math.

部份積分.

$$\int u v' dx = uv - \int u' v \, dx$$

$$\int u \, dv = uv - \int v \, du. \qquad du = \frac{du}{dx} dx = u' dx$$

$$\frac{d}{dx}(uv) = u'v + uv'$$

$$= \lim_{h \to 0} \left\{ \frac{u(x+h)\, v(x+h) - v(x)\, u(x)}{h} \right\}$$

$$= \lim_{h \to 0} \left\{ \left(\frac{u(x+h) - u(x)}{h} \right) v(x+h) + u(x) \left(\frac{v(x+h) - v(x)}{h} \right) \right\}.$$

ex $(x^3)' = (x^2 \cdot x)'$

$$3x^2 = 2x \cdot x + x^2 \cdot 1 = 3x^2$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c \quad n \neq 1 \qquad \int \frac{1}{x} dx = \ln|x| + c.$$

$$\int \ln x \, dx = x \ln x - \int x \, d\ln x$$

$$\cancel{= x\, d} \quad = x \ln x - \int x \frac{1}{x} dx.$$

$$= x \ln x - x + c.$$

$$\int f(x) \, dx = F(x) + c \qquad \frac{dF(x)}{dx} = f(x).$$

$$\int g'(x) f(g(x)) \, dx = \int f(g(x)) \, dg(x) = F(g(x)) + c$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\cos\theta}\cos\theta\,d\theta = \int d\theta = \theta + C$$

$x = \sin\theta \quad \theta = \arcsin x$

$dx = \cos\theta\,d\theta \quad \theta = \sin^{-1}x$

$\int \frac{1}{1+x^2}\,dx$ 令 $x = \tan\theta$.

$$\left(\frac{v}{u}\right)' = \frac{v'}{u} + \left(\frac{1}{u}\right)' v = \frac{v'}{u} - \frac{u'v}{u^2}$$

$$= \frac{uv' - u'v}{u^2}$$

$$f(g(x))' = \frac{d f(g(x))}{dx} = \frac{d f(g(x))}{d g(x)} \frac{d g(x)}{dx}$$

$$= f'(g(x))\, g'(x)$$

$$(\sin x^2)' = \frac{d \sin x^2}{d x^2} \frac{d x^2}{dx} = \cos x^2 \cdot 2x.$$

$$\int \frac{1+x+x^2}{(x-1)^3}\,dx$$

$$\frac{1+x+x^2}{(x-1)^3} = \frac{3}{(x-1)^3} + \frac{3}{(x-1)^2} + \frac{1}{x-1}$$

$\rightarrow$ partial fraction 部分分数

$d(x-1) = dx$

$$= -\frac{3}{2}\frac{1}{(x-1)^2} - \frac{3}{(x-1)} + \ln|x-1| + C$$

$1+x+x^2 = a_0 + a_1(x-1) + a_2(x-1)^2$

$x = 1 \quad 3 = a_0 = 3$

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$$\frac{1+x+x^2}{x^2(x-1)^2} = \frac{\overset{1}{a_0}}{x^2} + \frac{\overset{3}{a_1}}{x} + \frac{\overset{3}{b_0}}{(x-1)^2} + \frac{\overset{-3}{b_1}}{(x-1)}.$$

$x^2 \to (x-\alpha)^2,\ \alpha = 0.$

$$\frac{p(x)}{r(x)} = a_0 + a_1 x + x^2\{\cdots\}$$

$$a_0 = \frac{1}{0!}\left\{\frac{p(x)}{r(x)}\right\}_{x=0} = 1,\quad a_1 = \left[\frac{d}{dx}\frac{1+x+x^2}{(x-1)^2}\right]_{x=0}.$$

$$= \frac{1}{1!}\left[\frac{1+2x}{(x-1)^2} - 2\frac{(1+x+x^2)}{(x-1)^3}\right]_{x=0} = 3$$

$$\frac{\overset{p(x)}{1+x+x^2}}{x^2 r(x)} = (x-1)^2\left\{\frac{a_0}{x^2} + \frac{a_1}{x}\right\} + b_0 + b_1(x-1)$$

$$b_0 = \left\{\frac{1+x+x^2}{x^2}\right\}_{x=1} = 3.$$

$$b_1 = \frac{1}{1!}\left\{\frac{d}{dx}\frac{1+x+x^2}{x^2}\right\}_{x=1} = \frac{d}{dx}(x^{-2}+x^{-1}+1\}_{x=1} = (-2x^{-3} - x^{-2}\}_{x=1} = -3.$$

$$\int \frac{1+x+x^2}{x^2(x-1)^2}\,dx = \int \left[\frac{1}{x^2} + \frac{3}{x} + \frac{3}{(x-1)^2} - \frac{3}{x-1}\right]dx.$$

$$= -\frac{1}{x} + 3\ln|x| - \frac{3}{x-1} - 3\tan|x-1| + c$$

$$= -\frac{1}{x} - \frac{3}{x-1} + 3\ln\left|\frac{x}{x-1}\right| + c$$