MASTER EQUATION APPROACH AND QUANTUM TRAJECTORIES OF A QUANTUM MEASUREMENT PROCESS

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Abstract

We provide a unified view for the master equation (ME) approach and the quantum trajectory (QT) approach to the measurement problem of an electron tunnelling between two coherently coupled quantum dots (CQD's) using a low transparency point contact (PC) or tunnel junction as a detector. We show that the master equation for the reduced or "partially" reduced density matrix can be obtained when an average or "partial" average is taken on the conditional, stochastic density matrix over the possible outcomes of the measurement. Then we simulate $P(N,t)$, the probability of finding N electron that have tunnelled through the PC in time t, using 10000 QT's and measurement records. The simulation results of $P(N,t)$ are in very good agreement with those obtained from the Fourier analysis of the "partially" reduced density matrix.

The theory of QT's (also known as stochastic Schrödinger equation or stochastic ME) has been developed in last ten years mainly in the quantum optics community to describe open quantum system subject to continuous quantum measurements. But it was introduced to the context of solid-state mesoscopics only recently (e.g., Goan *et. al.* 2001 and references therein). The CQD system, considered here, forms a single charge quantum bit (qubit), and the measurement corresponds to a continuous in time readout of the occupancy of the quantum dots. The QT or stochastic quantum-jump ME of the density matrix operator $\rho_c(t)$, conditioned on the observed event in infinitesimal time dt for the case of efficient measurement for the CQD/PC model can be written as (Goan *et. al.* 2001)

$$d\rho_c(t) = dN_c(t)\left[\frac{J[T+Xn_1]}{P_{1c}(t)} - 1\right]\rho_c(t) - dt\{A[T+Xn_1]\rho_c(t) - P_{1c}(t)\rho_c(t) + \frac{i}{\hbar}[H_{CQD}, \rho_c(t)]\}. \tag{1}$$

Here the subscript c indicates that the quantity to which it is attached is conditioned on previous measurement results. $H_{CQD} = \hbar[\omega_1 c_1^+ c_1 + \omega_2 c_2^+ c_2 + \Omega(c_1^+ c_2 + c_2^+ c_1)]$ represents the effective tunnelling Hamiltonian for the measured CQD system. The superoperators J and A are defined as $J[B]\rho = B\rho B^+$ and $A[B]\rho = (B^+ B\rho + \rho B^+ B)/2$, and $n_1 = c_1^+ c_1$ is the occupation number operator for dot 1. The parameters T and X are given by $D = |T|^2$ and $D' = |T+X|^2$, where D and D' are respectively the average electron tunnelling rates through the PC barrier without and with the presence of the electron in dot 1. The stochastic point process $dN_c(t)$ represents the number (either zero or one) of tunnelling events seen in time dt in the PC detector, and its ensemble average is given by

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$$E[dN_c(t)] = P_{1c}(t)dt, \tag{2}$$

where $P_{1c}(t) = D + (D'-D)\langle n_1 \rangle_c(t)$, $\langle n_1 \rangle_c(t) = \mathrm{Tr}[n_1\rho_c(t)]$ and $E[Y]$ denotes an ensemble average of a stochastic process. Formally, we can write the current through the PC barrier as $I_c(t) = edN_c(t)/dt$.

The traditional (unconditional) ME approach for a system interacting with an environment (detector) is to trace over the environmental degrees of freedom to obtain the reduced density matrix for the system alone. The effect of tracing or integrating out the environmental (detector) degree of freedom is equivalent to that of completely ignoring or averaging over the results of all measurement records $dN_c(t)$. Hence the unconditional ME can be obtained by taking the ensemble average over the observed stochastic process on equation (1), by setting $E[dN_c(t)]$ equal to its expected value equation (2), as:

$$\dot{\rho}(t) = -(i/\hbar)[H_{CQD}, \rho(t)] + J[T + Xn_1]\rho(t) - A[T + Xn_1]\rho(t). \tag{3}$$

In this approach, the influence of the PC reservoirs, decoherence effect for example, on the CQD system can be analysed. But no information about the experimental observed quantity, namely the electron counts or current through the PC, is extracted.

An alternative approach (Gurvitz 1997, Makhlin *et. al.* 2001) referred here as the ME approach of "partially" reduced density matrix, is to take trace over environmental (detector) microscopic degrees of the freedom but keep track the number of electrons N that have tunnelled through the PC barrier during time t. The master (rate) equation for the "partially" reduced density matrix for the CQD qubit system measured by a PC detector was derived by Gurvitz (1997) from the so-called many-body Schrödinger equation. While it was derived by Makhlin *et. al.* (2001) by means of the diagrammatic technique in the Keldysh forward and backward in time contour, for a Cooper-pair-box charge qubit coupled capacitively to a single-electron transistor detector. Here we show that it can be simply obtained for the CQD/PC model by taking a "partial" average on the conditional (stochastic) master equation of the density matrix of the CQD system over the possible outcomes of the measurements of the PC detector. Identifying contributions from the two different possible measurement outcomes in time dt for the PC detector, namely *null* (no electron detected) and *detection* of an electron passing through the PC barrier, in equation (1), and then taking the ensemble average but keeping track of the number of electrons N that have tunnelled, we find

$$\dot{\rho}(N,t) = -(i/\hbar)[H_{CQD}, \rho(N,t)] + J[T + Xn_1]\rho(N-1,t) - A[T + Xn_1]\rho(N,t). \tag{4}$$

Note that the index $(N-1)$ appears in the *jump* superoperator J term of equation (4). The effect of the *jump* J term (Goan *et. al.* 2001) corresponds to an electron tunnelling through the PC in time interval $[t, t+dt)$. Hence if N electrons have tunnelled through the PC at time $t+dt$, then the accumulated number of electrons in the drain at the earlier time t, due to the contribution of the *jump* term, should be $(N-1)$. Equation (4), evaluated in the logical qubit charge state $|a\rangle$ and $|b\rangle$ (i.e., perfect localisation state of the electron in dot 1 and dot 2, respectively), is the same as the rate equations found by Gurvitz (1997). If the sum over all possible values of N is taken on the "partially" reduced density matrix, i.e., $\rho(t) = \sum_N \rho(N,t)$, equation (4) then reduces to equation

(3). While this approach provides us with information about the system state by means of measuring the number of accumulated electrons passing through the PC in time t, it is still in an ensemble and time average sense. In other words, the system dynamics is still deterministic in this approach. Hence, it cannot describe the conditional dynamics of the CQD qubit system in a single realisation of continuous measurements, which reflects the stochastic nature of electrons tunnelling through the PC barrier.

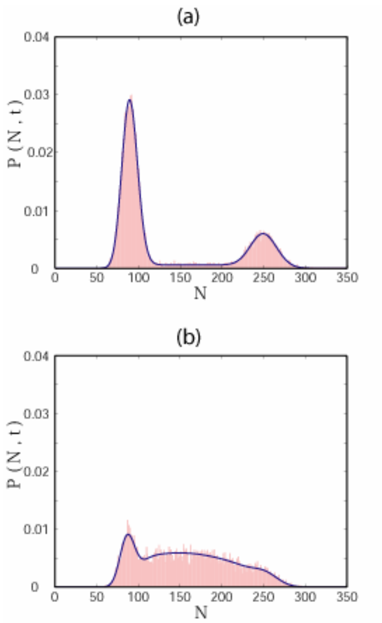

Figure 1. Probability distributions $P(N,t)$ for detection time $t = 20/\Gamma_d$ but with different (Ω/Γ_d) ratio of (a) 0.05 and (b) 0.2. Here Ω is the coherent coupling frequency between the CQDs, and Γ_d known as decoherence rate represents interaction between the CQD's and PC detector. The distribution $P(N,t)$ in (a) splits into two with weights corresponding respectively to the initial values of the qubit diagonal density matrix elements of $\rho_{aa}(0) = 0.75$ and $\rho_{bb}(0) = 0.25$. However, in (b) the valley between the two-peak structure of $P(N,t)$ is filled and hence no good readout measurement can be performed.

On the other hand, the QT approach provides us the most (all) information, and no average or trace over the environmental (detector) states is taken as far as the system evolution is concerned. Instead, repeated continuous in time measurements are made and recorded. Hence we are propagating *in parallel* the information of a conditioned (stochastic) state evolution [or conditional density matrix evolution $\rho_c(t)$], and a detection record $dN_c(t)$ in a continuous measurement process. We simulate $P(N,t)$ the probability distribution of finding N electron that have tunnelled through the PC barriers in time t (Makhlin *et. al.* 2001), using 10000 QT's and measurement records. The simulation results of $P(N,t)$ in shades in figure 1 are in very good agreement with those in solid lines obtained from the Fourier analysis of the partially reduced density matrix. Each QT (or stochastic state evolution) resembles a single history of the qubit state in a single run of the continuous measurement experiment. Hence more physical insights in the interpretation of ensemble and time averaged properties can be gained in the QT approach. We will discuss this appealing feature of the QT approach elsewhere.

References

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