approach consists of three major steps: the Hubbard-Stratonovich transformation, the chiral transformation, and the inclusion of chiral Jacobian (anomaly). Bosonization based on functional integration, first discussed by Fogedby⁴⁰ and later by Lee and Chen⁴¹ for the 1D Luttinger liquid, has been recently generalized and developed by Kopietz, Schönhammer, Hermisson and Castilla⁴², to bosonize interacting fermions in arbitrary dimensions. The difference between our functional bosonization and previous approaches is that instead of transforming interacting electrons to an entirely bosonic representation, after a Hubbard-Stratonovich transformation⁴³ we make a chiral transformation on the electron field, which allows us to decouple the Lagrangian (action) into a system of free fermions and an environment of the fluctuating bosonic phases. The 1D Luttinger-liquid correlation functions with anomalous power-law behavior can be obtained straightforwardly and reproduced exactly in our approach.

For simplicity, we restrict ourselves to the zero temperature spin- $\frac{1}{2}$ case but with only spin-independent forward scattering. In the terminology of g -ology^{5,7-10}, only g_2 and g_4 “charge sector” interactions are included. The Hamiltonian for this 1D Luttinger liquid model could be written as

$$\hat{H} = \int dx \left[\sum_{\sigma,a} \hat{\psi}_{a,\sigma}^{\dagger}(x,t) (-i\hbar av_F \frac{\partial}{\partial x}) \hat{\psi}_{a,\sigma}(x,t) + g_2 \hat{\rho}_+(x,t) \hat{\rho}_-(x,t) + \sum_{a=\pm} \frac{g_4}{2} \hat{\rho}_a(x,t) \hat{\rho}_a(x,t) \right], \quad (2.1)$$

where $\hat{\psi}_{a,\sigma}^{\dagger}(x,t)$, $\hat{\psi}(x,t)_{a,\sigma}$ are fermion creation and annihilation fields, $\sigma = \uparrow, \downarrow$ is the spin index, $a = \pm$ represents two branches of the dispersion varying linearly [$\varepsilon_a(k) = a\hbar v_F k$] about the two Fermi points $\pm k_F$, and energy is counted from the Fermi level. The density operator $\hat{\rho}_a(x,t)$ is defined as

$$\hat{\rho}_a(x,t) = \sum_{\sigma} \hat{\psi}_{a,\sigma}^{\dagger}(x,t) \hat{\psi}_{a,\sigma}(x,t). \quad (2.2)$$

In the functional integral formalism, the Hubbard-Stratonovich transformation⁴³ allows us to represent the density-density (four-fermion) interaction in terms of a functional integral of an action [the last two terms in Eq. (2.4)] over an auxiliary bosonic field ϕ_a . The partition functional for Hamiltonian (2.1) after introducing the Hubbard-Stratonovich transformation can be written as:

$$Z = \int \mathcal{D}\psi^{\dagger} \mathcal{D}\psi \mathcal{D}\phi e^{iS(\psi^{\dagger}, \psi, \phi)/\hbar}, \quad (2.3)$$

where the action, S , is given by

$$S = \int dt dx [L_0(\psi^{\dagger}, \psi) + L_1(\phi) + \sum_a \phi_a(x,t) \rho_a(x,t)]. \quad (2.4)$$

In Eq. (2.4), L_0 is the free fermion Lagrangian density:

$$L_0(\psi^{\dagger}, \psi) = \sum_{a,\sigma} \psi_{a,\sigma}^{\dagger}(x,t) i\hbar \left(\frac{\partial}{\partial t} + av_F \frac{\partial}{\partial x} \right) \psi_{a,\sigma}(x,t), \quad (2.5)$$

and L_1 can be written in a matrix form:

$$L_1(\phi) = \frac{1}{2} (\phi_+(x,t), \phi_-(x,t)) \hat{g}^{-1} \begin{pmatrix} \phi_+(x,t) \\ \phi_-(x,t) \end{pmatrix} \quad (2.6)$$

with the matrix \hat{g}^{-1} given by

$$\hat{g}^{-1} = \begin{pmatrix} g_4 & g_2 \\ g_2 & g_4 \end{pmatrix}^{-1} = \frac{1}{g_4^2 - g_2^2} \begin{pmatrix} g_4 & -g_2 \\ -g_2 & g_4 \end{pmatrix}. \quad (2.7)$$

The fermion field operator in the functional integral formalism, now becomes the Grassmann variable ψ_a . Physically, the bosonic Hubbard-Stratonovich auxiliary field variable ϕ_a describes collective fluctuations of the electron density and acts as a fluctuating chemical potential in the Luttinger liquid.

Traditional bosonization^{3,4,6,8-10,40-42} in 1D Luttinger liquid transforms the interacting fermionic field theory to an entirely bosonic field theory. In this sense, system and environment are not distinguishable. Here using an unconventional functional bosonization procedure, we represent the 1D Luttinger liquid in terms of a “system” of free electrons and an “environment” of fluctuating bosonic phases. Instead of transforming interacting electrons to an entirely bosonic representation, we make a chiral transformation on the electron field:

$$\psi_{a,\sigma}(x,t) = \tilde{\psi}_{a,\sigma}(x,t) e^{i\vartheta_a(x,t)}, \quad (2.8)$$

and decouple the fermions $\tilde{\psi}$ and the fluctuating bosonic fields ϕ . This procedure corresponds to eliminating the $\phi_a \rho_a$ term in action S (2.4), and leads to the equation for fluctuating phase ϑ_a :

$$\hbar \left(\frac{\partial}{\partial t} + av_F \frac{\partial}{\partial x} \right) \vartheta_a(x, t) = \phi_a(x, t). \quad (2.9)$$

At first sight, the transformed effective action after the chiral transformation (2.8) seems to be just the sum of the first two term in action S (2.4) with the substitution of the electron field $\psi_{a,\sigma}(x, t) \rightarrow \tilde{\psi}_{a,\sigma}(x, t)$ plus the relation (constraint) between ϕ_a and ϑ_a through Eq. (2.9). It is known, however, in path integral formulation that not only the action but also the functional measure in partition function may change under a chiral transformation⁴⁴. The change in the functional measure (Jacobian) is called chiral anomaly (non-invariance under chiral rotations). The Jacobian of this chiral transformation, defined as $\exp(iF/\hbar)$, can be calculated (calculation is given in Appendix A) and the result is:

$$F = \int dx dt \sum_a \frac{a}{2\pi} \phi_a(x, t) \frac{\partial \vartheta_a(x, t)}{\partial x}. \quad (2.10)$$

The final form of action after considering the Hubbard-Stratonovich transformation, chiral transformation, and the chiral anomaly (Jacobian) is

$$S_{1D} = S_0(\tilde{\psi}^\dagger, \tilde{\psi}) + S_2(\vartheta) \quad (2.11)$$

where $S_0 = \int dt dx L_0$ is the free electron action defined through (2.5), and the effective fluctuating phase action, $S_2 = S_1 + F$, is equal to the sum of original quadratic action term S_1 introduced by Hubbard-Stratonovich transformation and the Jacobian of fermion functional measure F due to the chiral transformation. Here $S_1 = \int dt dx L_1$ is defined through (2.6), and F , in (2.10). The fact that the auxiliary fields ϕ_a can be eliminated in terms of dynamical variables ϑ_a through Eq. (2.9) makes $S_2(\vartheta)$ quadratic in fluctuating phase ϑ . An explicit expression of $S_2(\vartheta)$ can be found in Appendix B. In the representation (2.11), the noninteracting fermions $\tilde{\psi}(x, t)$ represent the “system”, and the fluctuating phases $\vartheta(x, t)$ represent the “environment”. Physically, the fluctuations of $\vartheta(x, t)$ transmit the interaction between the original electrons.

Taking a functional integral over $\tilde{\psi}(x, t)$ and $\vartheta(x, t)$ exactly reproduces the well-known anomalous power-law correlation functions of the 1D Luttinger liquid^{7–10}. We briefly outline the procedure for calculating correlation functions here. A more detailed calculation is given in Appendix B. The Green function, two-point correlation function, following from the Hubbard-Stratonovich and chiral transformations (including Jacobian), can be written as

$$\begin{aligned} G_{a,\sigma}(z_1, z_2) &= -i \langle \psi_{a,\sigma}(z_1) \psi_{a,\sigma}^\dagger(z_2) \rangle_{S(\psi^\dagger, \psi)} \\ &= -i \langle \psi_{a,\sigma}(z_1) \psi_{a,\sigma}^\dagger(z_2) \rangle_{S(\psi^\dagger, \psi, \phi)} \\ &= -i \langle \tilde{\psi}_{a,\sigma}(z_1) \tilde{\psi}_{a,\sigma}^\dagger(z_2) \rangle_{S_0(\tilde{\psi}^\dagger, \tilde{\psi})} \\ &\quad \times \langle e^{i\vartheta_a(z_1) - i\vartheta_a(z_2)} \rangle_{S_2(\vartheta)} \end{aligned} \quad (2.12)$$

where $z_i = (x_i, t_i)$ for abbreviation. The first factorized term in (2.12), functional averaged over S_0 , is just the free electron Green function. The second factorized term, phase fluctuation average, which give the result of the product of the last two terms in (2.13), can also be calculated exactly, since the fluctuating phase action S_2 in Eq. (2.11) has quadratic nature. After some manipulations (details in Appendix B), the single-particle Green functions take the forms:

$$G_a(z_1, z_2) = \frac{a}{2\pi} \frac{1}{\zeta_{12}^a} \left(\frac{\xi_{12}^a}{\eta_{12}^a} \right)^{1/2} \left(\frac{\Lambda^2}{\eta_{12}^a \eta_{12}^{-a}} \right)^{\alpha/2} \quad (2.13)$$

where

$$\zeta_{ij}^a = x_{ij} - av_F t_{ij} + ia\delta \operatorname{sgn}(t_{ij}), \quad (2.14)$$

$$\xi_{ij}^a = \Lambda - ia(x_{ij} - av_F t_{ij}), \quad (2.15)$$

$$\eta_{ij}^a = \Lambda - ia(x_{ij} - av_F t_{ij}), \quad (2.16)$$

with $x_{ij} = x_i - x_j$, $t_{ij} = t_i - t_j$, $\delta > 0$ being infinitesimal, and Λ^{-1} the ultraviolet cut-off in the momentum. The quantity uv_F is the velocity of a “charge” excitation with

$$u = \frac{[(\pi\hbar v_F + g_4)^2 - g_2^2]^{1/2}}{\pi\hbar v_F}. \quad (2.17)$$

The parameter α , the so-called anomalous exponent of the 1D Luttinger-liquid correlation functions, is related to the strength of the interaction and is given by

$$\alpha = (1/K + K - 2)/4 \quad (2.18)$$

where K is the “stiffness constant” in traditional bosonization scheme:

$$K = \sqrt{(\pi\hbar v_F + g_4 - g_2)/(\pi\hbar v_F + g_4 + g_2)}. \quad (2.19)$$

The value of $K = 1$ corresponds to noninteracting case, and $K > 1$ represents the interaction being attractive and $K < 1$ the repulsive. In principle, any 1D Luttinger-liquid N -point correlation functions can be calculated by following the similar calculation for Green function (2.12). Taking the functional integral over $\tilde{\psi}$ and ϑ , we find, for example, the 4-point correlation function \mathcal{D}_{++++} has the expression:

$$\begin{aligned} \mathcal{D}_{++++}^{\sigma,\sigma}(z_1, z_2, z_3, z_4) &= (-i)^2 \langle \psi_{+,\sigma}(z_1) \psi_{+,\sigma}^\dagger(z_2) \psi_{+,\sigma}(z_3) \psi_{+,\sigma}^\dagger(z_4) \rangle_{S(\psi^\dagger, \psi)} \\ &= (-i)^2 \langle \tilde{\psi}_{+,\sigma}(z_1) \tilde{\psi}_{+,\sigma}^\dagger(z_2) \tilde{\psi}_{+,\sigma}(z_3) \tilde{\psi}_{+,\sigma}^\dagger(z_4) \rangle_{S_0(\tilde{\psi}^\dagger, \tilde{\psi})} \\ &\quad \times \langle e^{i(\theta_+(z_1) - \theta_+(z_2) + \theta_+(z_3) - \theta_+(z_4))} \rangle_{S_4(\theta)} \quad (2.20) \\ &= \frac{1}{4\pi^2} \left(\frac{1}{\zeta_{12}^+ \zeta_{34}^+} - \frac{1}{\zeta_{14}^+ \zeta_{32}^+} \right) \left(\frac{\xi_{12}^+ \xi_{34}^+ \xi_{32}^+ \xi_{41}^+ \eta_{42}^+ \eta_{31}^+}{\eta_{12}^+ \eta_{34}^+ \eta_{32}^+ \eta_{41}^+ \xi_{42}^+ \xi_{31}^+} \right)^{1/2} \\ &\quad \times \left(\frac{\Lambda^4 \eta_{31}^+ \eta_{31}^- \eta_{42}^+ \eta_{42}^-}{\eta_{12}^+ \eta_{12}^- \eta_{34}^+ \eta_{34}^- \eta_{32}^+ \eta_{32}^- \eta_{41}^+ \eta_{41}^-} \right)^{\alpha/2}. \end{aligned} \quad (2.21)$$

where ζ_{ij} , ξ_{ij} , η_{ij} , are defined in (2.14)–(2.16). The 4-point phase correlator, the last factorized term in (2.20),

useful for our later discussion to the problem of interchain tunneling coherence, gives the result of the product of the last two factorized terms in (2.21).

We next focus on the central issue of the physical role of a small interchain single-particle tunneling amplitude t_\perp in coupled Luttinger liquids. The action for this system can be written as

$$S_{2D} = \sum_n S_{1D,n} + S_\perp, \quad (2.22)$$

where $S_{1D,n}$ has the same decoupled ($\tilde{\psi}$ and ϑ) expression in (2.11) except that the field variables acquire additional transverse dependence n which labels the transverse chain (Luttinger liquid) number. The action, S_\perp , for transverse nearest-neighbor interchain tunnelings is

$$\begin{aligned} S_\perp &= \int dx dt \sum_{n,\sigma,a} t_\perp \psi_{a,\sigma}^\dagger(x, n+1, t) \psi(x, n, t)_{a,\sigma} + \text{H.C.} \\ &= \int dx dt \sum_{n,\sigma,a} t_\perp \tilde{\psi}_{a,\sigma,n+1}^\dagger(x, t) \tilde{\psi}_{a,\sigma,n}(x, t) \\ &\quad \times e^{-i[\vartheta_{a,n+1}(x,t) - \vartheta_{a,n}(x,t)]} + \text{H.C.} \end{aligned} \quad (2.23)$$

In this representation it is clear that we have the system of free fermions $\tilde{\psi}(x, t)$ and the environment of the fluctuating phases $\vartheta(x, t)$ both living in every chain. The transverse interchain tunneling action (2.23) then plays the role of the coupling between the system and environment.

III. SCHWINGER-KELDYSH CLOSED-TIME PATH FORMALISM

To address the question of interchain hopping coherence, we shall use perturbative expansion in powers of t_\perp within the Schwinger-Keldysh double-time or CTP formalism³⁹. Let us discuss briefly the basic idea of this CTP formalism (see also Feynman-Vernon influence functional formalism⁴⁸). This is the same formalism that was utilized to study the two-level model^{35,36}. In this formalism, the functional integral representation is applied to study the time evolution of the electron density matrix $\hat{R} = |\Psi(x, t)\rangle\langle\Psi(x, t)|$ rather than only the states in Hilbert space $|\Psi(x, t)\rangle$ where $\hat{H}|\Psi(x, t)\rangle = i\hbar(\partial|\Psi(x, t)\rangle/\partial t)$. Thus it is necessary to keep track of the two times, one representing the evolution of $|\Psi(x, t)\rangle$ forward in time, and another representing the evolution of $\langle\Psi(x, t)|$, which can be thought of as the evolution of $|\Psi(x, t)\rangle$ backward in time. In field theory, let us introduce $|\psi_t(x)\rangle$, a simultaneous common eigenstate of all the Heisenberg field operators $\hat{\psi}_H(x, t)$ at time t : $\hat{\psi}_H(x, t)|\psi_t(x)\rangle = \psi(x, t)|\psi_t(x)\rangle$. For a given value of (x, t) , $\psi(x, t)$ corresponds to a point in the field configuration space. The Schrödinger wave function $\langle\psi_t(x)|\Psi(x, t)\rangle$ is therefore a functional of $\psi(x, t)$. In this

way one can express the time-dependent density matrix element as a product of transition matrix elements from t_i to t_f and the time reversed (complex conjugate) matrix element from t_f to t_i times the initial density matrix element:

$$\begin{aligned} R(\psi_f, \psi'_f, t_f) &\equiv \langle\psi_f|\hat{R}(t_f)|\psi'_f\rangle \\ &= \langle\psi_f|\hat{U}(t_f, t_i)\hat{R}(t_i)\hat{U}^\dagger(t_f, t_i)|\psi'_f\rangle \\ &= \int \mathcal{D}\psi_i \mathcal{D}\psi'_i \langle\psi_i|\hat{R}(t_i)|\psi'_i\rangle \\ &\quad \times \langle\psi_f|\hat{U}(t_f, t_i)|\psi_i\rangle \langle\psi'_i|\hat{U}^\dagger(t_f, t_i)|\psi'_f\rangle. \end{aligned} \quad (3.1)$$

The spatial coordinate dependences are not shown explicitly in (3.1), but they can be recovered easily if one wishes.

Introducing the path integral representation for each transition matrix element in Eq. (3.1) results in the expression:

$$\begin{aligned} R(\psi_f, \psi'_f, t_f) &= \int \mathcal{D}\psi_i \mathcal{D}\psi'_i J(\psi_f, \psi'_f, t_f | \psi_i, \psi'_i, t_i) R(\psi, \psi', t_i), \end{aligned} \quad (3.2)$$

where J , the propagator of the density matrix, is given by

$$\begin{aligned} J(\psi_f, \psi'_f, t_f | \psi_i, \psi'_i, t_i) &= \int_{\psi_i}^{\psi_f} \mathcal{D}\psi e^{\frac{i}{\hbar} \int dx dt L[\psi]} \int_{\psi'_f}^{\psi'_i} \mathcal{D}\psi' e^{\frac{i}{\hbar} \int dx' dt' L[\psi']}, \end{aligned} \quad (3.3)$$

and L is the Lagrangian density. Hence, to incorporate both the statistical information carried by the density matrix and the dynamic evolution governed by the Lagrangian, it is physically necessary to deal with the propagation from t_i to t_f and then turns back to t_i , forming a double-time contour or CTP. Particularly, the interference between the forward- and backward-in-time field configuration is related to the off-diagonal component of the density matrix and then to the quantum coherence. It is known that G_{12} , the 1, 2-component of Keldysh Green-function matrix is directly related to the exact electron density matrix³⁹

$$R(x, x'; t) = -i\hbar G_{12}(x, x'; t, t). \quad (3.4)$$

The vanishing of the off-diagonal component of the density matrix is a characteristic of quantum incoherence.

IV. COHERENT/INCOHERENT INTERCHAIN TUNNELINGS

The perturbative expansion in t_\perp within the double-time functional integral for Luttinger liquids leads to the diagram shown in Fig. 1. The dashed line connecting points A and B represents the tunneling from the chain n to the chain $n+1$ that takes place at the spacetime point (x_n, t_n) . The top (bottom) solid lines with

the right (left) arrows represent the forward (backward) time propagation of $\tilde{\psi}$ along the corresponding chains n , $n + 1$, and $n + 2$. The left wavy line represents the correlation between the phases $\exp[i\vartheta_{a,n}(x_n, t_n)]$ and $\exp[-i\vartheta_{a,n}(x'_n, t'_n)]$, which mediates interaction between electrons on the chain n . In principle electrons on the same chain are correlated and are connected by wavy lines. By taking the chain $n + 1$ as an example, there are wavy lines (correlators) between points B and C, B and C', B and B', C and C', C and B', and B' and C'. However, for our purpose, only the relevant wavy lines are shown in Fig. 1.

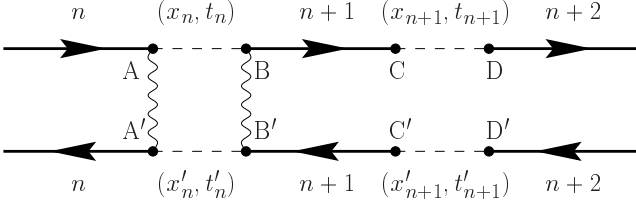


FIG. 1. Typical diagram of perturbative expansion in power of t_{\perp} within Schwinger-Keldysh double-time or closed-time path.

Strictly speaking the perturbative expansion in t_{\perp} within the closed-time path is carried out in the so-called single-dot approximation²⁷ in the inverse-self-energy Γ . This is exactly the same approximation as for the discussion^{11,13,20} of renormalization-group relevant-irrelevant condition for the t_{\perp} except that in the latter case only one forward-in-time path is considered. But when the intrachain interaction is not too strong (i.e., the anomalous correlation exponent α is small enough) and the interesting region of temperature or energy ($\omega \ll E_F$) is smaller than the effective tunneling amplitude (related to the so-called 1D crossover temperature $T_{1D} \approx t_{\text{eff}} \equiv (t_{\perp}/E_F^{\alpha})^{1/(1-\alpha)}$)^{12,20,27}, the perturbation t_{\perp} is relevant, i.e., each order in t_{\perp} carries a strongly divergent power of the energy $E_F^{-\alpha} \omega^{\alpha-1}$. In principle, one should consider all the higher-order diagrams, because they become of the same order of magnitude. However, since we are only interested in the behavior of the relative distance between the interchain tunnelings for the forward- and backward-in-time propagation in the neighboring-chain correlation functions, the inclusion of higher order diagrams would not change the power-law behavior, i.e., the exponent of this relative distance in correlation functions.

We next consider two extremal cases: noninteracting and strongly interacting electrons. For noninteracting, free electrons, the wavy-line correlator is equal to 1, so forward and backward electron lines are completely independent. Each of these lines (see Fig. 2(a)) collects the series in t_{\perp} independently and produces an extended, coherent Bloch field configuration for the transverse electron motion. On the other hand, the interaction between electrons introduces correlation between the forward- and backward-in-time electron interchain tunnelings. In the extreme case of very strong binding, the

wavy lines tie the two interchain tunneling events, one forward-in-time and the other backward-in-time, very tightly into a pair so that $(x_n, t_n) \approx (x'_n, t'_n)$. The closely bound pairs of forward- and backward-in-time electron tunnelings are called “collapsed blips”³⁶ for the two-level model⁴⁵. When similarly tightly bound pairs of forward- and backward-in-time electrons tunnel between chains for all neighboring chains as shown in Fig. 2(b), we say that the evolution of the field configuration $\psi_{a,\sigma,n}(x, t)$ and its complex conjugate component $\psi_{a,\sigma,n}^{\dagger}(x, t)$ form an independent bound pair combination $\psi_{a,\sigma,n}^{\dagger}(x, t)\psi_{a,\sigma,n}(x, t)$. In this case, the evolution of the probability from an initial field configuration to a final field configuration is equal to the sum of all possible evolutions of the pair field configuration $\psi_{a,\sigma}^{\dagger}(x, t)\psi_{a,\sigma}(x, t)$ rather than the square of the transition amplitude (the square of the sum over all possible evolutions of the field configuration $\psi_{a,\sigma,n}(x, t)$) from initial to final field configuration. In other words, the electron field density in Luttinger liquids, $\rho_{a,n}(x, t) = \sum_{\sigma} \psi_{a,\sigma,n}^{\dagger}(x, t)\psi_{a,\sigma,n}(x, t)$, rather than the electron field variable $\psi_{a,\sigma,n}(x_n, t_n)$, hops from the chain n to $n + 1$ (see Fig. 2). This regime corresponds to the incoherent tunneling between the chains. This picture of incoherence is similar to the case for the two-level model⁴⁵.

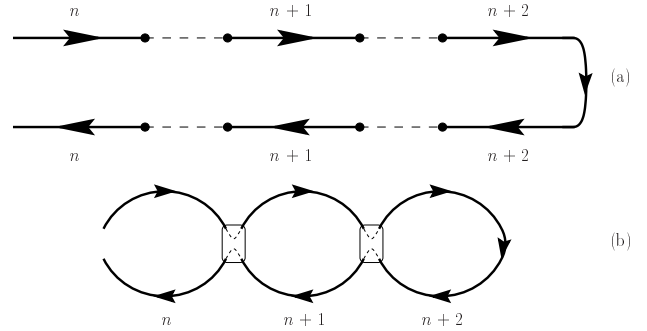


FIG. 2. (a) Coherent propagation of the electron field ψ_n ; uncorrelated interchain tunnelings for the forward and backward electron propagations in the case of noninteracting electrons. (b) Incoherent propagation of the electron probability density $\rho_n = \psi_n^{\dagger}\psi_n$; tightly bound, paired forward- and backward-in-time interchain tunnelings in the extreme case of strongly interacting electrons.

We give our criterion of coherent-incoherent transition in the following. We propose that the incoherent regime takes place when the integral of neighboring-chain correlators $\int d(x_n - x'_n) d(t_n - t'_n)$ over the relative distance between the interchain tunnelings for the forward and backward propagation starts to converge at the upper (infrared) limit⁴⁷. When the integral is convergent, the interchain tunneling events are bound into pairs called “collapsed blips”, which corresponds to the incoherent regime. If the integral does not converge, then the regime is coherent. Because the Luttinger-liquid correlators are power-law, it is straightforward to find a condi-

tion for the convergence in terms of the Luttinger-liquid exponents. Taking $a = +$ branch electrons as an example, tracing out functional average of the correlator $\langle \exp[i\vartheta_{+,n}(z_{n+1}) - i\vartheta_{+,n}(z_n) + i\vartheta_{+,n}(z'_n) - i\vartheta_{+,n}(z'_{n+1})] \rangle$ over the environment degree of freedom ϑ_n and using the infrared (large-distance) asymptotic forms of variables $\xi_{nn'}^a \approx \eta_{nn'}^a \approx (x_{nn'} - auv_F t_{nn'})$, we can write the relevant integral expression following from the expression of (2.21) as:

$$\int \frac{d\eta_{nn'}^+ d\eta_{nn'}^-}{(\eta_{nn'}^+ \eta_{nn'}^-)^\alpha}. \quad (4.1)$$

The integrand in (4.1) is a square of the relevant relative distance dependence in (2.21) due to the correlator contributions from both the wavy lines of the neighboring chains n and $n+1$. The spin-charge separation problem is not important to the issue of convergence of the integral expression (4.1) at the upper limit. As long as the spin and charge velocities are finite, they only introduce a finite factor²⁰. The convergence of the asymptotic integral (4.1) hence gives $\alpha > 1$ as the condition for the regime of the incoherence phase. This result is different from the result obtained before^{32,33}, where the confinement of coherence (incoherence regime) was found to occur at $\alpha > 1/2$. The condition for the t_\perp relevance in renormalization-group sense is also a condition on the exponents and is known³¹ to be irrelevant for $\alpha > 1$ and relevant for $\alpha < 1$. Our result indicates that the boundary value that separates coherent and incoherent regimes, $\alpha_{c/ic} = 1$, is the same as the value that separates relevant and irrelevant regimes. Thus the relevant but incoherent regime is not possible.

Our criterion of coherence-incoherence is formulated in exactly the same way as for the two-level model⁴⁵, ex-

cept there is no spatial coordinate x in the latter case. The closely bound pairs of forward- and backward-in-time electron tunnelings are called “collapsed blips”³⁶ for the two-level model. It is the extra dimension x that causes the difference between the results for the two-level model and the coupled Luttinger liquids.

V. CONCLUSION

In conclusion, we have introduced the functional bosonization approach which transforms the 1D Luttinger liquid into a representation of the “system” of free electrons and the “environment” of the fluctuating bosonic phase. The power-law correlation functions of the 1D Luttinger liquid are obtained straightforwardly and reproduced exactly by a functional average over free fermion action and fluctuating chiral phase action. We have examined the system of Luttinger liquids coupled by small transverse interchain single-particle tunneling t_\perp in terms of the CTP formalism. We find that the interchain tunneling t_\perp is relevant and coherent for $\alpha < 1$ and irrelevant and incoherent for $\alpha > 1$. There is no window for relevant but incoherent tunneling between 1D Luttinger liquids.

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APPENDIX A: CHIRAL JACOBIAN

In this Appendix we calculate the Jacobian (2.10) of the chiral transformation (2.8). Let us define the action S_3 as the sum of the first and third terms in (2.4):

$$S_3 = S_0(\psi^\dagger, \psi) + \int dt dx \sum_{a,\sigma} \phi_a(x, t) \rho_a(x, t). \quad (A1)$$

Then we define

$$e^{iF(\phi)/\hbar} \equiv \text{Jacobian of the chiral transformation} \\ = \frac{\int \mathcal{D}\psi^\dagger \mathcal{D}\psi e^{iS_3(\psi^\dagger, \psi, \phi)/\hbar}}{\int \mathcal{D}\tilde{\psi}^\dagger \mathcal{D}\tilde{\psi} e^{iS_0(\tilde{\psi}^\dagger, \tilde{\psi})/\hbar}}. \quad (A2)$$

Instead of calculating the Jacobian directly, let us calculate its variation with respect to a variation $\delta\phi_a$. By using the so called “point-splitting” method, the functional derivative of functional F with respect to ϕ_a from (A2) is given by

$$\frac{\delta F}{\delta\phi_a} = \lim_{\delta x, \delta t \rightarrow 0} \frac{\int \mathcal{D}\psi^\dagger \mathcal{D}\psi \sum_\sigma \psi_{a,\sigma}^\dagger(x + \delta x, t + \delta t) \psi_{a,\sigma}(x, t) e^{iS_3(\psi^\dagger, \psi, \phi)/\hbar}}{\int \mathcal{D}\psi^\dagger \mathcal{D}\psi e^{iS_3(\psi^\dagger, \psi, \phi)/\hbar}} \quad (A3)$$

$$= \lim_{\delta x, \delta t \rightarrow 0} \sum_\sigma \langle \psi_{a,\sigma}^\dagger(x + \delta x, t + \delta t) \psi_{a,\sigma}(x, t) \rangle_{S_3(\psi^\dagger, \psi, \phi)} \\ = \lim_{\delta x, \delta t \rightarrow 0} \sum_\sigma \langle \tilde{\psi}_{a,\sigma}^\dagger(x + \delta x, t + \delta t) \tilde{\psi}_{a,\sigma}(x, t) \rangle_{S_0(\tilde{\psi}^\dagger, \tilde{\psi})} e^{-i(\vartheta_a(x + \delta x, t + \delta t) - \vartheta_a(x, t))}. \quad (A4)$$

The advantage of Eq. (A3) is that, after we make the chiral transformation (2.8) on S_3 (A1) to obtain Eq. (A4), the Jacobian of the transformation cancel out in numerator and denominator of the functional average expression (A3). The 1D free fermion Green function is easy to obtained:

$$\langle \tilde{\psi}_{a,\sigma}(x,t)\tilde{\psi}_{a,\sigma}^\dagger(x',t') \rangle_{S_0(\tilde{\psi}^\dagger,\tilde{\psi})} = i \int \frac{dkd\omega}{(2\pi)^2} \frac{e^{ik(x-x')-i\omega(t-t')}}{\omega - av_F k + i\delta \text{sign}(av_F k)} \quad (\text{A5})$$

$$= \frac{1}{2\pi i} \frac{-a}{(x-x') - av_F(t-t') + ia\delta \text{sign}(t-t')}. \quad (\text{A6})$$

Making the expansion on the phase difference term of (A4), $\vartheta_a(x+\delta x, t+\delta t) - \vartheta_a(x, t) = (\partial_x \vartheta_a)\delta x + (\partial_t \vartheta_a)\delta t + O(\delta x^2, \delta t^2)$, and using (A5) for the free fermion Green function, we find

$$\frac{\delta F}{\delta \phi_a} = -2i \lim_{\delta x, \delta t \rightarrow 0} \int \frac{dkd\omega}{(2\pi)^2} \frac{e^{-i(k+\partial_x \vartheta_a)\delta x + i(\omega+\partial_t \vartheta_a)\delta t + O(\delta x^2, \delta t^2)}}{\omega - av_F k + i\delta \text{sign}(av_F k)}. \quad (\text{A7})$$

Changing the variables $k' = k + \partial_x \vartheta_a$, $\omega' = \omega + \partial_t \vartheta_a$, performing the frequency integration, and then taking the $\delta x, \delta t \rightarrow 0$ limits, we end up with the expression that

$$\frac{\delta F}{\delta \phi_a} = \int \frac{dk'}{\pi} n_F[av_F(k' - \partial_x \vartheta_a)]. \quad (\text{A8})$$

Expanding the Fermi distribution function and dropping the term of total number of fermions⁴⁹, we finally get

$$\frac{\delta F}{\delta \phi_a(x, t)} = \frac{a}{\pi} \partial_x \vartheta_a(x, t). \quad (\text{A9})$$

Since the auxiliary bosonic field ϕ_a is, from (2.9), proportional to the phase field ϑ_a , we arrive at the result that

the chiral Jacobian, the extra contribution to action due to the chiral transformation (2.8), is given by (2.10).

APPENDIX B: CALCULATIONS OF CORRELATION FUNCTIONS IN 1D LUTTINGER LIQUID

In this Appendix we calculate correlation functions in 1D Luttinger Liquid. Let us first find the explicit expression for the fluctuating phase action $S_2(\vartheta)$. After substituting (2.9) into (2.10) and (2.6), and Fourier transforming to momentum space, we find

$$\begin{aligned} S_2(\vartheta) &= F(\phi) + S_1(\phi) \\ &= \frac{\hbar}{2\pi v_F} \int \frac{dkd\omega}{(2\pi)^2} (\vartheta_+(-k, -\omega), \vartheta_-(-k, -\omega)) \left[\begin{pmatrix} -(\omega - v_F k)v_F k & 0 \\ 0 & (\omega + v_F k)v_F k \end{pmatrix} \right. \\ &\quad \left. + \frac{\pi\hbar v_F}{(g_4^2 - \bar{g}_2^2)} \begin{pmatrix} g_4(\omega - v_F k)^2 & -g_2(\omega^2 - v_F^2 k^2) \\ -g_2(\omega^2 - v_F^2 k^2) & g_4(\omega + v_F k)^2 \end{pmatrix} \right] \begin{pmatrix} \vartheta_+(k, \omega) \\ \vartheta_-(k, \omega) \end{pmatrix}, \\ &= \frac{-\hbar}{2i} \int \frac{dkd\omega}{(2\pi)^2} (\vartheta_+(-k, -\omega), \vartheta_-(-k, -\omega)) \hat{M}(k, \omega) \begin{pmatrix} \vartheta_+(k, \omega) \\ \vartheta_-(k, \omega) \end{pmatrix} \\ &\equiv -\frac{\hbar}{2i} [\vartheta, \hat{M}\vartheta] \end{aligned} \quad (\text{B1})$$

where the matrix \hat{M} given by

$$\hat{M} = \begin{bmatrix} \hat{M}_{++} & \hat{M}_{+-} \\ \hat{M}_{-+} & \hat{M}_{--} \end{bmatrix}, \quad (\text{B2})$$

with matrix elements:

$$\hat{M}_{aa} = \frac{-i}{\pi v_F (\bar{g}_4^2 - \bar{g}_2^2)} (\omega - av_F k) \times [\bar{g}_4(\omega + av_F k) - a(u^2 - 1)v_F k], \quad (\text{B3})$$

$$\hat{M}_{-aa} = \frac{i}{\pi v_F (\bar{g}_4^2 - \bar{g}_2^2)} \bar{g}_2(\omega + v_F k)(\omega - v_F k), \quad (\text{B4})$$

and normalized coupling constants:

$$\bar{g}_2 = \frac{g_2}{\pi v_F \hbar}, \quad \bar{g}_4 = \frac{g_4}{\pi v_F \hbar}. \quad (\text{B5})$$

The velocity of charge sector u is defined in (2.17).

Now we are in the position to calculate the correlation functions. Let us calculate the single-particle Green function first. Fourier transforming the phase fluctuation average term in (A6), we find

$$\langle e^{i\vartheta_a(z_1) - i\vartheta_a(z_2)} \rangle_{S_2(\vartheta)} = \frac{\int \mathcal{D}\vartheta_a \exp \left[i \int \frac{dkd\omega}{(2\pi)^2} \vartheta_a(k, \omega) (e^{ikx_1 - i\omega t_1} - e^{ikx_2 - i\omega t_2}) + i \frac{S_4(\vartheta_a)}{\hbar} \right]}{\int \mathcal{D}\vartheta_a \exp(iS_4(\vartheta_a)/\hbar)}. \quad (\text{B6})$$

Using the identity

$$e^{-\frac{1}{2}[\vartheta, \hat{M}\vartheta] + [B, \vartheta]} = e^{-\frac{1}{2}[\vartheta - \hat{M}^{-1}B, \hat{M}(\vartheta - \hat{M}^{-1}B)] + \frac{1}{2}[B, \hat{M}^{-1}B]} \quad (\text{B7})$$

with the definition

$$[B, \vartheta] \equiv \int \frac{dkd\omega}{(2\pi)^2} (b_+(-k, -\omega), b_-(-k, -\omega)) \begin{pmatrix} \vartheta_+(k, \omega) \\ \vartheta_-(k, \omega) \end{pmatrix}, \quad (\text{B8})$$

we obtain

$$\langle e^{i\vartheta_a(z_1) - i\vartheta_a(z_2)} \rangle_{S_2(\vartheta)} = e^{\frac{1}{2}[B, \hat{M}^{-1}B]} \quad (\text{B9})$$

where the transpose of B is:

$$B^T(k, \omega) = \begin{cases} (b(k, \omega), 0) & \text{for } a = + \\ (0, b(k, \omega)) & \text{for } a = - \end{cases} \quad (\text{B10})$$

with

$$b(k, \omega) = e^{-ikx_1 + i\omega t_1} - e^{-ikx_2 + i\omega t_2}. \quad (\text{B11})$$

The inverse of matrix \hat{M} (B2) can be calculated and the result is

$$\hat{M}^{-1} = \begin{bmatrix} (\hat{M}^{-1})_{++} & (\hat{M}^{-1})_{+-} \\ (\hat{M}^{-1})_{-+} & (\hat{M}^{-1})_{--} \end{bmatrix}, \quad (\text{B12})$$

with

$$(\hat{M}^{-1})_{aa} = \frac{\pi a i}{k} \left(\frac{1}{\omega - av_F k} - \frac{1}{\omega - av_F k} \right) + \frac{\pi i}{k} \alpha \left(\frac{1}{\omega - uv_F k} - \frac{1}{\omega + uv_F k} \right), \quad (\text{B13})$$

$$(\hat{M}^{-1})_{-aa} = \frac{\pi i}{k} \nu \left(\frac{1}{\omega - uv_F k} - \frac{1}{\omega + uv_F k} \right). \quad (\text{B14})$$

The parameter α , which turns out to be the anomalous exponent of the Green function in 1D Luttinger liquid, is defined in (2.18). Another exponent of correlation function ν which shows up, for example, in the 4-point correlation function $\langle \psi_{+, \sigma}(z_1) \psi_{+, \sigma}^\dagger(z_2) \psi_{-, \sigma}(z_3) \psi_{-, \sigma}^\dagger(z_4) \rangle_{S(\psi^\dagger, \psi)}$, has the expression

$$\nu = \frac{1}{4} \left(\frac{1}{K} - K \right) \quad (\text{B15})$$

where K , the ‘‘stiffness constant’’, is defined in (2.19).

From (B10) and (B11), we find that the exponent $(1/2)[B, \hat{M}^{-1}B]$ of (B9) only picks up one of the diagonal elements $(\hat{M}^{-1})_{aa}$ of matrix \hat{M}^{-1} :

$$\begin{aligned} \langle e^{i\vartheta_a(z) - i\vartheta_a(0)} \rangle_{S_2(\vartheta)} &= \exp \left\{ \frac{1}{2} [B, \hat{M}^{-1}B] \right\} \\ &= \exp \left\{ i\pi \int \frac{dkd\omega}{(2\pi)^2} [1 - \cos(kx - \omega t)] (\hat{M}^{-1})_{aa} \right\}. \end{aligned} \quad (\text{B16})$$

Following identity is very useful in our calculation:

$$\begin{aligned} &i\pi \int \frac{dkd\omega}{(2\pi)^2} \left[\frac{1 - \cos(kx - \omega t)}{k(\omega - av_F k + i\eta \text{sign}(ak))} \right] \\ &= \lim_{\Lambda \rightarrow 0} \frac{a}{2} \ln \left[\frac{\Lambda - ia(x - av_F t)}{\Lambda} \right]. \end{aligned} \quad (\text{B17})$$

In the derivation of the identity (B17), we regulate the integral at large momenta by introducing a convergence factor $\exp(-\Lambda|k|)$, where Λ^{-1} is the ultraviolet cut-off. As the physics does not involve large momenta, that is large energies, the introduction of a cut-off will not make any difference and, at the end of any calculation, the $\Lambda \rightarrow 0$ limit can be performed. Using Eq. (B17), one can easily figure out from (B13) and (B14) that

$$\begin{aligned} &\langle e^{i\vartheta_a(z) - i\vartheta_a(0)} \rangle_{S_2(\vartheta)} \\ &= \left[\frac{\Lambda - ia(x - av_F t)}{\Lambda - ia(x - av_F t)} \right]^{1/2} \\ &\times \left[\frac{\Lambda^2}{[\Lambda - i(x - uv_F t)][\Lambda + i(x + uv_F t)]} \right]^{\alpha/2}. \end{aligned} \quad (\text{B18})$$

Hence, by means of (2.12), (A6) and (B16), the single particle Green functions take the forms of (2.13). One can see that the parameter α (2.18) indeed is the anomalous exponent in Green function (2.13).

The similar procedure of calculating single particle Green function could be applied to compute N-point correlation functions. We consider the 4-point correlation function $\langle \psi_{+, \sigma}(z_1) \psi_{+, \sigma}^\dagger(z_2) \psi_{+, \sigma}(z_3) \psi_{+, \sigma}^\dagger(z_4) \rangle_{S(\psi^\dagger, \psi)}$ (2.20), which is useful in the discussion of coherent-incoherent transition. The free 4-point correlation functions in (2.20) can be easily computed and is equal to the product of two free fermion Green functions defined in (A6). The phase fluctuation average has the similar patterns as that in (B7). Therefore, instead of (B9), (B10) and (B11), we have:

$$\langle e^{i(\vartheta_+(z_1) - \vartheta_+(z_2) + \vartheta_\pm(z_3) - \vartheta_\pm(z_4))} \rangle_{S_4(\vartheta)} = e^{\frac{1}{2}[B, \hat{M}^{-1}B]} \quad (\text{B19})$$

where $B^T(k, \omega) = (b(k, \omega), 0)$, with

$$b(k, \omega) = e^{-ikx_1 + i\omega t_1} - e^{-ikx_2 + i\omega t_2} \\ + e^{-ikx_3 + i\omega t_3} - e^{-ikx_4 + i\omega t_4}. \quad (\text{B20})$$

The matrix \hat{M} and \hat{M}^{-1} are defined as before. Following the same procedure of deriving single particle Green functions (2.13), we find the expression (2.21) for the 4-point correlation functions $\mathcal{D}_{++++}^{\sigma, \sigma}(z_1, z_2, z_3, z_4)$. These correlation functions are reproduced exactly as those in Refs.⁷⁻¹⁰.

APPENDIX C: COHERENCE/INCOHERENCE OF TWO-LEVEL MODEL

The attractiveness of our approach for the coherence/incoherence condition of single-particle interchain tunnelings between Luttinger liquids is that it is formulated in exactly the same way as in the two level model, except there is no spatial coordinate x in the latter case. We present in this Appendix how the procedure works for the two-level model. The characteristic Hamiltonian for the two-level system coupled to a dissipative environment can be written as follows^{35,36}:

$$H_{\text{TLM}} = -\frac{1}{2}\hbar\Delta\sigma_x + \sum_i \left(\frac{1}{2}m_i\omega_i x_i^2 + \frac{1}{2m_i}p_i^2 \right) \\ + \frac{1}{2}\sigma_z \sum_i C_i x_i. \quad (\text{C1})$$

Here σ 's are the Pauli matrices, C_i is the coupling strength to the i th oscillator, and m_i, ω_i, x_i and p_i are the mass, frequency, position and momentum of the i th oscillator, respectively. The first term in Hamiltonian (C1) describes a quantum particle, the system, tunneling between two states connected by a matrix element Δ . The second term in (C1) represents the environment of a bath of oscillators. The third term in (C1) describes the interaction between the system and the dissipative environment with the coupling strength C_i . The bath of harmonic oscillators has distributed frequencies. We consider the Ohmic bath, for which the spectral density, $J(\omega)$, of the environment is given by

$$J(\omega) \equiv \frac{\pi}{2} \sum_i \frac{C_i^2}{m_i\omega_i} \delta(\omega - \omega_i) = 2\pi\hbar\alpha_\eta\omega e^{-\omega/\omega_c} \quad (\text{C2})$$

where ω_c is a cutoff frequency and α_η is a positive constant measuring the strength of the coupling to the environment and the stronger the interaction with the environment, the larger the value of α_η .

If initially the particle is in one of the two states, say, in the $\sigma_z = \uparrow$ state for all $t < 0$, $|\psi(t < 0)\rangle = |\sigma_z, \uparrow\rangle$, then in the absence of interaction with the environment, i.e. $C_i = 0$ ($\alpha_\eta = 0$), the wave function according to elementary quantum mechanics can be easily written as follows:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left[e^{i\phi_+} e^{i\Delta t/2} |\sigma_x, \uparrow\rangle + e^{i\phi_-} e^{-i\Delta t/2} |\sigma_x, \downarrow\rangle \right]. \quad (\text{C3})$$

In this case, the relative phase between these two eigenstates remains well defined indefinitely and the probability to find the particle in the other state oscillates in time, $p(t) = (\langle \sigma_z(t) \rangle + 1)/2 = (1 + \cos \Delta t)/2$. When interaction with the environment is sufficiently strong (when $\alpha_\eta \geq 1/2$), this probability does not oscillate, but changes in time exponentially, saturating at the probability $1/2$, $p(t) \sim \frac{1}{2}(1 + e^{-\Gamma t})$. This behavior is called incoherent, because it corresponds to a classical, rather than quantum-mechanical, coupling of the two states. At a stronger interaction with the environment (when $\alpha_\eta \geq 1$), the particle gets indefinitely localized in the initial state, and does not make a transition to the other state at all. In this regime, Δ is irrelevant^{37,38}.

We shall apply perturbation theory in the tunneling matrix element Δ rather than in the system-environment interaction. To diagonalize the last three terms in H_{TLM} , it is simply necessary to shift each oscillator through a distance $\sigma_z \delta x_i$, where $\delta x_i = -\frac{1}{2} \frac{C_i}{m_i \omega_i^2}$. The appropriate unitary operator is

$$\hat{U} = \exp \left(-\frac{i}{2} \sigma_z \sum_i \frac{C_i}{m_i \omega_i^2} \hat{p}_i \right). \quad (\text{C4})$$

The transformed Hamiltonian

$$H'_{\text{TLM}} = \hat{U} H_{\text{TLM}} \hat{U}^{-1} \\ = -\frac{1}{2}\hbar\Delta(\sigma^+ e^{-i\Omega} + h.c.) + H_{\text{oscillators}} \quad (\text{C5})$$

where $\Omega = \sum_i \frac{C_i}{m_i \omega_i^2} p_i$, and $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$. The tunneling operator between the two states has been replaced by an operator which creates and destroys excitations of the oscillator bath, as well as changing the state of the spin. The two-point correlation function of $\sigma^+ e^{-i\Omega}$ can then be found:

$$\langle \sigma^+ e^{-i\Omega(t)} \sigma^- e^{i\Omega(0)} \rangle = \exp \left\{ -\int_0^\infty \frac{1 - e^{-i\omega t}}{\omega^2} J(\omega) \right\} \\ = \exp \left\{ -2\alpha_\eta \int_0^\infty \frac{1 - e^{-i\omega t}}{\omega} e^{-\omega/\omega_c} \right\} \\ \sim e^{i\pi\alpha_\eta} (\omega_c t)^{-2\alpha_\eta}. \quad (\text{C6})$$

Next we discuss the coherent-incoherent transition for the two-level model within the Schwinger-Keldysh CTP formalism. Suppose that the top and bottom diagrams in Fig. 3(a) represent one of the possible paths forward- and backward-in-time respectively in the two-level model. The paths can jump only between two discrete values + and - which correspond to the state in the right and left well or $\sigma_z = \uparrow\downarrow$, respectively. One can visualize the double path integrals in CTP formalism as a single integral

over a single path (see Fig. 3(b)) that jumps between the four states corresponding to $\{\uparrow, \uparrow\}$, $\{\uparrow, \downarrow\}$, $\{\downarrow, \uparrow\}$, $\{\downarrow, \downarrow\}$ (the first element in the braces $\{ \}$ is the system state in the forward-in-time path and the second element is the system state in the backward-in-time path). Blips, as shown in Fig. 3(b), are the periods when the system is in either of two off-diagonal states corresponding to $\{\uparrow, \downarrow\}$ or $\{\downarrow, \uparrow\}$ state. It was argued³⁶ that for $\alpha_\eta \rightarrow 1/2$ from below, only blips of effective vanishing length, that is “collapsed” blips, contribute. In this case, interactions among different collapsed blips vanish and noninteracting-blip approximation^{35,36} becomes exact. The probability or the time dependent expectation value of σ_z , $P(t) = \langle \sigma_z(t) \rangle = (2p(t) - 1)/2 = \exp[-\pi\Delta^2 t/(2\omega_c)]$, has pure damped behavior without any oscillation, which is just the signature of incoherence. The fact that blips collapse and become noninteracting is related to the situation that interference becomes negligible. The particle states and their complex conjugates in two different time branches of the CTP are bound and hop together, i.e., the change of the state from \uparrow to \downarrow in one of the two time branches immediately causes the corresponding state of the particle in the other time path changes from \downarrow to \uparrow state. Therefore the evolution of probability density from an initial state to a final state is equal to the sum of all possible evolutions of the bound pairs, collapsed blips, rather than the square of the transition amplitude (the square of the sum over all possible evolutions of the wave function) from initial to final state.

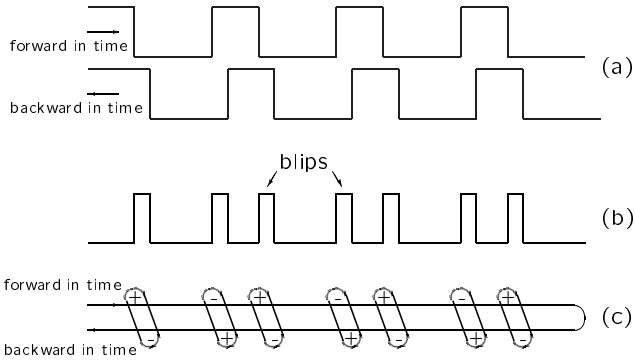


FIG. 3. (a) Possible forward- and backward-in-time paths for the two-level model. (b) The corresponding visualized single path. (c) Coulomb charges lying in the Schwinger-Keldysh CTP and under certain condition charges at different branches of the CTP pairing into noninteracting or independent neutral dipoles with vanishing dipole moments.

The formation of “collapsed” blips in two-level model can be understood in terms of Coulomb gas (CG)^{50,51} language, i.e., Coulomb charges lying in the Schwinger-Keldysh closed-time path or “double-time” contour. In terms of CG language, the times spin flips correspond to the locations of charges. We assume that the time moment at which spin flips from \uparrow to \downarrow (kink) corresponds to the position of positive charged particle in a 1D space and

the moment from \downarrow to \uparrow (anti-kink) corresponds to that of negative charged particle. The locations of charges corresponding to the times that spin flips in Fig. 3(a) in the CTP, for example, are shown in Fig. 3(c). Under certain conditions (i.e., when particle-environment interaction is strong enough in the two-level model), the charges at different branches of the CTP pair into noninteracting or independent neutral dipoles (see Fig. 3(c)) with vanishing dipole moments (vanishing distances: $t \approx t'$). In this case, the CG is said to be in the incoherent (confined) phase. These independent dipoles with vanishing dipole moments develop from the short distance singularity (when approaching from the plasma phase) of the attractive interaction between nearest-neighbor opposite charges in two different time branches of the CTP. The formation of these “collapsed” dipoles play a role similar to the “collapsed” blips in the dissipative two-level model, where particle states and their complex conjugates in two different time paths are tightly bound and hop together. This corresponds to the condition for the disappearance of coherence.

We next discuss the condition of coherent-incoherent transition for the two-level model in a similar way as in the case for the interchain tunneling between coupled Luttinger liquids. We propose that the incoherent regime takes place when the integral $\int d(t-t')$ over the relative time distance between the spin flips for the forward and backward propagation starts to converge at the upper (infrared) limit. The N-point correlation function can be found by the similar procedure for obtaining Eq. (C6). Tracing out the environment degrees of freedom, p_i and x_i , we find that the N-point correlator has the power law dependence of $2\alpha_\eta$ on the relative time distance between the spin flips for the forward and backward propagation. We write the relevant integral expression as the following:

$$\int \frac{d(t-t')}{(t-t')^{2\alpha_\eta}}. \quad (C7)$$

The convergence of the integral (C7) at large distance when approaching from the dipole phase or the short distance singularity of the integral (C7) when approaching from the plasma phase corresponds to the incoherent regime. Therefore the regime, $\alpha_\eta > 1/2$, in which the blips “collapse” or Coulomb charges in different time branches become tightly bound into independent dipoles with vanishing dipole moments (i.e., interference become negligible), corresponds to the regime of incoherence in the two-level model.

The picture of the Coulomb charges lying in the Schwinger-Keldysh closed-time-path and tending to reorganize as a dipole gas has been used to describe the tunneling statistics of Luttinger liquids through narrow barriers⁵². It was shown that an independent or noninteracting dipole approximation gives a Poisson distribution for the locations of the dipole centers of mass, which corresponds to the situation when the tunneling events in either direction are independent or shot noise is uncorrelated. The independent or noninteracting dipoles with

vanishing dipole moments (vanishing distances), or with the positions of the charges in forward- and backward-time path and dipole centers being not distinguished in

the low-frequency range ($\omega \ll \omega_0 = e^*V/\hbar$) in Ref.⁵² are similar to the “collapsed” blips in the two-level model.

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- ¹ S. Tomonaga, Prog. Theor. phys. **5** 349 (1950).
- ² J. M. Luttinger, J. Math. Phys. **4**, 1154 (1963).
- ³ D. C. Mattis et E. H. Lieb, J. Math. Phys. **6**, 304 (1965).
- ⁴ A. Luther et J. Peshel, Phys. Rev. B **9**, 2911 (1974).
- ⁵ I. E. Dzyaloshinski et A. I. Larkin, Sov. Phys. JETP **38**, 202 (1974).
- ⁶ F. D. M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980); J. Phys. C **14**, 2585 (1981); Phys. Lett. **81A**, 153 (1981); Phys. Rev. Lett. **47**, 1840 (1981).
- ⁷ For a review, see J. Solyom, Adv. Phys. **28**, 201 (1979).
- ⁸ For a review, see V. J. Emery in *Highly Conducting One-Dimensional Solids*, eds J. T. Devreese, R. P. Evrard et V. E. van Doren (Plenum 1979), p. 247.
- ⁹ For a review, see H. J. Schulz, “*Correlated Electron Systems*”, p. 199, ed. V. J. Emery (World scientific, Singapore, 1993); H. J. Schulz, “*Strongly Correlated Electronic Materials: The Los Alamos Symposium - 1993*”, p. 187, ed. K. S. Bedell, Z. Wang, D. E. Meltzer, A. V. Balatsky, E. Abrahams (Addison-Wesley, Reading, Massachusetts, 1994).
- ¹⁰ For a more recent review, see J. Voit, Reports on Progress in Physics **58**, p. 977 (1995).
- ¹¹ X.G. Wen, Phys. Rev. B **42**, 6623 (1990);
- ¹² C. Bourbonnais and L. G. Caron, Int. J. Mod. Phys. B **5**, 11033 (1991);
- ¹³ H.J. Schulz, Int. J. Mod. Phys B **5**, 57 (1991).
- ¹⁴ M. Fabrizio, A. Parola and E. Tosatti, Phys. Rev. B **46**, 3159 (1992).
- ¹⁵ C. Castellani, C. di Castro and W. Metzner, Phys. Rev. Lett. **69**, 1703 (1992); Phys. Rev. Lett. **72**, 316 (1994).
- ¹⁶ F. V. Kusmartsev, A. Luther and A. Nersesyan, JETP Lett. **55**, 692 (1992).
- ¹⁷ V. M. Yakovenko, JETP Lett. **56**, 510 (1992).
- ¹⁸ A. Finkelstein and A. I. Larkin, Phys. Rev. B **47**, 10461 (1993).
- ¹⁹ A. A. Nersesyan, A. Luther and F. V. Kusmartsev, Phys. Lett. A **176**, 363 (1993).
- ²⁰ D. Boies, C. Bourbonnais and A. M. S. Tremblay, Phys. Rev. Lett. **74**, 968 (1995).
- ²¹ P. Kopietz, V. Meden and K. Schönhammer, Phys. Rev. Lett. **74**, 2997 (1995); L. Bartosch and P. Kopietz, Phys. Rev. B **55**, 15360 (1997).
- ²² H. J. Schulz, Phys. Rev. B **53**, 2955 (1996).
- ²³ A. M. Tselik, cond-mat/9710343 preprint.
- ²⁴ N. Shannon, Y. Li and N. d’Ambrumenil, Phys. Rev. B **55**, 12963 (1997).
- ²⁵ D. Schmeltzer and A. R. Bishop, Phys. Rev. B **57**, 5419 (1998).
- ²⁶ K. Schönhammer, Phys. Rev. B **58**, 3494 (1998).
- ²⁷ E. Arrigoni, Phys. Rev. Lett. **80**, 790 (1998); E. Arrigoni, to appear in Phys. Rev. Lett., cond-mat/9903431.
- ²⁸ F. Mila and D. Poilblanc, Phys. Rev. Lett. **76**, 287 (1996); see also D. G. Clarke and S. P. Strong, Phys. Rev. Lett. **78**, 563 (1997); F. Mila and D. Poilblanc, Phys. Rev. Lett. **78**, 564 (1997).
- ²⁹ D. Poilblanc, H. Endres, F. Mila, M. Zacher, S. Capponi and W. Hanke, Phys. Rev. B **54**, 10261 (1996); S. Capponi, D. Poilblanc and F. Mila, Phys. Rev. B **54**, 17547 (1996).
- ³⁰ S. Capponi, D. Poilblanc, and E. Arrigoni, Phys. Rev. B **57**, 6360 (1998).
- ³¹ see, for example, Refs.^{11,13,17,19,20}.
- ³² D. G. Clarke, S. P. Strong and P. W. Anderson, Phys. Rev. Lett. **72**, 3218 (1994); D. G. Clarke, S. P. Strong and P. W. Anderson, Phys. Rev. Lett. **74**, 4499 (1995).
- ³³ S. P. Strong, D. G. Clarke, J. Phys.: Condens. Matter **8**, 10089 (1996); D. G. Clarke and S. P. Strong, Ferroelectrics **177**, 1 (1996). D. G. Clarke and S. P. Strong, J. Phys.: Condens. Matter **9**, 3853 (1997); D. G. Clarke and S. P. Strong, Adv. Phys., **46**, 545 (1997).
- ³⁴ D. G. Clarke, S. P. Strong and P. M. Chaikin and E. I. Chashechkina, cond-mat/9708081.
- ³⁵ A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987); *ibid*, **67**, 725 (Errat.) (1995).
- ³⁶ M. Sasseti and U. Weiss, Phys. Rev. A **41**, 5383 (1990); U. Weiss, *Quantum Dissipative Systems* in Series Modern Condensed Matter Physics, Vol. 2 (World Scientific, Singapore, 1993)
- ³⁷ A. J. Bray and M. A. Moore, Phys. Rev Lett. **49**, 1546 (1982); S. Chakravarty, Phys. Rev Lett. **50**, 1811 (1982).
- ³⁸ P. W. Anderson and G. Yuval, Phys. Rev. Lett. **23**, 89 (1969); G. Yuval and P. W. Anderson, Phys. Rev. B **1**, 1522 (1970). P. W. Anderson, G. Yuval and D.R. Hamann, Phys. Rev. B **1**, 4464 (1970); G. Yuval and P. W. Anderson, J. Phys. C **4**, 607 (1971).
- ³⁹ J. Schwinger J. Math. Phys. **2** 407 (1961); L.V. Keldysh, Sov. Phys. JETP **20**, 1018, (1965); V. Korenman, Ann. Phys. **39**, 72 (1966); See, also, E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981); G.-Z. Zhou, Z.-B. Su, B.-L. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985).
- ⁴⁰ H. C. Fogedby, J. Phys. C **9**, 3757 (1976).
- ⁴¹ D.K.K. Lee and Y. Chen, J. Phys. A **21**, 4155 (1988).
- ⁴² P. Kopietz and K. Schönhammer, Z. Phys. B **100**, 259 (1996); P. Kopietz, J. Hermisson, and K. Schönhammer, Phys. Rev. B **52**, 10877 (1995); P. Kopietz, in the Proceedings of the Raymond L. Orbach Symposium, p.101, edit by D. Hone (World Scientific, Singapore, 1996); P. Kopietz and G.E. Castilla, Phys. Rev. Lett. **76** 4777 (1996); for a recent textbook on the subject, see P. Kopietz, *Bosonization of Interacting Fermions in Arbitrary Dimensions*, (Springer-

- Verlag, Berlin, 1997).
- ⁴³ J. Hubbard, Phys. Rev Lett. **3**, 77 (1959); R.L. Stratonovich, Sov. Phys.-Dokl. **2**, 416 (1957).
- ⁴⁴ K. Fujikawa, Phys. Rev. Lett. **42** 1195 (1979). for a recent textbook on the subject of anomalies, see R.A. Bertlmann, *Anomalies in Quantum Field Theory*, (Oxford, New York, 1996).
- ⁴⁵ see Refs.^{35,36} and Appendix C.
- ⁴⁶ S. A. Brazovskii and V. M. Yakovenko, J. Physique Lett. (Paris) **46**, L111 (1985); S. A. Brazovskii and V. M. Yakovenko, Sov. Phys. JETP **62**, 1340 (1985).
- ⁴⁷ The idea that the correlation function decays faster than the square of the electron-hole or electron-electron separations and then this fast-decay behavior leads effectively to a contraction of the two coordinates (a bound-pair tunneling) was applied to the discussion of the relevance of interchain tunneling in coupled Luttinger liquids in Refs.^{13,20,46}.
- ⁴⁸ R.P. Feynman and F.L. Vernon Jr., Ann. Phys. (N.Y.) **24**, 118 (1963); A.D. Caldera and A.J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983); *ibid* **153**, 445(E) (1983).
- ⁴⁹ The fact that we drop the term of total number of fermions in (A7), is equivalent to normally ordering the fermion density (2.2). Formally, the number of fermions in Luttinger liquid model (2.1) is infinite because of the linearization of the electron dispersion law near the Fermi energy. Nevertheless, the *variation* of the fermion number is finite and can be calculated unambiguously. Calculating the change of the fermion density is equivalent to subtracting the infinite vacuum average from it, *i.e.*, normal ordering of the fermion density.
- ⁵⁰ J.M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973); J.M. Kosterlitz, J. Phys. C **7**, 1046 (1974); for a review, see P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).
- ⁵¹ H. Schultz, J. Phys. A **14**, 3277 (1981); S. A. Bulgadaev, Phys. Lett. A **86**, 213 (1981); P.J. Forrester, J. Stat. Phys. **45**, 153 (1986).
- ⁵² C. de C. Chamon, D.E. Freed and X.-G. Wen, Phys. Rev. B **51**, 2363 (1995).