Temperature evolution of the quantum Hall effect in the FISDW state: Theory vs Experiment

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Abstract. We discuss the temperature dependence of the Hall conductivity σ_{xy} in the magnetic-field-induced spin-density-wave (FISDW) state of the quasi-one-dimensional Bechgaard salts $(TMTSF)_2X$. Electronic thermal excitations across the FISDW energy gap progressively destroy the quantum Hall effect, so $\sigma_{xy}(T)$ interpolates between the quantized value at zero temperature and zero value at the transition temperature T_c, where FISDW disappears. This temperature dependence is similar to that of the superfluid density in the BCS theory of superconductivity. More precisely, it is the same as the temperature dependence of the Fröhlich condensate density of a regular CDW/SDW. This suggests a two-fluid picture of the quantum Hall effect, where the Hall conductivity of the condensate is quantized, but the condensate fraction of the total electron density decreases with increasing temperature. The theory appears to agree with the experimental results obtained by measuring all three components of the resistivity tensor simultaneously on a $(TMTSF)_2PF_6$ sample and then reconstructing the conductivity tensor.

In a magnetic field H, quasi-one-dimensional organic conductors of the $(TMTSF)_2X$ family experience a cascade of magnetic-field-induced spin-density-wave (FISDW) phase transitions (see review [1]). At zero temperature, each FISDW phase exhibits an integer quantum Hall effect (see review [2]). As temperature T increases, the Hall effect decreases and virtually vanishes at T_c, the transition temperature of FISDW. (In the normal state without FISDW, the Hall effect is very small and can be approximated as zero compared to the Hall effect in the FISDW state.) It was shown in Refs. [2, 3, 4] that the temperature evolution of the Hall conductivity (per one layer) is given by the following formula:

$$\sigma_{xy}(T) = f(T)\,\frac{2Ne^2}{h}, \qquad (1)$$

where e is the electron charge, h is the Planck constant, N is an integer number that characterizes the FISDW, and $f(T)$ is the dimensionless condensate density of FISDW. The condensate density $f(T)$ interpolates between 1 at zero temperature (all electrons are in the condensate) and 0 at T_c (the condensate density vanishes when FISDW disappears). An explicit expression for $f(T)$ depends on the order in which the limits of zero frequency $\omega \to 0$ and zero momentum $q \to 0$ are taken [5, 6]. In the dynamic limit, where the $q \to 0$ is taken first and then $\omega \to 0$ is taken, the condensate density is

$$f_d(T) = 1 - \int_{-\infty}^{\infty} \frac{dp_x}{v_{\rm F}} \left(\frac{\partial E}{\partial p_x}\right)^2 \left[-\frac{\partial n_{\rm F}(E)}{\partial E}\right], \qquad (2)$$

where $E = \sqrt{(v_{\rm F} p_x)^2 + \Delta^2}$ is the electron energy dispersion law in the FISDW state (v_F is the Fermi velocity, p_x is the electron momentum along the chains, and Δ is the energy gap), and $n_{\rm F}(\epsilon) = (e^{\epsilon/k_{\rm B}T} + 1)^{-1}$ is the Fermi distribution function ($k_{\rm B}$ is the Boltzmann constant). In the static limit

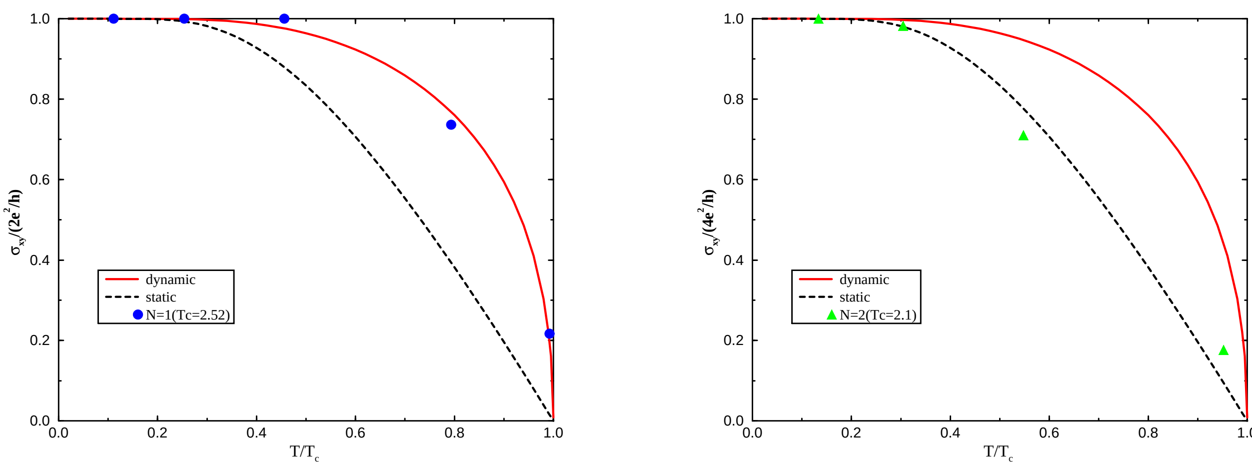

Figure 1: Hall conductivity σ_{xy} normalized to its zero-temperature value as a function of temperature normalized to the transition temperature of FISDW. The solid and dashed lines represent the temperature dependences of the condensate density in the dynamic and static limits, respectively, calculated from Eqs. (2) and (3). The left panel shows experimental points for $(TMTSF)_2PF_6$ under pressure 8.5 kbar for the $N = 1$ Hall plateau at $H = 15.5$ T assuming $T_c = 2.52$ K, and the right panel for $N = 2$ at $H = 13.25$ T assuming $T_c = 2.1$ K.

($\omega \to 0$ first, then $q \to 0$), the condensate density is

$$f_s(T) = 1 - v_{\mathrm{F}} \int_{-\infty}^{\infty} dp_x \left[-\frac{\partial n_{\mathrm{F}}(E)}{\partial E} \right]. \qquad (3)$$

The static limit (3) is appropriate for calculation of the magnetic field penetration depth in superconductors, which comes from the truly static Meissner effect in thermodynamic equilibrium. On the other hand, the Hall effect is kinetic and not thermodynamic, so we believe that the dynamic limit (2) is appropriate for Eq. (1).

In Fig. 1, we compare theory and experiment. Hall conductivity σ_{xy} normalized to its zero-temperature value is shown as a function of temperature normalized to the transition temperature of FISDW. The solid and dashed lines represent the temperature dependences of the condensate density in the dynamic and static limits, respectively, calculated from Eqs. (2) and (3) presuming that the temperature dependence of the energy gap Δ is the same as in the BCS theory of superconductivity [7]. The points are obtained experimentally by measuring the three components of the resistivity tensor, ρ_{xx}, ρ_{yy}, and ρ_{xy}, simultaneously on a sample of $(TMTSF)_2PF_6$ under pressure 8.5 kbar and then calculating the conductivity tensor. The left panel shows the experimental points for the $N = 1$ Hall plateau at $H = 15.5$ T assuming $T_c = 2.52$ K, and the right panel for $N = 2$ at $H = 13.25$ T assuming $T_c = 2.1$ K. The left panel demonstrates a good agreement with the dynamic limit. The right panel seems to agree with the static limit. However, the right panel has only four experimental points, and the discrimination between the static and dynamic fits is effectively controlled by only one point at about $T/T_c = 0.6$. So, we believe that there is not enough data to make a conclusion for the $N = 2$ plateau, but for the $N = 1$ plateau the agreement with the dynamic limit appears to be convincing. Nevertheless, more data is necessary to make a firmer conclusion. We also wish to point out that Eq. (1) provides a unique opportunity to determine the condensate density of FISDW by measuring the Hall conductivity. This is a linear-response measurement, uncomplicated by depinning, phase slips, and other nonlinear effects, which characterize sliding of a regular charge- or spin-density wave.

The last terms in Eqs. (2) and (3) reflects the fact that normal quasiparticles, thermally excited above the energy gap, reduce the quantum Hall effect. We can write the condensate density in the form

$$f(T) = 1 - \rho_n/\rho, \tag{4}$$

where the second term is the density of the normal component, ρ_n, normalized to the total electron density ρ. Thus, $f(T)$ is analogous to the superfluid density in superconductors. Given that normal electrons exhibit virtually zero Hall effect, Eq. (1) can be interpreted in a two-fluid manner: The condensate carries the dissipationless quantum Hall current, and normal electrons carry the longitudinal, dissipative electric current. The Hall response of the condensate remains quantized even at a finite temperature, but the Hall conductivity decreases because the condensate density decreases with increasing temperature. In this picture, the quantum Hall current in the FISDW state is analogous to the supercurrent in a superconductor, which is also carried only by a condensate, whose density decreases with increasing temperature. This analogy continues an amazing sets of parallels between the quantum Hall effect in the FISDW state and superconductivity. The thermodynamics of both states is described by the BCS theory of a mean-field energy gap Δ opening at the Fermi surface [7]. Both states are characterized by dissipationless currents: the quantum Hall current and the supercurrent. At a finite temperature, both systems have two-fluid electrodynamics.

The temperature dependence discussed in this paper was obtained under assumption that the quantum Hall effect originates from the bulk of the crystal. The results have implications for the edge states picture of the quantum Hall effect. Theory says that in the FISDW state there must be N gapless chiral electron states at the sample edge [2]. At zero temperature, the quantized Hall conductivity can be equivalently obtained in terms of either bulk or edge states [2]. However, it does not seem possible to describe the Hall conductivity at a finite temperature in terms of only the edge states. Indeed, the edge states are gapless, so they cannot produce Eqs. (2) and (3), which involve the gap Δ. Moreover, because of thermal excitations across the energy gap in the bulk, it does not seem possible to ignore the bulk contribution compared to the edge contribution at a finite temperature. By any means, at $T \to T_c$ distinction between the bulk and edge states must go away. So, the bulk theory of the quantum Hall effect appears to be more general and robust than the edge one. It is worth mentioning that the question where the quantum Hall current actually flows, in the bulk or in the edges, was never studied experimentally in quasi-one-dimensional conductors and does not have a clear answer even in the case of the quantum Hall effect in semiconductors.

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