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AN ANALYSIS OF READING OUT THE STATE OF A CHARGE QUANTUM BIT

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We provide a unified picture for the master equation approach and the quantum trajectory approach to a measurement problem of a two-state quantum system (a qubit), an electron coherently tunneling between two coupled quantum dots (CQD's) measured by a low transparency point contact (PC) detector. We show that the master equation of "partially" reduced density matrix can be derived from the quantum trajectory equation (stochastic master equation) by simply taking a "partial" average over the all possible outcomes of the measurement. If a full ensemble average is taken, the traditional (unconditional) master equation of reduced density matrix is then obtained. This unified picture, in terms of averaging over (tracing out) different amount of detection records (detector states), for these seemingly different approaches reported in the literature is particularly easy to understand using our formalism. To further demonstrate this connection, we analyze an important ensemble quantity for an initial qubit state readout experiment, $P(N,t)$, the probability distribution of finding N electron that have tunneled through the PC barrier(s) in time t. The simulation results of $P(N,t)$ using 10000 quantum trajectories and corresponding measurement records are, as expected, in very good agreement with those obtained from the Fourier analysis of the "partially" reduced density matrix. However, the quantum trajectory approach provides more information and more physical insights into the ensemble and time averaged quantity $P(N,t)$. Each quantum trajectory resembles a single history of the qubit state in a single run of the continuous measurement experiment. We finally discuss, in this approach, the possibility of reading out the state of the qubit system in a single-shot experiment.

Keywords: charge qubit, quantum measurement, single-shot readout, quantum trajectories, stochastic Schödinger equation
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1. Introduction

In condensed matter physics measurements are usually performed in two ways. Either a measurement is performed on an ensemble of quantum systems. Or for individual quantum system, only one measurement is made in each experimental run, but the same experiments are repeated many times. In the above two cases, only ensemble average properties are studied. Hence, the traditional quantum open system approach, namely the (unconditional) master equation of the reduced density matrix obtained by tracing out the environmental (measurement apparatus) degrees of freedom, is sufficient to describe the ensemble average

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time evolution of the system of interest. However, when a sequence of measurements in an experiment is made upon a single quantum system, the system state conditioned upon the measurement result is important when its subsequent time evolution is concerned. So if one wants to map out the system state evolution conditioned on the continuous in time measurement results for a single quantum system (i.e. quantum trajectory of the measured system), the conditional, stochastic Schrödinger (master) equation or quantum trajectory theory [1] should be employed. This is indeed sometimes the case for the readout process of a single quantum bit (qubit) in various proposed solid-state quantum computer architectures [2, 3, 4, 5, 6].

The theory of quantum trajectories or stochastic Schrödinger equations [1] has been developed in last ten years mainly in the quantum optics community to describe open quantum system subject to continuous quantum measurements. But it was introduced to the context of solid-state mesoscopics only recently [7, 8, 9, 10] to discuss a two-state quantum system (a charge qubit), an electron coherently tunneling between two coupled quantum dots (CQD's), interacting with an environment (a detector), a low transparency point contact (PC) or tunnel junction [11]. One of the main purposes of the paper is to provide a unified picture for the quantum trajectory approach and the master equation approach of reduced and "partially" reduced density matrix. Here we refer to the master equation approach of the "partially" reduced density matrix as the approach recently developed in Refs. [11, 12, 13, 14, 15], mainly for the purpose of reading out an initial qubit state. In that approach, one takes a trace over environmental (detector) microscopic degrees of the freedom but keeps track of the number of electrons, $N(t)$, that have tunneled through the PC barrier during time t in the reduced density matrix. In Refs.[7, 8, 9, 10], the difference and connection between the (conditional) quantum trajectory approach and the (unconditional) master equation approach of reduced density matrix were emphasized. But, to our knowledge, no direct connection between the quantum trajectory approach and the master equation approach of the "partially" reduced density matrix has been formally established and reported in the literature so far. One of the reasons may be that the methods used to derive the stochastic quantum trajectory equations and master equation of the "partially" reduced density matrix are so different. The master equation (rate equations) of the "partially" reduced density matrix for the CQD qubit system measured by a PC detector was derived in Ref. [11] from the so-called many-body Schrödinger equation. While it was derived in Refs. [12, 14] and [15], by means of the diagrammatic technique in the Keldysh forward and backward in time contour, for a Cooper-pair-box charge qubit coupled capacitively to a single-electron transistor detector. On the other hand, the prescriptions for the quantum trajectories (or conditional, stochastic system state evolution) in Refs. [7] and [8] were based on the Bayesian formalism, while they were derived in Refs. [9] and [10] starting from unconditional master equation.

However, in this paper we provide a direct connection between the quantum trajectory approach and the "partially" reduced density matrix approach. We show that the master equation of the "partially" reduced density can be obtained for the CQD/PC model by taking a "partial" average on the conditional, stochastic master equation of the CQD qubit system density matrix over the possible outcomes of the measurements of the PC detectors. The procedure to achieve this connection is particularly easy to understand using our formalism. In fact, the quantum trajectories provide us with the most (all) information as far as the system evolution is concerned. When more and more information of the detector states (detection records) is traced (averaged) out from the quantum trajectories, we obtain first the "partially" reduced density matrix and then the reduced density matrix. This provides a unified picture for these seemingly different approaches reported in the literature.

To further demonstrate the connection between the quantum trajectory approach and the "partially" reduced density matrix approach, we consider an experiment of reading out an initial CQD qubit state. We analyze an important ensemble quantity in the readout experiment, $P(N,t)$, the probability distribution of finding N electron that have tunneled through the PC in time t. The quantity $P(N,t)$ was discussed in terms of the "partially" reduced density matrix in Ref. [13] for a special case of the CQD/PC model, and it was analyzed in Refs. [12] and [15] for a Cooper-pair-box charge qubit measured by a single-electron transistor. Here we present a detailed analysis of $P(N,t)$ for the CQD/PC model using both of the "partially" reduced density matrix and quantum trajectory approaches. We show that the simulation results of $P(N,t)$ using 10000 quantum trajectories and measurement records are, as expected, in very good agreement with those obtained from the Fourier analysis of the "partially" reduced density matrix. However, each single quantum trajectory and corresponding measurement record mimics a single run of the measurement experiment. Hence, even though we are interested in the time or ensemble average properties [such as $P(N,t)$] rather than conditional dynamics, the possible individual realizations of quantum trajectories and their corresponding measurement records do provide insight into, and aid in the interpretation of, the average properties. This appealing feature of the quantum trajectories is also illustrated in this paper.

As just mentioned above, each single quantum trajectory and corresponding measurement record resembles a possible single run of the continuous in time measurement experiment. We finally discuss, in this approach, the possibility of reading out the qubit state in a single-shot measurement. In particular, we clarify and provide conditions for single-shot readout of an initial charge qubit state. Generally speaking, in order to obtain a confident CQD qubit state readout in the charge state basis, it is better to (a) reduce the coherent coupling between the charge state, (b) increase the interaction with the PC detector and (c) switch on the energy mismatch of the CQD's for the readout measurement.

In Sec. 2, the CQD/PC model is briefly described and the stochastic master equation or quantum trajectory equation is presented. In Sec. 3, it is shown that quantum trajectory theory contains the most (all) information as far as the system evolution is concerned. A close connection between the quantum trajectory approach and master equation approach of the reduced or "partially" reduced density matrix is established so that a unified picture for these seemingly different approaches can be obtained. Then we demonstrate, further, this connection by considering an experiment of reading out the initial CQD qubit state in Sec. 4. We show that the quantum trajectory approach not only give the simulation results of $P(N,t)$ in an good agreement with those obtained from the "partially" reduced density matrix approach, but also provide more physical insight into the ensemble quantity $P(N,t)$. Then in Sec. 5, the issues of different time scales regarding to the quantum trajectory approach, the possibility of reading out the state of the qubit in a single-shot experiment, and the possible experimental implementation of the CQD/PC device are discussed. Finally, a short conclusion is given in Sec. 6.

2. Mesoscopic model and quantum trajectories

The CQD's measured by (interacting with) a low-transparency PC detector (bath) has been extensively studied in Refs. [11, 7, 14, 8, 9, 10]. Basically, when the electron in the CQD system is near the PC (i.e., dot 1 is occupied), the effective energy independent tunneling amplitude of the PC detector changes from $T_{00} \to T_{00} + \chi_{00}$ (see, e.g., Fig. 1 of Ref. [9]). As a consequence, the current through the PC is also modified. This changed current can be detected, and thus a measurement of the location of the electron in the CQD system (qubit) is effected. The unconditional and conditional master equation (or rate equations for all the reduced-density-matrix elements) for the CQD system (qubit) has been derived and analyzed in Refs. [11, 9] and Refs.[7, 8, 9, 10]. But the connection between the quantum trajectory (conditional, stochastic master equation) approach and the master equation approach of the "partially" reduced density matrix has not yet been formally established. It is one of the major purposes of this paper to provide such a connection between them, in terms of averaging over (tracing out) different amount of detection records (detector states).

For completeness, the stochastic quantum-jump master equation of the density matrix operator, conditioned on the observed event in the case of efficient measurement in time dt in Ref. [9, 10] is given here again as:

$$\begin{aligned} d\rho_c(t) \quad &= \quad dN_c(t)\left[\frac{\mathcal{J}[\mathcal{T}+\mathcal{X}n_1]}{\mathcal{P}_{1c}(t)} - 1\right]\rho_c(t) \\ &\quad + dt\left\{-\mathcal{A}[\mathcal{T}+\mathcal{X}n_1]\rho_c(t) + \mathcal{P}_{1c}(t)\rho_c(t) - \frac{i}{\hbar}[\mathcal{H}_{CQD}, \rho_c(t)]\right\}. \end{aligned} \tag{1}$$

The subscript c indicates that the quantity to which it is attached is conditioned on previous measurement results. $d\rho_c(t) = \rho_c(t+dt) - \rho_c(t)$ represents the change of the conditional density matrix operator in time dt. $\mathcal{H}_{\rm CQD} = \hbar[\omega_1 c_1^\dagger c_1 + \omega_2 c_2^\dagger c_2 + \Omega(c_1^\dagger c_2 + c_2^\dagger c_1)]$ represents the effective tunneling Hamiltonian for the measured CQD system (charge qubit), where c_i ($c_i^\dagger$) and $h\omega_i$ represent the electron annihilation (creation) operator and energy for a single electron state in each dot respectively, and the coherent coupling between these two dots is given by $h\Omega$. The superoperators $\mathcal{J}$ and $\mathcal{A}$ are defined as $\mathcal{J}[B]\rho = B\rho B^\dagger$, and $\mathcal{A}[B]\rho = (B^\dagger B\rho + \rho B^\dagger B)/2$, and $n_i = c_i^\dagger c_i$ is the occupation number operator for dot $i = 1, 2$. The parameters $\mathcal{T}$ and $\mathcal{X}$ are given by $D = |\mathcal{T}|^2 = 2\pi e|T_{00}|^2 g_L g_R V_{\rm sd}/\hbar$, and $D' = |\mathcal{T} + \mathcal{X}|^2 = 2\pi e|T_{00} + \chi_{00}|^2 g_L g_R V_{\rm sd}/\hbar$. Here D and D' are respectively the average electron tunneling rates through the PC barrier without and with the presence of the electron in dot 1 (see, e.g., Fig. 1 of Ref. [9]). The external source-drain bias applied across the PC is given by $eV_{\rm sd} = \mu_L - \mu_R$ (μ_L and μ_R stand for the chemical potentials in the left and right reservoirs respectively), and g_L and g_R are the energy-independent density of states for the left and right reservoirs. In the *quantum-jump* case [9], the stochastic point processes $dN_c(t)$ in Eq. (1) represents the number (either zero or one) of tunneling events seen in an infinitesimal time dt in the PC. If no tunneling electron is detected, the result is *null*, i.e., $dN_c(t) = 0$. If there is *detection* of a tunneling electron in time dt, then $dN_c(t) = 1$. We can think of $dN_c(t)$ in Eq. (1) as the increment in the number of electrons $N_c(t) = \sum dN_c(t)$ passing through PC barrier in the infinitesimal time dt. It is the variable $N_c(t)$, the accumulated electron number transmitted through the PC barrier, which is used in Refs. [11, 12, 15]. Since the nature of detection of electrons tunneling through the PC is stochastic, $dN_c(t)$ should represent a classical random process.

In addition, the ensemble average $E[dN_c(t)]$ of the classical stochastic process $dN_c(t)$ should equal the probability (quantum average) of detecting electrons tunneling through the PC barrier in time dt. Hence, we have (see, e.g., Eqs. (22), (29) and (30) of Ref. [9])

$$E[dN_c(t)] = \mathcal{P}_{1c}(t)dt, \tag{2}$$

where

$$\mathcal{P}_{1c}(t) = D + (D' - D)\langle n_1\rangle_c(t), \tag{3}$$

$\langle n_1\rangle_c(t) = \mathrm{Tr}[n_1\rho_c(t)]$, and $E[Y]$ denotes an ensemble average of a classical stochastic process Y. Formally we can write the current through the PC as $I_c(t) = e\,dN_c(t)/dt$. It is then easy to see that Eqs. (2) and (3) simply state that when dot 1 is empty the average current through the PC is eD. While it is eD' when dot 1 is occupied. In the quantum trajectory approach, the instantaneous system state $\rho_c(t)$ conditions the measured current [see Eqs. (2) and (3)], while the measured current [$dN_c(t)$ in Eq. (1)] conditions the future evolution of the measured system in a self-consistent manner.

3. Connections to master equation approach

We show next that quantum trajectory theory or conditional, stochastic density matrix contains the most (all) information as far as the system evolution is concerned, and the master equation for the reduced or "partially" reduced density matrix simply results when an average or "partial" average is taken on the conditional, stochastic master equation (1) over the possible outcomes of the measurements on the PC detector. This provides a unified picture, in terms of averaging over (tracing out) different amount of detection records (detector states), for these seemingly different approaches reported in the literature.

The traditional, unconditional master equation approach is to define a total system-environment density matrix. Then tracing over the states of the environments (PC detectors) leads to the reduced density matrix for the CQD qubit system alone. The effect of integrating or tracing out the environmental (detector) degrees of the freedom to obtain the reduced density matrix is equivalent to that of completely ignoring or averaging over the results of all measurement records $dN_c(t)$. Hence the unconditional master equation can be obtained as in Refs. [9] and [10] by taking the ensemble average over the observed stochastic process on Eq. (1). By setting $E[dN_c(t)]$ equal to its expected value Eq. (2), and replacing $E[\rho_c(t)] = \rho(t)$ in Eq. (1), we then find the resultant unconditional master equation as

$$\dot\rho(t) = -\frac{i}{\hbar}[\mathcal{H}_{\mathrm{CQD}}, \rho(t)] + \mathcal{J}[\mathcal{T} + \mathcal{X}n_1]\rho(t) - \mathcal{A}[\mathcal{T} + \mathcal{X}n_1]\rho(t) \tag{4}$$

where $\dot\rho(t) = d\rho(t)/dt$. In this approach of master equation of the reduced density matrix, the influence of the PC environments on the CQD system can be analyzed [9, 11]. For example, the total decoherence or dephasing rate due to the PC environment (detector) can be found to be $\Gamma_d = |\mathcal{X}|^2/2$. But this approach or Eq. (4) does not tell us anything about the experimental observed quantity, namely the electron counts or current through the PC. Hence, the PC detector in this approach are treated as a pure environment for the system, rather than a measurement device.

An alternative approach recently developed in Refs. [11, 12, 13, 14, 15] is to take trace over environmental (detector) microscopic degrees of the freedom but keep track the number of electrons, N, that have tunneled through the PC barriers during time t in the reduced density matrix. This allows one to extract information about the quantum state of the qubit, by measuring electron number counts N through the PC in time t. This approach is regarded here as the “partially” reduced density matrix approach. The master (rate) equation of the “partially” reduced density matrix for the CQD qubit system measured by a PC detector was derived in Refs. [11] and [13] from the so-called many-body Schrödinger equation. While it was derived in Refs. [12, 14] and [15], by means of the diagrammatic technique in the Keldysh forward and backward in time contour, for a Cooper-pair-box charge qubit coupled capacitively to a single-electron transistor detector. Here we show that it can be obtained for the CQD/PC model by simply taking a “partial” average on the conditional, stochastic master equation (1) of the CQD qubit system density matrix over the possible outcomes of the measurements of the PC detectors. For the case of *quantum jumps*, [9] there are two possible measurement outcomes of the PC detector in time dt, namely *null* (no electron detected) and *detection* of an electron passing through the PC barrier. As discussed in Ref. [9, 16] and [17], the effect of the *detection* of an electron passing through the PC barrier in time interval $[t,t+dt)$ on the CQD qubit density matrix is described by the $\mathcal{J}$ superoperator term (see, e.g., Eq. (29) of Ref. [9]). This is why sometimes $\mathcal{J}$ is called a *jump* superoperator. The procedure to take the “partial” average can now be described as follows. First, taking the ensemble average on Eq. (1), we obtain Eq. (4). Then in order to keep track of the number of electrons N that have tunneled, we need to identify the effect of the *jump* superoperator $\mathcal{J}$ term in Eq. (4). If N electrons have tunneled through the PC at time $t + dt$, then the accumulated number of electrons in the drain of the PC at the earlier time t, due to the contribution of the *jump* term of the PC, should be $(N - 1)$. Hence, after writing out the number dependence N or $(N - 1)$ explicitly for the density matrix in Eq. (4), we obtain the master equation for the “partially” reduced density matrix as:

$$\dot{\rho}(N,t) = -\frac{i}{\hbar}[\mathcal{H}_{\rm CQD}, \rho(N,t)] + \mathcal{J}[\mathcal{T}_u + \mathcal{X}_u n_1]\rho(N-1,t) - \mathcal{A}[\mathcal{T}_u + \mathcal{X}_u n_1]\rho(N,t) \quad (5)$$

Note that the index $(N - 1)$ appearing in the *jump* superoperator $\mathcal{J}$ terms of Eq. (5). If the sum over all possible values of N is taken on the “partially” reduced density matrix [i.e., tracing out the detector states completely, $\rho(t) = \sum_N \rho(N,t)$], Eq. (5) then reduces to Eq. (4). This procedure of reducing Eq. (1) to Eq. (5) and then to Eq. (4) by tracing out more and more available detector information is one of the main results of the paper. This procedure is particularly simple if the master equations are expressed in a form in terms of superoperators $\mathcal{J}$ and $\mathcal{A}$, and the effect of the *jump* superoperator $\mathcal{J}$ term is identified.

In order to see our approach does reproduce the result in Refs. [11] and [13], we consider the case of real tunneling amplitudes for the PC. In this case the relative phase between the two tunneling amplitudes $\theta = \pi$. This is because the presence of the electron of the CQD system near the PC detector raises the effective tunneling barrier of the detector, so the average electron tunneling rate through the PC, $D' = |\mathcal{T} + \mathcal{X}|^2 < D = |\mathcal{T}|^2$. After evaluating Eq. (5) in the logical qubit charge state $|a\rangle$ and $|b\rangle$ (i.e., perfect localization state of the charge in dot 1 and dot 2, respectively), we obtain the rate equations as:

$$\dot{\rho}_{aa}(N,t) = i\Omega[\rho_{ab}(N,t) - \rho_{ba}(N,t)] + |\mathcal{T} + \mathcal{X}|^2\rho_{aa}(N-1,t) - |\mathcal{T} + \mathcal{X}|^2\rho_{aa}(N,t)\ , \quad (6)$$

$$\dot{\rho}_{bb}(N,t) = i\Omega[\rho_{ba}(N,t) - \rho_{ab}(N,t)] + |\mathcal{T}|^2\rho_{bb}(N-1,t) - |\mathcal{T}|^2\rho_{bb}(N,t)\ , \quad (7)$$

$$\begin{aligned}\dot{\rho}_{ab}(N,t) &= i\mathcal{E}\rho_{ab}(N,t) + i\Omega[\rho_{aa}(N,t) - \rho_{bb}(N,t)] + (|\mathcal{T}|\ |\mathcal{T}+\mathcal{X}|)\rho_{ab}(N-1,t) \\ &\quad -\frac{1}{2}(|\mathcal{T}+\mathcal{X}|^2 + |\mathcal{T}|^2)\rho_{ab}(N,t)\,, \end{aligned} \tag{8}$$

where $\rho_{ij} = \langle i|\rho|j\rangle$ with $i, j = a, b$ refer to the qubit charge basis, and $\hbar\mathcal{E} = \hbar(\omega_2 - \omega_1)$ is the energy mismatch between the two dots. By setting $|\mathcal{T}|^2 = D$, $|\mathcal{T}+\mathcal{X}|^2 = D'$ and $(|\mathcal{T}|\ |\mathcal{T}+\mathcal{X}|) = \sqrt{D\,D'}$, we find that Eqs. (6)–(8) are the same as Eq.(3.3) of Ref. [11]. While the "partially" reduced density matrix approach provides us with information about the number of accumulated electrons passing through the PC, it is still in an ensemble and time average sense. In other words, the system dynamics is still deterministic in this approach. Hence, it cannot describe the conditional dynamics of the CQD qubit system in a single realization of continuous measurements, which reflects the stochastic nature of electrons tunneling through the PC barrier.

On the other hand, in the quantum trajectory or stochastic Schrödinger equation approach, no average or trace over the bath states is taken as far as the system evolution is concerned. Instead, repeated continuous in time measurements are made, and the measurement results are recorded. So in this approach, we are propagating *in parallel* the information of a conditioned (stochastic) state evolution $|\psi_c(t)\rangle$ [or equivalently a conditioned density matrix evolution $\rho_c(t)$] and a detection record $dN_c(t)$ in a single run of a continuous measurement process. The stochastic element in the quantum trajectory corresponds exactly to the consequence of the random outcomes of the detection record.

In summary, the master equations of the reduced or "partially" reduced density matrix can be obtained as a result of taking an ensemble average or partial average over the possible measurement records in the quantum trajectory approach. This procedure of reducing Eq. (1) to Eq. (5) and then to Eq. (4), by averaging over (tracing out) more and more information of the detection records (detector states), provides us with a unified picture for these seemingly different approaches. If only one measurement value is recorded in each run of experiments (for example, the number of electrons N that have tunneled in time t) and ensemble average properties [for example, $P(N,t)$, see Sec.] are studied over many repeated experiments, the quantum trajectory approach will give the same result as the master equation approach of the reduced or "partially" reduced density matrix. However, more physical insights in the interpretations of the ensemble average properties can be gained in terms of different realizations of quantum trajectories and their corresponding measurement records. We will discuss this appealing feature of the quantum trajectory approach in next section by considering an example of a readout measurement experiment of the initial CQD qubit state.

4. Reading out the initial qubit state

To read out the initial qubit state $\rho_{aa}(0) = |a|^2$ and $\rho_{bb}(0) = |b|^2 = 1 - |a|^2$, we may define, as in Refs. [12, 14] and [15], the probability distribution of finding N electron that have tunneled during time t:

$$P(N,t) = \mathrm{Tr}_S[\rho(N,t)], \tag{9}$$

where Tr_S means tracing the density matrix over the degrees of freedom of the qubit system. The quantity $P(N,t)$ has been analyzed in terms of the "partially" reduced density matrix in Refs. [12, 14] and [15] for a Cooper-pair-box charge qubit coupled capacitively to a single-electron transistor. Here, to demonstrate the connection between quantum trajectory and "partially" reduced density matrix approaches, we present the case for the CQD/PC model first using the same method and then the quantum trajectory approach.

4.1. *Partially reduced density matrix approach*

The quantity $P(N,t)$, defined in Eq. (9) in the CQD charge basis, can be written as $P(N,t) = \rho_{aa}(N,t) + \rho_{bb}(N,t)$. To obtain the solution of $\rho_{ij}(N,t)$ with $i,j = a,b$ in the "partially" reduced density matrix approach, we can first apply a Fourier transform [12, 13, 14, 15] $\rho_{ij}(k,t) = \sum_N e^{-ikN}\rho_{ij}(N,t)$ to Eqs. (6)–(8) since these equations are translationally invariant in N space. Solving the resultant coupled first-order differential equations for $\rho_{ij}(k,t)$ and then performing a inverse Fourier transform, we obtain the probability distribution $P(N,t) = \rho_{aa}(N,t) + \rho_{bb}(N,t)$. The result is illustrated in Fig. 1. At $t = 0$, no electrons have tunneled so $P(N,0) = \delta_{N,0}$. Then the probability distribution (for parameters in the Zeno regime, discussed later) moves to positive N values with two different velocities (slopes in the N, t space) D' and D, which correspond to the average electron tunneling rates through PC for CQD qubit state being in $|a\rangle$ and $b\rangle$ states, respectively. Simultaneously, the width, $\sqrt{2D't}$ or $\sqrt{2Dt}$, of the respective distribution due to shot noise widens with time. Due to this intrinsic shot noise on the measurement output, the readout of the initial qubit state necessarily needs finite time duration in order to have an acceptable signal-to-noise ratio. So if after some time (larger than the measurement time $t_{\rm m}$ or localization time [9] $t_{\rm loc}$), the two distributions become sufficiently separated and distinguishable, and their weights still correspond closely to the initial values of the diagonal elements of the qubit density matrix in the qubit charge basis, then the initial qubit state probability can be read out. After a longer time (the mixing time $t_{\rm mix}$) due to transitions between the two charge states, taken into account the back action of the measurements on the qubit, the valley of the two distributions would fill up and a single broad plateau would develop. Hence there is an optimal time window $t_{\rm m} \ll t \ll t_{\rm mix}$ for which a confident readout measurement can be performed.

Of course, the measurement of the PC, which results in a back action on the qubit system, will influence the qubit system evolution. For fixed $\mathcal{E}$, $\mathcal{T}$, and Ω, increasing the value of $\mathcal{X} = \sqrt{2\Gamma_d}$, we increase the interaction with the PC. As shown for typical conditional evolutions or quantum trajectories for symmetric (i.e., $\mathcal{E} = 0$) CQD's in Fig. 4 of Ref. [10], the period of coherent oscillations between the two CQD qubit states increases with increasing (Γ_d/Ω), while the time of a transition (switching time) decreases. Now in order to read out the initial CQD qubit state, we want the qubit during the detection process to stay in one of its charge states much longer than $t_{\rm m} \sim t_{\rm loc}$, but much smaller than the average time between state-changing transitions (the mixing time $t_{\rm mix}$), i.e., $t_{\rm m} \ll t \ll t_{\rm mix}$. It was shown [9, 12] that in the regime of small (Ω/Γ_d) ratio [9, 12], the measurement (localization) time $t_{\rm m} \sim t_{\rm loc} \approx (1/\Gamma_d)$ is much smaller than the mixing time $t_{\rm mix} \approx (\Gamma_d^2 + \mathcal{E}^2)/(4\,\Omega^2\,\Gamma_d)$. This is the (Zeno) regime that we will consider. Figure 2 shows the probability distributions $P(N,t)$ of symmetric CQD's for a fixed value of Γ_d but with different ratios of (Ω/Γ_d). The detection time intervals are up to $t = 10/\Gamma_d$, or $t = 20/\Gamma_d$. We can see from Fig. 2(a)-(d) that for smaller ratio of $(\Omega/\Gamma_d) = 0, 0.05$, the distribution splits into two and their weights corresponds closely to the initial values of the diagonal elements of $\rho_{aa}(0) = 0.75$ and $\rho_{bb}(0) = 0.25$ [the latter is supported by the fact that the ensemble average $\rho_{aa}(t)$ deviates from $\rho_{aa}(0) = 0.75$ only a little bit for all times within $t = 20/\Gamma_d$]. In this case, a good quantum readout measurement

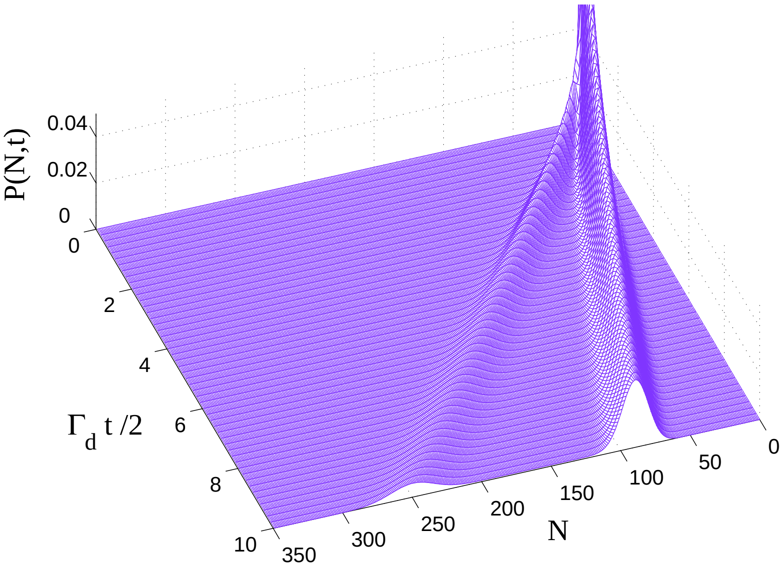

Fig. 1. Probability distribution of finding N electrons that have tunneled through the PC during time t, $P(N,t)$, plotted vs t and N with the initial qubit state being $\sqrt{3/4}|a\rangle + (1/2)|b\rangle$. Other parameters are: $\mathcal{E} = 0$, $\theta = \pi$, $|\mathcal{T}|^2 = 25|\mathcal{X}|^2/4 = 25\Gamma_d/2 = 250\Omega$, and time is in unit of $2(\Gamma_d)^{-1}$. In the plot, in order to have a clear contrast for the distribution in the specified time region, the range of $P(N,t)$ values is limited to be smaller than 0.05. Note that $P(N,0) = \delta_{N,0}$.

for the CQD qubit initial state can be performed by repeatedly measuring $N(t)$ in these time durations. For larger ratio of $\Omega/\Gamma_d = 0.2$, the valley between the two-peak structure is already partially filled for $t = 10/\Gamma_d$, as shown in Fig. 2(e). After a longer time at $t = 20/\Gamma_d$, the two peaks evolve into a broad plateau (see Fig. 2(f)). This indicates that (on average) mixing transitions take place on the same time scale as that of peak separation. Therefore, no good quantum measurement of the qubit initial state can be performed in this case.

The effect of energy mismatch $\mathcal{E}$ of asymmetric CQD's on the $P(N,t)$ is that the mixing rate $\gamma_{\rm mix} = t_{\rm mix}^{-1}$ is smaller than that of symmetric CQD's. This can be observed in Fig. 3. The weight in the overlap region of $P(N,t)$ in Fig. 3(b) for asymmetric CQD's is smaller than that in Fig. 3(a) for symmetric CQD's up to the same detection time interval $t = 20/\Gamma_d$. This suggests that if the CQD qubit undergoes additional coherent evolution [i.e., $\Omega \neq 0$, the source of mixing behavior in $P(N,t)$] during the readout process of its initial state, it is better to switch on the energy mismatch $\mathcal{E} \neq 0$ of the CQD's right before the readout measurement as in the case of Cooper-pair-box charge qubit in Ref. [18].

4.2. *Quantum trajectory approach*

In the approach of quantum trajectories, more physical insights can, however, be gained. Each single trajectory resembles a single history of the system state in a single run of a continuous in time measurement experiment. We can therefore use quantum trajectories and their corresponding detection records to simulate measurement experiments on a single

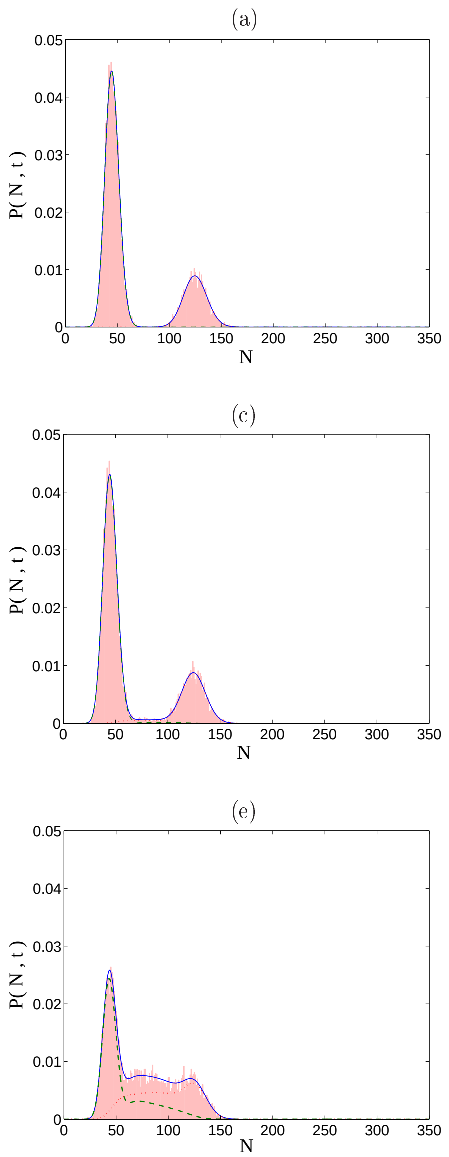

Fig. 2. Probability distribution $P(N,t)$ for a fixed value of Γ_d but with different ratios of $(\Omega/\Gamma_d) = 0, 0.05, 0.2$. The initial amplitudes of the qubit state are $\rho_{aa}(0) = |a|^2 = 0.75$ and $\rho_{bb}(0) = |b|^2 = 0.25$. The ratios of (Ω/Γ_d) for (a) and (b) is 0, for (c) and (d) is 0.05, and for (e) and (f) is 0.2. The detection time (in unit of $2(\Gamma_d)^{-1}$) for (a), (c) and (e) is up to $t = 10/\Gamma_d$, and for (b), (d) and (f) is to $t = 20/\Gamma_d$. Other parameters are: $\mathcal{E} = 0$, $\theta = \pi$, $|\mathcal{T}|^2 = 25|\mathcal{X}|^2/4 = 25\Gamma_d/2$. The quantities $\rho_{aa}(N,t)$, $\rho_{bb}(N,t)$ and $P(N,t) = \rho_{aa}(N,t) + \rho_{bb}(N,t)$ obtained from the Fourier analysis are represented in each plot as dashed, dotted, and solid line, respectively. The shaded region in each plot is the simulation result of using 10000 quantum trajectories and their corresponding detection records. The expected average number of electron counts Dt and $D't$ of the two peaks for (a), (c) and (e) with detection time up to $t = 10/\Gamma_d$ is 45 and 125 respectively, and it is 90 and 250 for (b), (d) and (f) with $t = 20/\Gamma_d$.

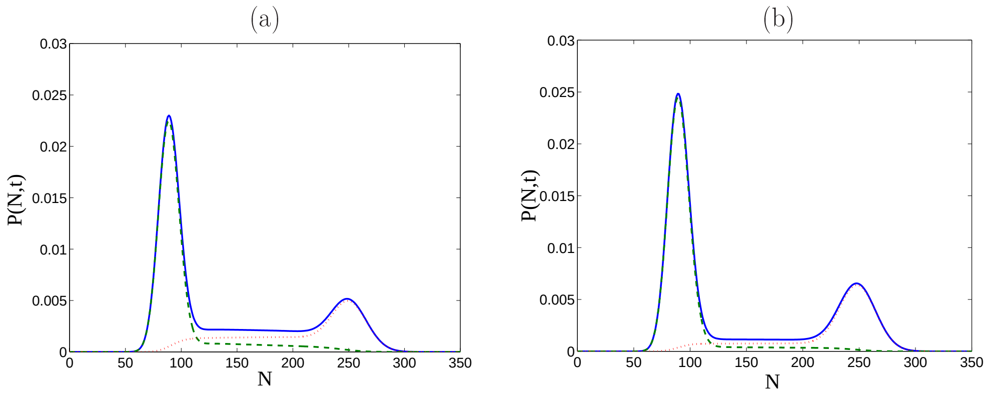

Fig. 3. The quantities $\rho_{aa}(N,t)$, $\rho_{bb}(N,t)$ and $P(N,t) = \rho_{aa}(N,t) + \rho_{bb}(N,t)$ obtained from the Fourier analysis of the "partially" reduced density matrix, and represented as dashed, dotted, and solid line, respectively, in each plot for detection time interval $t = 20/\Gamma_d$: (a) symmetric CQD's with $\mathcal{E} = 0$ and (b) asymmetric CQD's with $\mathcal{E} = \Gamma_d$. Other parameters are: $\theta = \pi$, $|\mathcal{T}|^2 = 25|\mathcal{X}|^2/4 = 25\Gamma_d/2 = 125\Omega$.

quantum system. For example, the quantity $P(N,t)$ defined in Eq. (9) can be simulated by building the histogram of the accumulated number of electrons $N_c(t) = \sum dN_c(t)$ up to time t for many realizations of the detection records (generated together with their corresponding quantum trajectories), and then normalizing the distribution to one. Figure 2 shows that the probability distributions $P(N,t)$, each constructed from 10000 possible realizations (in shades), are, as expected, in very good agreement with those obtained from the Fourier analysis of the partially reduced density matrix (in solid lines).

Note that the time scales $t_{\rm m}$ (or $t_{\rm loc}$) and $t_{\rm mix}$ are basically statistical ensemble average quantities. We can gain more physical insight, for example, on the occurrence of the mixing, i.e., the occurrence of the overlap region between the two distributions in $P(N,t)$. The fundamental origin of the mixing is due to the presence of the non-zero coherent coupling term between two preferred basis states of the qubit in the Hamiltonian. In our case, it is the non-zero Ω term which causes transitions between the two qubit charge states. For $\Omega = 0$, the conditional state becomes localized in one of the two dots, i.e., either $\rho^c_{aa}(t) = 1$ or $\rho^c_{aa}(t) = 0$ [see typical quantum trajectories in Fig. 4(a) and (b)]. Hence no overlap between the two distributions in $P(N,t)$ occurs for $t > t_{\rm m}$ [see Fig. 2(a) and (b)]. Besides, it can be easily shown that when $\Omega = 0$ the ensemble averaged $\rho_{aa}(t)$ and $\rho_{bb}(t)$ are constants of motion, i.e., $\rho_{aa}(t) = \rho_{aa}(0)$ and $\rho_{bb}(t) = \rho_{bb}(0)$. However, when $\Omega \neq 0$, the overlap between the two distributions develops, and the weight in the overlap region increases with increasing (Ω/Γ_d) ratio as illustrated in Fig. 2. The typical quantum trajectories whose accumulated electron detection number falls in the peak regions are shown in Fig. 4(a)-(c). Those in overlap regions are shown in Fig. 4(d)-(f). Note that the accumulated detected electron number of the first half of the time evolution in Fig. 4(d) (i.e., for detection time up to $t = 10/\Gamma_d$), actually falls in the peak region of $\rho_{aa}(N,t)$. Within the readout time t, if the qubit changes its state, then its accumulated electron number is very probably falling into the valley region between the peaks of the two distributions. For fixed Γ_d, as the values of Ω increase, the chance for the

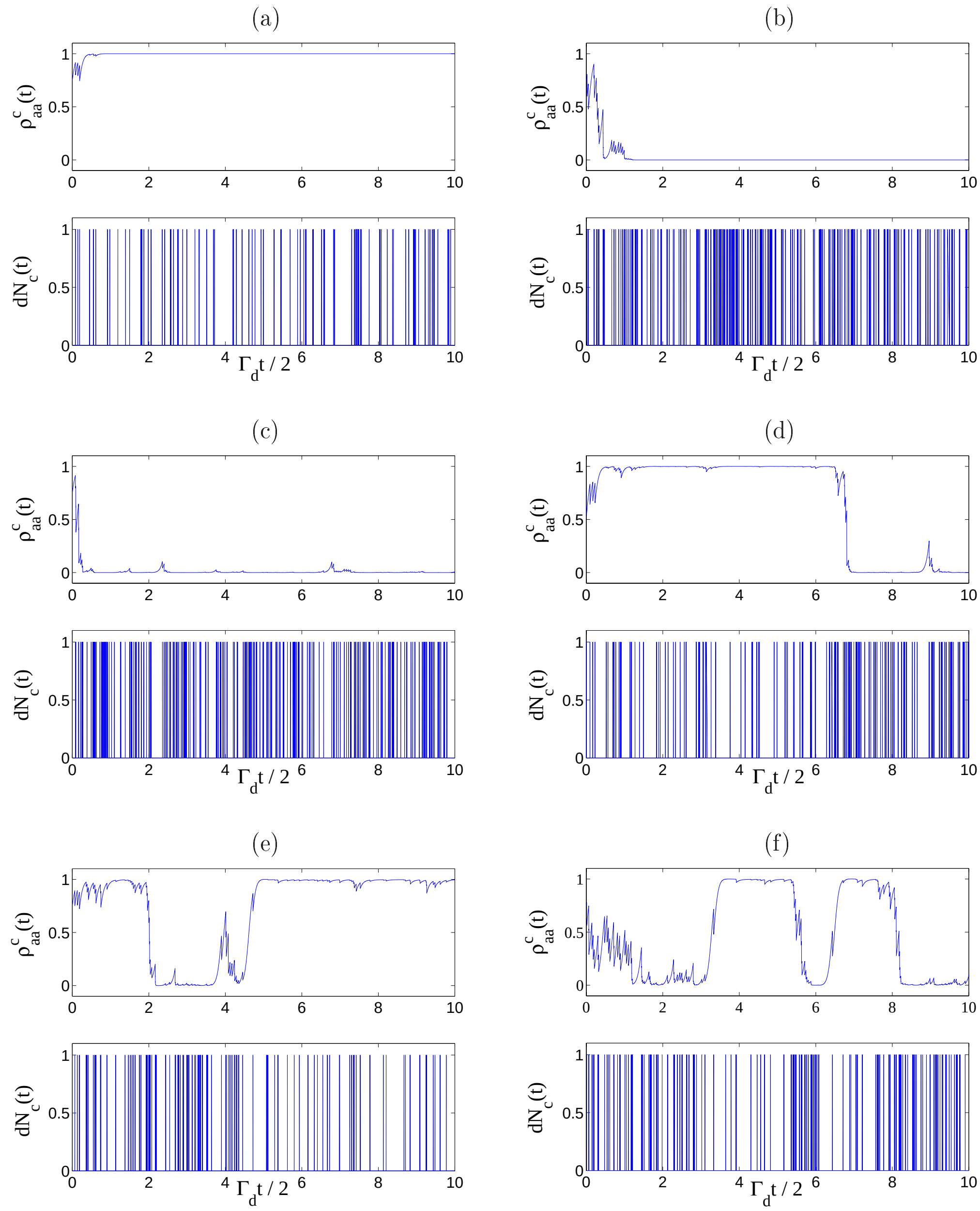

Fig. 4. Typical quantum trajectories and corresponding detection records for $P(N,t)$ shown in Fig. 2. The ratios of (Ω/Γ_d) for (a) and (b) is 0, for (c) and (d) is 0.05, and for (e) and (f) is 0.2. Other parameters are the same as in Fig. 2. The number of electron $N_c(t) = \sum dN_c(t)$ that have tunneled through PC up to time $t = 10/\Gamma_d$ and $t = 20/\Gamma_d$ is respectively 43 and 93 in (a), 132 and 237 in (b), 129 and 248 in (c), 50 and 157 in (d), 83 and 122 in (e), and 75 and 168 in (f).

state-changing transitions to happen in each individual realization of the quantum trajectory increases. If the ratio (Ω/Γ_d) is increased further, the number of state-changing transitions within the same readout time interval also increase [see, for example, Fig. 4(e) and (f)]. As a result, more weight of the distributions move to fill the valley and the two peaks transform into a broad plateau [see Fig. 2(e) and (f)]. These typical quantum trajectories indeed help us understand the occurrence of the mixing.

In summary, many different individual realizations of quantum trajectories and measurement records provide more information and do give physical insight into, and aid in the interpretation of, the ensemble average properties, for example, the quantity $P(N,t)$ discussed here.

5. Discussions

5.1. *Different time scales*

Next we discuss briefly the relationship between different time scales involved: (a) the required detection time interval dt for the quantum trajectory equations to be valid, (b) the actual detection time interval, $(1/\Delta\omega)$, where $\Delta\omega$ is the bandwidth of the measurement device. (c) the PC bath correlation time $\tau_B \approx \hbar/eV_{\rm sd}$, (d) the typical system evolution or response time $t_{\rm res} = \min(1/\Omega, 1/\mathcal{E}, 1/\Gamma_d)$, where $\tau_d = 1/\Gamma_d$ is known as the decoherence (dephasing) time in the reduced density matrix approach, (e) the typical time $\min(1/D, 1/D')$ for an individual electron tunneling through the PC, (f) the measurement time $t_{\rm m}$ for being able to distinguish the two probability distributions in $P(N,t)$, (g) the mixing time $t_{\rm mix}$ after which the two distributions in $P(N,t)$ transform into a single, broad distribution so that the information about the initial qubit state is lost.

The detection time interval dt for the quantum trajectory approach should be much smaller than $t_{\rm res}$ so that effectively the system is under continuous measurement. But it cannot be arbitrarily small. This is because we should allow the PC baths sufficient time to relax. If not, then the buildup of PC bath electron population in other number (Fock) state will be suppressed by the continuous measurement, and one would encounter the situation analogous to the quantum Zeno effect. For the Born-Markov quantum-jump stochastic master equation[9, 10] to be valid, we should require $\tau_B \ll dt = (1/\Delta\omega) \leq \min(1/D, 1/D') \ll t_{\rm res}$, and for the quantum-diffusive case [7, 9, 10], $\tau_B \ll \min(1/D, 1/D') \ll dt = (1/\Delta\omega) \ll t_{\rm res}$.

5.2. *Single-shot readout*

One of the interesting questions in the measurement experiment is whether it is possible to read out the qubit state in a single-shot measurement. We shall discuss this question using quantum trajectory approach for the following cases: (I) to read out the qubit state, $\rho_{aa}(t)$, at the detection time t, (II) to read out the initial qubit state, $\rho_{aa}(0)$, (III) to read out the two possible charge states of a classical charge signal undergoing random jumps, or to observe the incoherent charge transfer between two quantum dots. It was stated [19, 20] that if the ratio between the mixing time and the measurement time is much larger than unity $(t_{\rm mix}/t_{\rm m}) \gg 1$, it should be possible to read out the qubit state in a single measurement. Since the time scales $t_{\rm m}$ and $t_{\rm mix}$ are basically statistical ensemble average quantities, strictly speaking the single-shot readout is possible for case I only when $(t_{\rm mix}/t_{\rm m}) \to \infty$. For case II, it is possible provided that $(t_{\rm mix}/t_{\rm m}) \to \infty$, and $\rho_{aa}(0) = 1$ or $\rho_{aa}(0) = 0$, i.e., the initial qubit state is in one of the two perfectly localized states. In practice, provided certain required measurement fidelity threshold is met, one may relax the condition $(t_{\rm mix}/t_{\rm m}) \to \infty$ for cases I and II to $(t_{\rm mix}/t_{\rm m}) \gg 1$. For case III, it is possible as long as the largest frequency for the charge state change is smaller than the bandwidth of the detector. We discuss further these three cases in the following.

Case I: If the bandwidth and other time scales satisfy the requirement for the quantum trajectory equations to be valid, then given that the initial state is known, we can, in principle, know the state of the qubit conditioned on the measurement record after each detection time interval in each single run of experiment. If the bandwidth is not large enough $\Delta\omega < (1/t_{\rm res})$, the measurement device would perform a time average over the time interval $(1/\Delta\omega)$, which is larger than the typical system evolution or response time $t_{\rm res}$. In this case, even with the detection record from the measurement device, we lose some information about the changes of the qubit state. The decrease in our knowledge of the qubit state would then lead to decoherence (dephasing) for the qubit state. In other words, the conditional stochastic master equation would not be valid, and we do not know for sure the state of the qubit after each detection time interval.

In the present CQD/PC model, when $\Omega = 0$, the CQD qubit will eventually (typically after $t > t_{\rm m} \sim t_{\rm loc}$) localize in one of the two dots and stay there (fixed points). One can then determine the qubit state, $\rho^c_{aa}(t)$, right after the measurement [see Fig. 4(a) and (b)], based on a single-shot measurement result of $N_c(t)$ falling into one of the two peak regions of $P(N,t)$ [see Fig. 2(a) and (b)] even if the initial qubit state is unknown. This perfect projective readout procedure, however, cannot be applied exactly to the case of non-vanishing Ω. For $\Omega \neq 0$, there is always a chance for CQD qubit not being exactly in the perfectly localized charge states after a single-shot measurement of time $t > t_{\rm m}$. This can be seen from Fig. 4(c)-(f), which show the appearance of wiggling or noisy portions and the possibility of switching or making transitions in $\rho^c_{aa}(t)$ at some times $t > t_{\rm m}$ [i.e., $\rho^c_{aa}(t)$ does not stay and remain *exactly* at the value of 0 or 1]. The reason is that the occupation number operator of dot 1, $n_1 = c_1^\dagger c_1$ (the measured quantity), does not commute with the Ω term in the Hamiltonian describing the free evolution of the qubit. Hence the free evolution, for $\Omega \neq 0$ case, will interfere with the process of projection into the qubit charge states (logical 0 and 1 states) during the readout measurement. Only when $\Omega = 0$, $[n_1(t), n_1(t')] = 0$ in the interaction picture. This condition ensures that in this case, if the qubit is projected into one of the charge states [an eigenstate of $n_1(t_0)$], it remains in this state for all subsequent times. This is referred to as a quantum nondemolition measurement. [21]

In reality, one may regard the single-shot readout to be possible either when the state-changing probability is lower than or the fidelity of the measurement is greater than their respective certain required thresholds. In the present CQD/PC model, the minimal condition to satisfy the requirement might be $t_{\rm mix} \gg t \gg t_{\rm m}$. For $\Omega \neq 0$, we can switch on the energy mismatch $\mathcal{E} \gg \Omega$ so that the eigenstates of the qubit free-evolution Hamiltonian are close to the qubit charge states. In this case, the probability of the qubit making transitions between the two charge states will be greatly reduced. If the coupling between the detector and the qubit $\Gamma_d = |\mathcal{X}|^2/2$ can also be increased, the ratio of $(t_{\rm mix}/t_{\rm m})$ will increase further. As a result, the fidelity of the single-shot readout can be improved significantly.

Case II: To be able to read out the initial qubit state, the bandwidth of the detector must be at least larger than the mixing rate, $\Delta\omega > t_{\rm mix}^{-1}$. As to read out the initial state of the CQD qubit in a single shot, it is possible provided that (a) $(t_{\rm mix}/t_{\rm m}) \to \infty$ and (b) $\rho_{aa}(0) = 1$ or $\rho_{aa}(0) = 0$, i.e., the initial qubit state is in one of the two perfectly localized states. Note that condition (a) for symmetric CQD's ($\mathcal{E} = 0$) is equivalent to $(\Omega/\Gamma_d) \to 0$. It is possible to distinguish the the two distributions in $P(N, t)$ for $(\Omega/\Gamma_d) = 0$ and $(\Omega/\Gamma_d) = 0.05$ cases in Fig. 2(a)-(d). But there is still some overlap between the two distributions in the $(\Omega/\Gamma_d) = 0.05$ case where $(t_{\rm mix}/t_{\rm m}) \approx 25$. If the single-shot measurement result of $N_c(t)$ falls into the overlap region, we will not be able to tell that the result should contribute to the qubit being in dot 1 or dot 2 [see Fig. 2(c) and (d)]. Although the probabilities for the "ambiguous" results to occur (or equivalently the state-changing or switching probabilities) for the $(\Omega/\Gamma_d) = 0.05$ case, in Fig. 2(c) with $t_{\rm mix} : t : t_{\rm m} \approx 25 : 10 : 1$ and in Fig. 2(d) with $t_{\rm mix} : t : t_{\rm m} \approx 25 : 20 : 1$, are rather small, they do not vanish since $\Omega \neq 0$. Again, in practice, one may neglect these probabilities if they are smaller than certain required probability thresholds.

However, even we can obtain a distinct result of $N_c(t)$ falling into one of the two peak regions rather than the valley region [see Fig. 2(a) and (b)] for $\Omega = 0$ case where $(t_{\rm mix}/t_{\rm m}) \to \infty$. But if the initial qubit state is not in one of the two perfectly localized states [i.e., $\rho_{aa}(0) = 1$ or $\rho_{aa}(0) = 0$], we still do not know the initial state based on the single-shot measurement result. This is because in this case repeating the experiment many times to build up the probability distributions with weights corresponding closely to the initial values of the diagonal elements of $\rho_{aa}(0)$ and $\rho_{bb}(0)$ is needed. We should also mention that even though we have determined $\rho_{aa}(0)$ and $\rho_{bb}(0)$ from the readout measurement, we still do not know if the initial state is a mixed or pure state. We have to prepare the same initial state, and perform the readout in different measurement basis, or perform single qubit rotations and then repeat the experiment in the same charge basis of the measurement. [22] After this procedure, we may then be able to distinguish a pure initial state from a mixed one.

Case III: Another situation is the measurement of a charge signal that undergoes periodical classical probabilistic jumps, or random telegraph jumps, between two different charge states. In this case the charge signal is in one of the two different charge states at almost all times except at some very small times where the sudden switches of the charge state take place. As long as the charge signal would stay in one of the two charge states long enough to provide enough signal-to-noise ratio ($t > t_{\rm m}$) and the largest frequency for charge state change (analogous to mixing rate $t_{\rm mix}^{-1}$) is smaller than the bandwidth $\Delta\omega$, the single-shot charge state readout for the charge signal may be possible. This charge signal could be generated by usual electronics device or by a charge defect trap which could capture and emit an electron or other charged particles. This case of classical charge signal is similar to the case of CQD charge qubit in the limit of vanishing (Ω/Γ_d) ratio and with (initial) state in one of the two charge states. We can apply similar reasonings to the case of an incoherent electron charge transfer between two quantum dots when an appropriate bias voltage is applied between them. As long as the bandwidth is larger than the sweeping bias frequency, it is possible for this incoherent charge transfer to be observed in a single-shot measurement.

5.3. *Possible experimental implementation*

Experimentally, coherent coupling between two CQD's has been reported. It has been shown [23, 24] that if the inter-dot tunneling barrier is low and the strength of the coupling of two CQD's is strong, the two CQD's behave as a large single dot in a Coulomb blockade phenomenon. In addition, the energy splitting between bonding and anti-bonding states of two CQD's has been confirmed by microwave absorption measurements [25, 26, 27]. The CQD qubit system discussed here may thus be fabricated as in Ref. [25, 26] and [27] in a AlGaAs/GaAs heterostructure system with external gates to control the energy levels in each dot and tunneling coupling between the two dots. We require strong inner- and inter-dot Coulomb repulsion, so the double dot system can be tuned into a regime [25, 26, 27] such that effectively only one electron can occupy and tunnel between the two dots and only one level in each dot contributes to the dynamics. Typical experimental values [25, 27] for parameters Ω and $\mathcal{E}$ could be, for example, in the range of $0 - 200\mu\text{eV}/\hbar$. It is also possible to fabricate the PC detector [28] in the same heterostructure system. It was reported in a experiment[28] with a "which-path" interferometer that Aharonov-Bohm interference is suppressed owing to the measurement of which path an electron takes through the double-path interferometer. A biased quantum point contact (QPC) with arbitrary transparency located close to a quantum dot, which is built in one of the interferometer's arms, acts as a measurement device. The change of transmission coefficient (probability) of the QPC, which depends on the electron charge state of the quantum dot, can be detected. The typical transmission probability $T_{\rm t}$ and average change in the transmission probability $\Delta T_{\rm t}$ for a low transparency PC at a source-drain voltage of $eV_{sd} = 100\mu\text{eV}$ could be [28] in the range of $0.05 - 0.1$ and $0.013 - 0.028$, respectively. The average tunneling rates D and D' is related to the transmission probability as $D = |\mathcal{T}|^2 = T_{\rm t} eV_{\rm sd}/\hbar$ and $D' = |\mathcal{T} + \mathcal{X}|^2 = (T_{\rm t} - \Delta T_{\rm t})eV_{\rm sd}/\hbar$. Note that the relative phase between $\mathcal{T}$ and $\mathcal{X}$ is $\theta = \pi$.

After a coherent manipulation, the qubit state is ready for readout. We can then apply fast pulses[18] on the external gates to tune, for example, $\Omega = 1\mu\text{eV}/\hbar$, $\mathcal{E} = 20\mu\text{eV}/\hbar$, and $eV_{\rm sd} = 5\text{meV}$. If assuming $T_{\rm t} = 0.05$ and $\Delta T_{\rm t} = 0.028$, we find $D = 250\mu\text{eV}/\hbar$, $D' = 110\mu\text{eV}/\hbar$ and $t_{\rm m}^{-1} \approx \Gamma_d = (|\mathcal{X}|^2/2) \approx (14\mu\text{eV}/\hbar) \gg t_{\rm mix}^{-1} \approx 0.09\mu\text{eV}/\hbar$. It thus appears feasible to build a device to read out the initial qubit state, as discussed in this paper.

However, the difficulty in realizing the quantum trajectories experimentally is mainly due to the requirement of large bandwidth of the detector signal coming out of the cryostat, i.e., $\Delta\omega \gg t_{\rm res}^{-1}$. We have not included the effect of other environments (besides PC detectors) on the qubit dynamics during the measurement process. It is assumed that the relaxation time induced by these environments is much larger than the measurement time, $t_{\rm rel} \gg t_{\rm m}$. But the environment-induced decoherence (dephasing) time may be comparable to $(1/\Gamma_d)$, the detector-induced decoherence time in the reduced density matrix approach. For example, the typical decoherence time for a single Cooper-pair-box charge qubit [18] is on the order of $10 - 100$ ns, which implies that the bandwidth of the detectors should be at least of order of 1GHz.

6. Conclusions

We have discussed the connection between the quantum trajectory approach and master equation approach of the reduced or "partial" reduced density matrix. The quantum trajectory or stochastic Schrödinger equation approach provides us with all the information as far as the system state evolution is concerned. The reduced or "partially" reduced density matrix can be obtained as a result of ensemble average or "partial" average on the conditional, stochastic system density matrix over all possible detection records. Each Quantum trajectory and corresponding detection record, mimics each possible single continuous in time measurement experiment. Their possible individual realizations can provide insight into, and aid in the interpretation of, the ensemble average properties. Especially, we have shown that the probability distribution $P(N,t)$ constructed from 10000 realizations, is, as expected, in very good agreement with that obtained from the Fourier analysis of the "partially" reduced density matrix. In addition, the mixing behavior of $P(N,t)$ is illustrated and explained in terms of different individual realizations of the quantum trajectories and corresponding detection records. These results provide a unified picture for these seemingly different approaches reported in the literature.

We have also discussed the possibility of a single-shot readout of the qubit state. In the present CQD/PC model, for $\mathcal{E}=0$ it may be possible in the limit of vanishing Ω/Γ_d ratio. Generally speaking, in order to obtain a confident CQD qubit state readout in the charge state basis, it is better to (a) reduce the coherent coupling Ω between the charge state, (b) increase the interaction Γ_d with the PC detector and (c) switch on the energy mismatch $\mathcal{E}$ for the readout measurement.

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