数学

不定積分 $\int f(x)\,dx = F(x) + C$

定積分 $\int_a^b f(x)\,dx$ 又 $\sum_{i=1}^{n} f(\xi_i)\,\Delta x_i$

$$\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \left\{ \sum_{i=1}^{n} f(\xi_i)\,\Delta x_i \right\} \text{（黎曼積分）}$$

$\Delta x_i = x_{i+1} - x_i \quad \xi_i \in \{ x_i \ x_{i+1} \}$

$\Delta x \equiv \min \{ \Delta x_i \ \ i = 1 \cdots n \}$

$\frac{\Delta f}{\Delta x} \xrightarrow[\Delta x \to 0]{} \frac{df}{dx}$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \qquad \Delta f = f(x+\Delta x) - f(x)$

$$\int_a^b \{ c f(x) + \hat{c} g(x) \}\,dx = c \int_a^b f(x)\,dx + \hat{c} \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad c \in [a, b].$$

$$\int_a^a f(x)\,dx = 0$$

$F(x) = \int_a^x f(x')\,dx' \qquad \int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

$$F'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\int_a^{x+\Delta x} f(x)\,dx - \int_a^x f(x)\,dx}{\Delta x}$$

$(= f(x))$

$$= \lim_{\Delta x \to 0} \frac{\left(\int_x^{x+\Delta x} f(x)\,dx \right)}{\Delta x} = \lim_{\Delta x \to 0} f(\xi) = f(x).$$

$(f(x)\,\Delta x) \qquad \xi \in (x,\ x+\Delta x)$

$F(x) = \int_a^x f(x')\,dx' \Rightarrow F(b) = \int_a^b f(t)\,dt + C$ ~~$F(a) = \int_a^a f(t)\,dt$~~

~~$C = F(a)$~~

$$F(a) = \int_a^a f(x)\,dx + C$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int f(g(x))\,g'(x)\,dx = \int f(g(x))\,d\,g(x) = F(g(x)) + C$$

$$\int_0^{\pi/2} \sin^4 x \cos x\,dx = \int_0^1 u^4\,du = \frac{1}{5}u^5\Big|_0^1 = \frac{1}{5}$$

$u = \sin x$

$du = \cos x\,dx$

中間值定理

$$\int_x^{x+\Delta x} f(x)\,dx = f(\xi)\,\Delta x \qquad \xi \in [x,\ x+\Delta x]$$

$$\int_0^1 x\,e^{2x}\,dx = [uv]_0^1 - \int_0^1 vu'\,dx$$

$$\int_0^1 uv'\,dx + \int_0^1 u'v\,dx = \int_0^1 (uv)'\,dx$$

$$= \left[x\,\frac{e^{2x}}{2}\right]\Big|_0^1 - \int_0^1 \frac{1}{2}e^{2x}\,dx$$

$$= \left[x\,\frac{e^{2x}}{2}\right]\Big|_0^1 - \left[\frac{e^{2x}}{4}\right]_0^1$$

橢圓 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b > 0.$

$$y = f(x) = b\sqrt{1 - \frac{x^2}{a^2}}.$$

$$A = 2\int_{-a}^{a} f(x)\, dx.$$

$$= 2ba \int_{-a}^{a} \sqrt{1 - \frac{x^2}{a^2}} \left(\frac{dx}{a}\right) \qquad \frac{dx}{a} = d\left(\frac{x}{a}\right).$$

$$\frac{x}{a} = \sin\theta = u.$$

$$= 2ba \int_{-1}^{1} \sqrt{1-u^2}\, (du).$$

$$= 2ba \int_{-1}^{1} \cos\theta \, d\sin\theta = 2ba \int_{-1}^{1} \cos^2\theta \, d\theta.$$

$$= 2ba \int_{\pi}^{\pi} \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta.$$

$$= 2ba \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right) \Big|_{-\pi/2}^{\pi/2} = \pi ab.$$

$$\int \frac{1 + x + x^2}{(x-1)^3}\, dx$$ → 把 拆成部分分式 即可積分.

$$= \int \frac{3}{(x-1)^3} + \frac{3}{(x-1)^2} + \frac{1}{(x-1)}\, dx$$