

CONDITIONAL QUANTUM EVOLUTION INDUCED BY CONTINUOUS MEASUREMENT FOR A MESOSCOPIC QUBIT

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We consider the problem of an electron tunneling between two coupled quantum dots, a two-state quantum system (qubit), using a low-transparency point contact (PC) or tunnel junction as a detector continually measuring the position of the electron. We focus on the qubit dynamics conditioned on a particular realization of the actual measured current through the PC device. We illustrate the conditional evolutions by numerical simulations. The different behaviors between unconditional and conditional evolutions are demonstrated. The conditional qubit dynamics evolving from quantum jumps to quantum diffusion is presented.

1 Introduction

In condensed matter physics, usually many identical quantum systems are prepared at the same time when a measurement is made upon the systems. For example, in nuclear or electron magnetic resonance experiments, generally an ensemble of systems of nuclei and electrons are probed to obtain the resonance signals. This implies that the measurement result in this case is an average response of the ensemble. On the other hand, for various proposed condensed-matter quantum computer architectures, the need to readout physical properties of a single electronic qubit, such as charge or spin at a single electron level, is demanding. It is particularly important to take account of the decoherence introduced by the measurements on the qubit as well as to understand how the quantum state of the qubit, conditioned on a particular single realization of measurement, evolves in time for the purpose of quantum computing.

We consider, in this paper, the problem of an electron tunneling between two coupled quantum dots (CQDs), a two-state quantum system (qubit), using a low-transparency point contact (PC) or tunnel junction as a detector (environment) continuously measuring the position of the electron. This problem has been extensively studied in Refs. ^{1,2,3,4}. The master equation (or rate equations for all the reduced-density-matrix elements) for the CQD system (qubit) has been derived and analyzed in Refs. ^{1,4}. This (unconditional) master equation is obtained when the results of all measurement records (electron current records in this case) are completely ignored or averaged over, and describes only the ensemble average property for the qubit. However, if a measurement is made on the system and the results are available, the state or density matrix is a conditional state conditioned on the measurement results. Hence the deterministic, unconditional master equation cannot describe the conditional dynamics of the qubit in a single realization of continuous measurements which reflects the stochastic nature of an electron tunneling through the PC barrier.

Korotkov ² has obtained the Langevin rate equations for the CQD system measured by a ideal PC detector in the quantum-diffusive limit⁴. These rate equations describe the random evolution of the density matrix that both conditions, and is conditioned by, the PC detector output. Recently, we ⁴ presented a *quantum trajectory* measurement analysis of the same system. We found that the conditional dynamics of the qubit can be described

by the stochastic Schrödinger equation for the conditioned state vector, provided that the information carried away from the qubit by the PC reservoirs can be recovered by the perfect detection of the measurements. We also analyzed the localization rates at which the qubit becomes localized in one of the two states when the coupling frequency Ω between the states is zero. We showed that the localization time discussed there is slightly different from the measurement time defined in Refs. ³. The mixing rate at which the two possible states of the qubit become mixed when $\Omega \neq 0$ was calculated as well and found in agreement with the result in Ref. ³. In this paper, we focus on the qubit dynamics conditioned on a particular realization of the actual measured current through the PC device. We illustrate the conditional evolutions by numerical simulations. The different behavior between unconditional and conditional evolutions are demonstrated. Furthermore, the conditional qubit dynamics evolving from quantum jumps to quantum diffusion ⁴ is presented.

2 Conditional quantum evolution under continuous measurement

The whole CQD and PC model is described in Refs. ^{1,2,4}. Basically, when the electron in the CQD system is near the PC (i.e., dot 1 is occupied), there is a change in the PC tunneling barrier, which then results in a change in the effective tunneling amplitude. As a consequence, the current through the PC is also modified. This changed current can be detected, and thus a measurement of the location of the electron in the CQD system (qubit) is effected. We describe parameters used in this paper in the following. We denote the logical qubit states as $|a\rangle$ (i.e., dot 1 is occupied) and $|b\rangle$ (i.e., dot 2 is occupied). The coupling and energy mismatch between the qubit states are $\hbar\Omega$, and $\hbar\mathcal{E}$ respectively. $\Gamma_d = |\mathcal{X}|^2/2$ is the decoherence (dephasing) rate generated by the PC reservoirs in the unconditional dynamics. The parameters \mathcal{X} and \mathcal{T} are given by: $|\mathcal{T} + \mathcal{X}|^2 = D'$, and $|\mathcal{T}|^2 = D$, where D' and D are the average electron tunneling rates through the PC barrier with and without the presence of the electron in dot 1 respectively. Physically, the presence of the electron in dot 1 raises the effective tunneling barrier of the PC due to electrostatic repulsion. As a consequence, the effective tunneling amplitude becomes lower, i.e., $D' = |\mathcal{T} + \mathcal{X}|^2 < D = |\mathcal{T}|^2$. This sets a condition on the relative phase θ between \mathcal{X} and \mathcal{T} : $\cos\theta < -|\mathcal{X}|/(2|\mathcal{T}|)$. For simplicity, in this paper we consider the case $\theta = \pi$ as in Refs. ^{1,2}.

The unconditional and conditional master equations of the qubit density matrix, $\rho(t)$, have been obtained and written respectively as sets of coupled stochastic differential equations in terms of the Bloch sphere variables, $x(t)$, $y(t)$, and $z(t)$, in Ref. ⁴. We will use these equations to describe the qubit dynamics. In the Bloch sphere representation,

$$\rho(t) = \frac{1}{2} \begin{pmatrix} 1 + z(t) & x(t) - iy(t) \\ x(t) + iy(t) & 1 - z(t) \end{pmatrix}. \quad (1)$$

Here the variable $z(t)$ represents the population difference between the two dots. Especially, $z(t) = 1$ and $z(t) = -1$ indicate that the electron is localized in dot 2 and dot 1 respectively. The value $z(t) = 0$ corresponds to an equal probability for the electron to be in each dot.

Fig. 1(a) shows the unconditional (ensemble average) time evolution of the population difference $z(t)$ with the initial qubit state being in state $|a\rangle$. The unconditional population difference $z(t)$, rises from -1 , undergoing some oscillations, and then tends towards zero, a steady (maximally mixed) state. On the other hand, the conditional time evolution, conditioned on one possible individual realization of the sequence of measurement results,

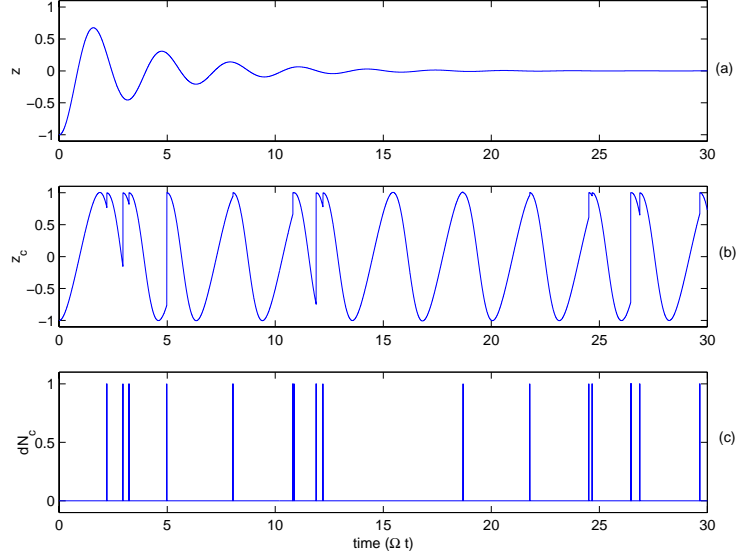


Figure 1. Illustration for different behaviors between unconditional and conditional evolutions. The initial qubit state is $|a\rangle$. The parameters are: $\mathcal{E} = 0$, $\theta = \pi$, $|\mathcal{T}|^2 = |\mathcal{X}|^2 = 2\Gamma_d = \Omega$, and time is in unit of Ω^{-1} . (a) unconditional, ensemble-averaged time evolution of $z(t)$. (b) conditional evolution of $z_c(t)$. (c) randomly distributed moments of detections, which correspond to the quantum jumps in (b).

behaves quite differently. We consider first the situation, where $D' = |\mathcal{T} + \mathcal{X}|^2 = 0$, discussed in Ref. ¹. In this case, due to the electrostatic repulsion generated by the electron, the PC is blocked (no electron passing through) when dot 1 is occupied. As a consequence, whenever there is a detection of an electron tunneling through the PC barrier, the qubit state is collapsed into state $|b\rangle$, i.e., dot 2 is occupied. The conditional, quantum-jump evolution of $z_c(t)$ shown in Fig. 1(b) (using the same parameters and initial condition as in Fig. 1(a)) is quite strikingly different from the unconditional one in that the time evolution is not smooth, but exhibits jumps, and it does not tend towards a steady state. One can see that initially the system starts to undergo a oscillation. As the population difference grows in time so does the probability for an electron tunneling through the PC barrier. This oscillation is then interrupted by the detection of an electron tunneling through the PC barrier, which bring $z_c(t)$ to the value 1, i.e., the qubit state is collapsed into state $|b\rangle$. Then the whole process starts again. The randomly distributed moments of detections, $dN_c(t)$, corresponding to the quantum jumps in Fig. 1(b) is illustrated in Fig. 1(c). The time evolutions in Fig. 1(a) and (b) show little similarity; however, averaging over many individual realizations shown in (b) leads to a closer and closer approximation of the ensemble average in (a).

Next we illustrate how the transition from the quantum-jump picture to the quantum-diffusive picture takes place. In Ref. ⁴, we have seen that the quantum-diffusive equations can be obtained from the quantum-jump description under the assumption of $|\mathcal{T}| \gg |\mathcal{X}|$. In Fig. 2(a)–(d) we plot conditional, quantum-jump evolution of $z_c(t)$ and the corresponding moments of detections $dN_c(t)$, with different $(|\mathcal{T}|/|\mathcal{X}|)$ ratios. Each jump (discontinuity) in the $z_c(t)$ curves corresponds to the detection of an electron through the PC barrier. One

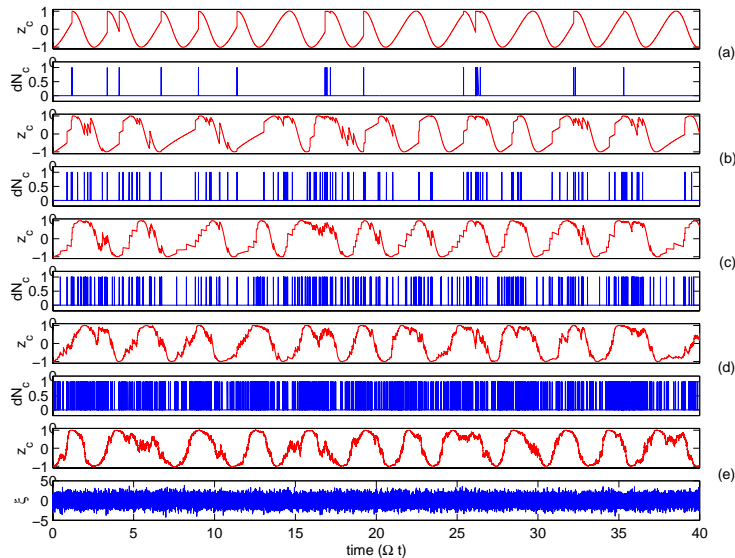


Figure 2. Transition from quantum jumps to quantum diffusion. The initial qubit state is $|a\rangle$. The parameters are: $\mathcal{E} = 0$, $\theta = \pi$, $|\mathcal{X}|^2 = 2\Gamma_d = \Omega$, and time is in unit of Ω^{-1} . (a)–(d) are the quantum-jump, conditional evolutions of $z_c(t)$ and corresponding detection moments with different $|\mathcal{T}|/|\mathcal{X}|$ ratios: (a) 1, (b) 2, (c) 3, (d) 5. With increasing $|\mathcal{T}|/|\mathcal{X}|$ ratio, jumps become more frequent but smaller in amplitude. (e) represents the conditional evolutions of $z_c(t)$ in quantum-diffusive limit. $\xi(t)$, appearing in the expression of current through PC in quantum-diffusive limit, is a Gaussian white noise with zero mean and unit variance.

can clearly observe that with increasing $(|\mathcal{T}|/|\mathcal{X}|)$ ratio, the number of jumps increases. The amplitudes of the jumps of $z_c(t)$, however, decrease from $D' = 0$ with the certainty of the qubit being in state $|b\rangle$ to the case of $(D - D') \ll (D + D')$ with a smaller probability of finding the qubit in state $|b\rangle$. Nevertheless, the population difference $z_c(t)$ always jumps up since $D = |\mathcal{T}|^2 > D' = |\mathcal{T} + \mathcal{X}|^2$. In other words, whenever there is a detection of an electron passing through PC, dot 2 is more likely occupied than dot 1. The case using quantum-diffusive equations is plotted in Fig. 2(e). We can see that the behavior of $z_c(t)$ for $|\mathcal{T}| = 5|\mathcal{X}|$ in the quantum-jump case shown in Fig. 2(d) is already very similar to that of quantum diffusion shown in Fig. 2(e).

In conclusion, we have discussed the qubit dynamics and illustrated the conditional evolutions by numerical simulations. The difference between unconditional and conditional evolutions is demonstrated. Furthermore, the conditional qubit dynamics evolving from quantum jumps to quantum diffusion is presented. HSG is grateful to useful discussions with A. N. Korotkov, H. M. Wiseman and H. B. Sun.

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