date 99.07.01 No.

Indefinite integral 不定積分.

$\int f(x) = F(x)+C \Leftrightarrow \frac{dF(x)}{dx} = f(x)$.

Definite integral 定積分.

$\int_a^b f(x)dx = F(b)-F(a) \equiv [F(x)]_a^b$.

Proper integral (恰當積分).

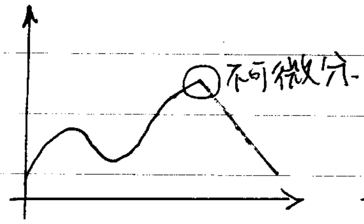
$a, b \neq \pm\infty$, $f(x)$ is continueus.

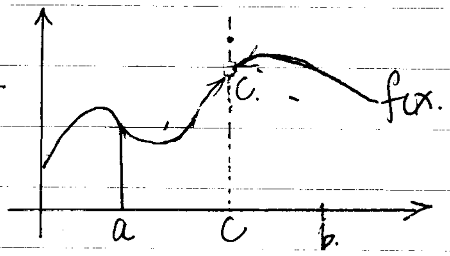
Inproper integral (瑕積分).

$\begin{cases} f(x) \text{ isn't continueus. at some point. } C \in (a,b). \\ a \text{ or } b = \infty \end{cases}$

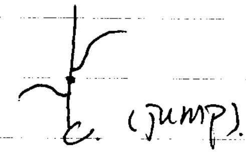
If $f(x)$ is continueus on $[a,b]$, except at $C \in (a,b)$.

$\int_a^b f(x)dx$.

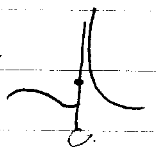




$\lim_{x\to C^+} f(x) = \lim_{x\to C^-} f(x)$, 又 if.

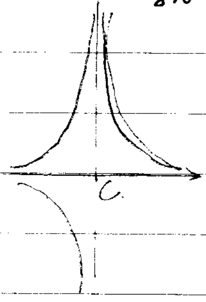


or



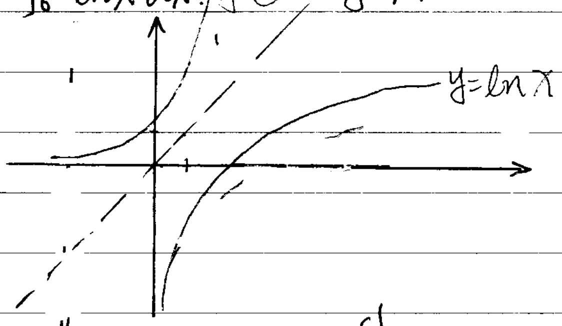
If at point C. continues.

$\int_a^b f(x)dx = \lim_{\delta\to 0}\int_a^{c-\delta} f(x)dx + \lim_{\delta\to 0}\int_{c+\delta}^b f(x)dx$.



date 99.07.01. No.

$\int_0^1 \ln x\,dx$



$* \log_e x = \ln x.$

$\delta \ln \delta \xrightarrow[\delta\to 0]{} 0.$

$\delta > 0.$

$\delta^2 \ln \delta = \ln\delta / \delta^{-2} \to \frac{(\ln\delta)'}{(\delta^{-2})'} \to 0.$

$\to \frac{1/\delta}{-2\delta^{-3}} = -\frac{1}{2}\delta^2$

$\int_0^1 \ln x\,dx = \lim_{\delta\to 0}\int_\delta^1 \ln x\,dx.$

$u = \ln x,\ v = x$

$\int v'u\,dx = uv - \int vu'\,dx.$

$\int \ln x\,dx = x\ln x - \int x(\ln x)'\,dx.$

$(\ln x)x.$

$= x\ln x - x + C.$

$\int_0^1 \ln x\,dx = \lim_{\delta\to 0}\int_\delta^1 \ln x\,dx = \lim_{\delta\to 0}[x\ln x - x]_\delta^1$

$= (1\ln 1) - \lim_{\delta\to 0}[\delta\ln\delta - \delta]$

$= -1 - \lim_{\delta\to 0}[\delta\ln\delta]$

$\lim_{\delta\to 0}(\delta\ln\delta) = \lim_{\delta\to 0}\left(\frac{\ln\delta}{1/\delta}\right) = \lim_{\delta\to 0}\left[\frac{(\ln\delta)'}{(1/\delta)'}\right].$

$\lim_{\delta\to 0}(-\delta) = 0.$ $\quad = \left[\frac{1}{\delta}\Big/\left(-\frac{1}{\delta^2}\right)\right]$

$* \delta\ln\delta.$

$= \ln\delta/\left(\frac{1}{\delta}\right) \to \lim_{\delta\to 0}\frac{(\ln\delta)'}{(1/\delta)'} \to 0.$

$\delta\ln\delta = \frac{\delta}{(1/\ln\delta)} \to \frac{1}{-\frac{1}{\delta(\ln\delta)^2}}$

$= -\delta(\ln\delta)^2.$

L'Hospital's rule.

$\lim_{x\to\xi}\frac{f(x)}{g(x)}$ $\quad f(\xi) = g(\xi) = 0. \to \lim_{x\to\xi}\frac{f(x)}{g(x)} = \lim_{x\to\xi}\frac{\overbrace{[f(x)-f(\xi)]/(x-\xi)}^{f'(\xi)}}{[g(x)-g(\xi)]/(x-\xi)}\ g'(\xi)$

$= \frac{0}{0}!$ when $\frac{\sin x}{x} \to 1.$

$= \lim_{x\to\xi}\frac{f'(\xi)}{g'(\xi)}.$

$* \lim_{x\to\xi}\frac{f(x)}{g(x)},\ f(\xi) = g(\xi) = 0,$

$$\boxed{\Rightarrow \lim_{x\to\xi}\frac{f(x)}{g(x)} = \lim_{x\to\xi}\frac{f(x)}{g'(x)}}$$

date 99. 07. 01.

$\frac{\sin x}{x} \xrightarrow[x\to 0]{} \frac{(\sin x)'}{(x)'} \Rightarrow \frac{\cos x}{1} \to 1.$

$\frac{\sin x}{|x|} \to \frac{(\sin x)'}{\cancel{(|x|)'}}$

$\frac{x}{1-\cos x} \longrightarrow \frac{(x^2)'}{(1-\cos x)'} \longrightarrow \frac{2x}{\sin x} \to \frac{(2x)'}{(\sin x)'} = \frac{2}{\cos x} = 2.$ #.

$\lim_{x\to\xi} \frac{f(x)}{g(x)} = \lim_{x\to\xi}\left[\frac{1/g(x)}{1/f(x)}\right] = \lim_{x\to\xi}\left[\frac{-\frac{1}{g^2(x)}g'(x)}{-\frac{1}{f^2(x)}f'(x)}\right] = \left[\frac{f^2(x)}{g^2(x)}\frac{g'(x)}{f'(x)}\right]_{x\to\xi}.$

$f(\xi), g(\xi) \to 0.$

$\lim_{x\to\xi}\frac{f(\xi)}{g(\xi)} = \lim_{x\to\xi}\frac{f'(\xi)}{g'(\xi)}$, $\quad l = l^2 \lim_{x\to\xi}\frac{g'(x)}{f'(x)}$

$l = \lim_{x\to\xi}\frac{f'(x)}{g'(x)}$