A QUBIT-STATE READOUT ANALYSIS: QUANTUM TRAJECTORIES VS. MASTER EQUATION APPROACH

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We demonstrate the connection between the quantum trajectory approach and master equation approach of "partially" reduced density matrix by studying an initial charge qubit state readout experiment by a low-transparency point contact (PC) detector. We analyze an important ensemble quantity for the readout experiment, $P(N,t)$, the probability distribution of finding N electrons that have tunneled through the PC barrier in time t. The simulation results of $P(N,t)$ using 10000 quantum trajectories and corresponding measurement records are, as expected, in very good agreement with those obtained from the Fourier analysis of the "partially" reduced density matrix. However, the quantum trajectory approach provides more information and more physical insight into the ensemble and time averaged quantity $P(N,t)$.

The problem of a charge qubit measured by a detector (or interacting with an environment) has attracted much attention recently.[1,2,3,4] In this paper, we consider the case of an electron coherently tunneling between two coupled quantum dots (CQD's) subject to a measurement using a low-transparency point contact (PC) or tunnel junction.[1,3,4] The double dot system forms a single charge qubit, and the measurement corresponds to a continuous in time readout of the occupancy of the quantum dot. This problem has been discussed using traditional (unconditional) master equation approach, [1] by tracing over the environmental (detector) degrees of freedom to obtain the reduced density matrix for the system alone. In that approach, the influence of the PC reservoirs, decoherence effect for example, on the CQD system can be analyzed. But no information about the experimental observed quantity, namely the electron counts or current through PC, is extracted. An alternative approach, recently developed in Ref. 2 and referred here as the master equation approach of "partially" reduced density matrix, is to take a trace over the environmental (detector) microscopic degrees of freedom but keep track of the number of electrons N that have tunneled through the PC barrier during time t. While this "partially" reduced density matrix approach provides us with information about the initial qubit state by measuring the number of accumulated electrons passing through the PC, it is still in an ensemble and time average sense. In other words, the system dynamics is still deterministic in this approach. Hence, it can not describe the conditional dynamics of the CQD qubit system in a single realization of continuous measurements, which reflects the stochastic nature of electrons tunneling through the PC barrier.

On the other hand, in the quantum trajectory approach [3,4] no average or trace over the environmental (detector) states is taken as far as the system evo-

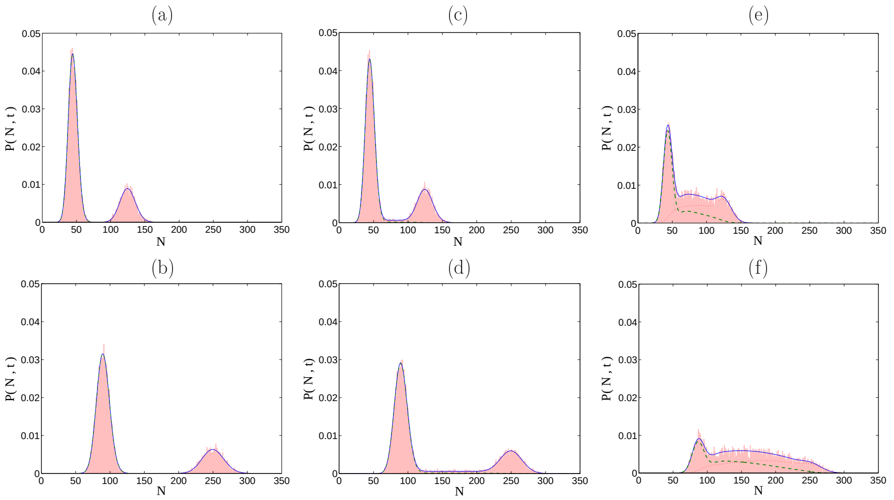

Figure 1. Probability distribution $P(N,t)$ for $\mathcal{E} = 0$ and a fixed value of Γ_d, but with different ratios of $(\Omega/\Gamma_d) = 0$ for (a) and (b), 0.05 for (c) and (d), and 0.2 for (e) and (f). The detection time for (a), (c) and (e) is up to $t = 10/\Gamma_d$, and for (b), (d) and (f) is to $t = 20/\Gamma_d$. The quantities $\rho_{aa}(N,t)$, $\rho_{bb}(N,t)$ and $P(N,t)$ obtained from the Fourier analysis are represented in each plot as dashed, dotted, and solid line, respectively. The shaded region in each plot is the simulation result of 10000 quantum trajectories and their corresponding detection records.

lution is concerned. Instead, repeated continuous in time measurements are made. The stochastic element in the quantum trajectory corresponds exactly to the consequence of the random outcomes of the detection record. Hence we are propagating *in parallel* (self-consistently) the information of a conditioned (stochastic) state evolution [or conditional density matrix evolution $\rho_c(t)$], and a detection record $dN_c(t)$ in a continuous measurement process. In Refs. 3 and 4, the difference and connection between the (conditional) quantum trajectory approach and the (unconditional) master equation approach of reduced density matrix were emphasized. But, to our knowledge, no direct connection between the quantum trajectory approach and the master equation approach of the "partially" reduced density matrix has been formally established and reported until recently. [5] In Ref. 5, we have shown that the master equation for the reduced or "partially" reduced density matrix can be obtained when an average or "partial" average is taken on the conditional, stochastic master equation (quantum trajectory equation) over the possible outcomes of the measurement. This provides a unified picture for these seemingly different approaches in the literature.

Here we demonstrate further the connection between the quantum trajectory approach and the "partially" reduced density matrix approach, by analyzing an important quantity for the CQD initial qubit state readout measurement, $P(N,t)$ the probability distribution [2] of finding N electrons that have tunneled through the PC barrier in time t. Each quantum trajectory resembles a single history of the system state in a single run of a continuous in time measurement experiment. We can, therefore, use many possible realizations of quantum trajectories and their corresponding detection records to simulate measurement experiments on a single quantum system. In our case, the quantity $P(N,t)$ can be simulated by building the histogram of the accumulated number of electrons $N_c(t) = \sum dN_c(t)$ up to time t for many realizations of the detection records, and then normalizing the distribution to one. Figure 1 shows that the probability distributions $P(N,t)$, each constructed from 10000 realizations (in shades), are, as expected, in very good agreement with those obtained from the Fourier analysis of the partially reduced density matrix (in solid lines). It was shown [4,2] that there is an optimal time window $t_{\rm m} \ll t \ll t_{\rm mix}$ for a confident readout measurement can be performed. In the regime of small (Ω/Γ_d) ratio, [4,2] the measurement (localization) time $t_{\rm m} \sim t_{\rm loc} \approx (1/\Gamma_d)$ is much smaller than the mixing time $t_{\rm mix} \approx (\Gamma_d^2 + \mathcal{E}^2)/(4\,\Omega^2\,\Gamma_d)$. Here Ω and $\mathcal{E}$ are the coherent tunneling frequency and energy mismatch between the CQD's respectively, and Γ_d known as decoherence rate in the reduced density matrix approach represents interaction strength between CQD qubit and PC detector. For symmetric ($\mathcal{E}=0$) CQD's, Fig. 1(a)-(d) show that the peak of the $P(N,t)$ distribution splits into two with weights closely corresponding to the initial values of the qubit diagonal density matrix elements of $\rho_{aa}(0) = 0.75$ and $\rho_{bb}(0) = 0.25$, where subscripts a, b refer respectively to the two charge states of the CQD qubit. In Fig. 1(e) and (f) the valley between the two-peak structure of $P(N,t)$ is gradually filled up and a single broad plateau develops [i.e., $t_{\rm m}$ ($t_{\rm loc}$) and $t_{\rm mix}$ are on the same time scales]. Thus, no good initial qubit state readout measurement can be performed in this case.

Note that $t_{\rm m}$ ($t_{\rm loc}$) and $t_{\rm mix}$ are basically statistical ensemble average quantities. Many different individual realizations of quantum trajectories and their corresponding measurement records can provide physical insight into and aid in the interpretation of the ensemble average properties. For example, we can gain more physical insight on the occurrence of the mixing, i.e., the occurrence of the overlap region between the two distributions in $P(N,t)$. The typical quantum trajectories whose accumulated electron detection number falls in the two peak regions are shown in Fig. 2(a)-(c). Those in the overlap regions are shown in Fig. 2(d)-(f). Note that the accumulated electron detection number of the first half of the time evolution in Fig. 2(d) (i.e., for detection time up to $t = 10/\Gamma_d$), actually falls in the peak region of $\rho_{aa}(N,t)$. Within the readout time t, if the qubit changes its state, then its accumulated electron number is very probably falling into the valley region between the peaks of the two distributions. For fixed Γ_d, as the values of Ω increase, the chance for the state-changing transitions to happen in each individual

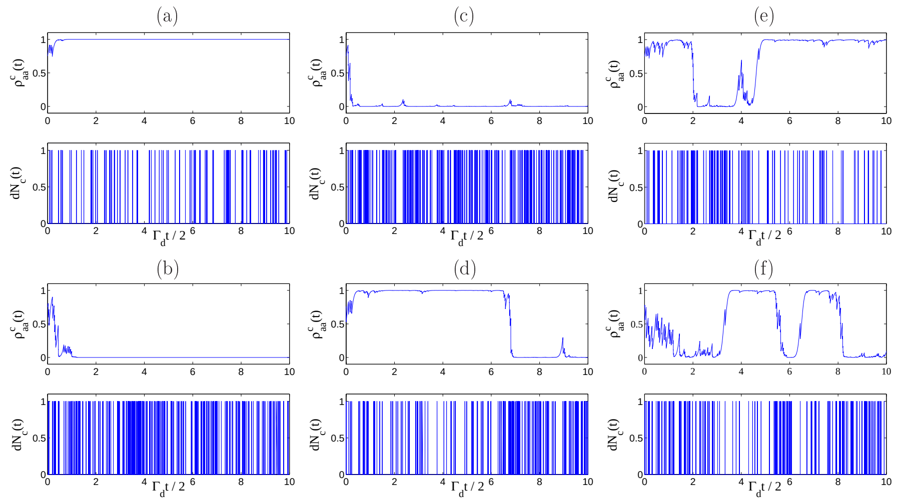

Figure 2. Typical quantum trajectories and corresponding detection records for $P(N,t)$ shown in Fig. 1. The ratios of (Ω/Γ_d) for (a) and (b) is 0, for (c) and (d) is 0.05, and for (e) and (f) is 0.2. Other parameters are the same as in Fig. 1.

realization of quantum trajectories increases. If the ratio (Ω/Γ_d) is increased further, the probability for more numbers of state-changing transitions within the same readout time interval also increases [see, e.g., Fig. 2(e) and (f)]. As a result, more weights of the distributions move to fill the valley and the two peaks transform into a broad plateau [see Fig. 1(e) and (f)].

In summary, we have demonstrated that the quantum trajectory approach not only gives the same results as the master equation approach of the "partially" reduced density matrix, but more appealingly provides more information and aids in the interpretation of the ensemble average properties, such as the occurrence of the mixing behavior in $P(N,t)$ discussed here.

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