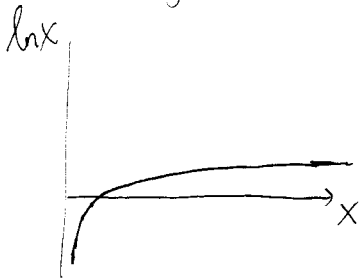


$$\frac{d}{dx} e^{kx} = k e^{kx} \Rightarrow \int e^{kx} dx = \frac{1}{k} e^{kx} + C \quad (k \neq 0)$$

$$\frac{d \ln x}{dx} = \frac{1}{x} \Rightarrow \int \frac{1}{x} dx = \ln|x| + C$$

$$\text{If } a > 0 \quad \lim_{x \rightarrow \infty} \frac{\ln x}{x^a} = 0.$$



$$\frac{d}{dx} \sin(kx) = k \cos(kx) \Rightarrow \int \cos kx dx = \frac{1}{k} \sin kx + C$$

$$\frac{d}{dx} \cos(kx) = -k \sin kx \Rightarrow \int \sin kx dx = -\frac{1}{k} \cos kx + C$$

$$\frac{d}{dx} \sin(\cos x) = \left[\frac{d}{d \cos x} \sin(\cos x) \right] \frac{d \cos x}{dx}$$

$$= \cos(\cos x) [-\sin x]$$

$$= -\cos(\cos x) \sin x$$

$$\int \sin(x^2) dx = ?$$

$$\text{but } \int x \sin(x^2) dx = \frac{1}{2} \int \sin(x^2) dx^2$$

$$\text{ex: } \int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{1}{x^2+1} d(x^2+1)$$

$$= \frac{1}{2} \ln(x^2+1) + C$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = - \int \frac{1}{\cos x} d \cos x = -\ln |\cos x| + C$$

Integration by part 部分積分.

$$\int \frac{d}{dx} \{u(x)v(x)\} dx = \int u'(x)v(x) dx + \int u(x)v'(x) dx.$$

$$\Rightarrow \int u'(x)v(x) dx = u(x)v(x) - \int u(x)v'(x) dx.$$

$$\text{ex: } \int x e^x \, dx = x e^x - \int e^x \, dx = (x-1)e^x + C$$

$$\int u v' \, dx = \int u \, dv = u \cdot v - \int v \, du$$

ex:

$$\begin{aligned} I = \int e^{2x} \sin x \, dx &= \frac{1}{2} \int \sin x \, dx \, de^{2x} \\ &= \frac{1}{2} \left\{ \sin x e^{2x} - \int e^{2x} \cos x \, dx \right\} \\ &= \frac{1}{2} \sin x e^{2x} - \frac{1}{2^2} \int \cos x \, de^{2x} \\ &= \frac{1}{2} \sin x e^{2x} - \frac{1}{4} \left\{ \cos x e^{2x} + \underbrace{\int e^{2x} \sin x \, dx}_{I} \right\} \end{aligned}$$

$$\frac{5}{4} I = \frac{1}{2} \sin x e^{2x} - \frac{1}{4} \cos x e^{2x}$$

$$I = \frac{2}{5} \sin x e^{2x} - \frac{1}{5} \cos x e^{2x}$$

$$= \frac{1}{5} (2 \sin x - \cos x) e^{2x}$$