轉頁從此開始寫起。

第四頁

課程編號：

國立臺灣大學 期中 學期 考試答案卷

系碼		課號	

科目 ________ 教學班組（ ）________學院 ________學系 ________組 ____年級

考試日期：____年____月____日 學號________ 姓名（中文）________

記分	教師簽名或蓋章

從此處開始寫起。試卷用紙務須節用。非經主試認可不得續用其他紙張作答。

敎學

反矩陣 $A^{-1}A = I$, $AA^{-1} = I$

$$A \downarrow$$

$$\left[\begin{array}{ccc|ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & b_{11} & b_{12} & b_{13} \\ 0 & 1 & 0 & b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 & b_{31} & b_{32} & b_{33} \end{array}\right] \Rightarrow \text{cp } A^{-1}$$

$$A^{-1} = \frac{C^t}{|A|} = \frac{1}{|A|}\begin{bmatrix} C_{11} & \cdots & C_{n1} \\ C_{12} & & \\ \vdots & & \\ C_{1n} & & C_{nn} \end{bmatrix}$$

C = cofactor matrix

pf

$$\Rightarrow A C^t = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{pmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{pmatrix}$$

$$[A^{-1}]_{11} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = |A|.$$

$[A^{-1}]_{12} = 0$ （某行 or 列會相等）

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

$$C = \begin{pmatrix} -2 & 2 & 0 \\ 3 & -3 & 1 \\ 4 & -2 & 0 \end{pmatrix} \qquad A^{-1} = \frac{1}{2}\begin{pmatrix} -2 & 3 & 4 \\ 2 & -3 & -2 \\ 0 & 1 & 0 \end{pmatrix}$$

$ABC = I$

$A^{-1}(ABC)C^{-1} = A^{-1}(I)C^{-1}$

$\hookrightarrow IBI$

$B = A^{-1}C^{-1} = (CA)^{-1}$

$(AB)^t = B^t A^t$

$(CA)^{-1} \neq C^{-1}A^{-1}.$

$\parallel$

$A^{-1}C^{-1}$

$(CA)(CA)^{-1} = I.$

$CAA^{-1}C^{-1} = I.$

記分欄　轉頁從此開始寫起

$$U A U^{-1} = \begin{pmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & & a_{nn} \end{pmatrix} \equiv D$$ 對角線矩陣

$$D = U A U^{-1} \qquad A^2 = U^{-1} D U U^{-1} D U \qquad A^n = (U^{-1} D U)^n$$

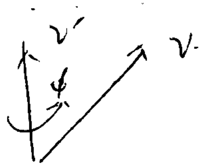
$$A = U^{-1} D U \qquad \quad = U^{-1} D^2 U \qquad \quad = U^{-1} D^n U$$

$A v = \lambda v$　　eigenvalue => λ

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

eigenvector => v

旋轉



$v' = R(\phi) v$　　$R(\phi \hat{k}) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

$$\begin{pmatrix} v_1' \\ v_2' \\ v_3' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \lambda \begin{pmatrix} v_x \\ v_y \end{pmatrix}.$$

$(A-\lambda I)V=0$.

$\rightarrow \underbrace{\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix}}_{D}\begin{pmatrix} v_x \\ v_y \end{pmatrix}=0$ $\quad \begin{pmatrix} v_x \\ v_y \end{pmatrix}\neq 0$ 時 $\det\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix}=0$.

pf

$\Rightarrow$ $D^{-1}DV=D^{-1}0=0$. $V=0 \Rightarrow$ 沒意義.

代表 D^{-1} 不存在 $\det(D)=0$.

$A^{\dagger}=A \Rightarrow \lambda$ 都是實數. $\quad A^{\dagger} \Rightarrow A$ 的共軛轉置 matrix.

↳ $A \Rightarrow$ 巴米特矩陣.

$\hbar=\frac{h}{2\pi}$ $\quad h=6.626\times10^{-34}$ J·s.

$L^2 \longrightarrow l(l+1)\hbar^2$ $\quad l=0,1,2,3\ldots$

$L_z \longrightarrow m\hbar$ $\quad m=-l,-l+1,\ldots,0,1,2,\ldots,l-1,l$

角動量量子化

以 假

關 轉頁從此開始寫起。

關 轉頁從此開始寫起。

$\Delta x \cdot \Delta p \geq \hbar/2$ 測不準原理

$m_e = 9.1 \times 10^{-31}$ kg

电子自旋 spin $1/2$ $\quad s_\uparrow = 1/2 \quad s_\downarrow = -1/2$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \hbar/2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\uparrow\rangle \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\downarrow\rangle$$

spin up $\qquad$ spin down

$$S_z \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\hbar/2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$ (看看就好)

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\alpha = (1 \quad 0) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \langle \uparrow | \psi \rangle$$

$$\beta = (0 \quad 1) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \langle \downarrow | \psi \rangle.$$

$$S_x = \begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \lambda \begin{pmatrix} v_x \\ v_y \end{pmatrix}.$$

$$\begin{pmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = 0. \qquad \lambda^2 = \left(\frac{\hbar}{2}\right)^2 \quad \lambda = \pm \hbar/2.$$

$$\begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \hbar/2 \begin{pmatrix} v_x \\ v_y \end{pmatrix} \qquad \begin{matrix} \hbar/2\, v_y = \hbar/2\, v_x \\ \hbar/2\, v_x = \hbar/2\, v_y \end{matrix} \quad \Rightarrow \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

歸一化

$$\begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = -\hbar/2 \begin{pmatrix} v_x \\ v_y \end{pmatrix} \qquad \begin{matrix} \hbar/2\, v_y = -\hbar/2\, v_x \\ \hbar/2\, v_x = -\hbar/2\, v_y \end{matrix} \quad \Rightarrow \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

$$\langle v | v \rangle = \frac{1}{\sqrt{2}} (1 \quad 1) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1.$$

歸一化條件為內積為1.

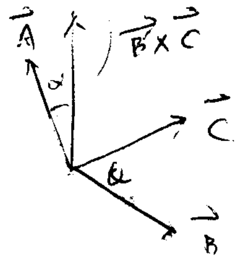
內積

三重純量積

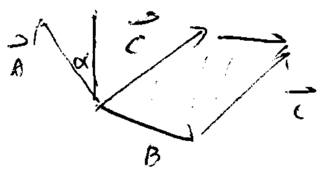
$\vec{A}\cdot(\vec{B}\times\vec{C})$

$$\vec{A}\cdot\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ B_i & B_j & B_z \\ C_i & C_j & C_z \end{vmatrix} = \begin{vmatrix} A_i & A_j & A_z \\ B_i & B_j & B_z \\ C_i & C_j & C_z \end{vmatrix} = \begin{vmatrix} B_i & B_j & B_z \\ C_i & C_j & C_z \\ A_i & A_j & A_z \end{vmatrix} = \begin{vmatrix} C_i & C_j & C_z \\ A_i & A_j & A_z \\ B_i & B_j & B_z \end{vmatrix}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$



$\vec{A}\cdot(\vec{B}\times\vec{C}) = A\,\underbrace{(BC\sin\theta)}_{\Downarrow}\cos\alpha \Rightarrow$ 即3向量構成的平行六面体的体積



$A\cos\alpha$ 為高

若 $\vec{A}\cdot(\vec{B}\times\vec{C})=0$ 則 $\vec{A}, \vec{B}, \vec{C}$ 在同平面

triple vector product

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$

$$\parallel$$

pf.

$$x\vec{B} + y\vec{C} \quad \Rightarrow \quad \vec{A}\cdot(x\vec{B} + y\vec{C}) = 0.$$

$$x\,\vec{A}\cdot\vec{B} + y\,\vec{A}\cdot\vec{C} = 0.$$

$$\frac{x}{\vec{A}\cdot\vec{C}} = \frac{-y}{\vec{A}\cdot\vec{B}} = \lambda.$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \lambda\left[(\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}\right]$$

if $\vec{A}=\vec{B}$

$$\vec{C}\cdot(\vec{A}\times(\vec{A}\times\vec{C})) = \vec{C}\cdot\left(\lambda(\vec{A}\cdot\vec{C})\vec{A} - \lambda(\vec{A}\cdot\vec{A})\vec{C}\right)$$

$$\parallel$$

$$(\vec{C}\times\vec{A})\cdot(\vec{A}\times\vec{C})$$

$$\parallel$$

$$-(AC\sin\beta)^2 = \lambda A^2C^2\cos^2\beta - \lambda A^2C^2 \qquad \lambda = 1$$

橢圓 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$f'(x) = \frac{df(x)}{dx} \leftarrow$ 导函数.

$= \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$.

不定積分

$\int f(x)\,dx = F(x) + c \qquad \frac{d\,F(x)+c}{dx} = f(x)$

c 為常數.

$\int a f(x) + b\,y(x)\,dx = a\int f(x)\,dx + b\int y(x)\,dx$ => 线性时

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\int [f(x)]^{-a} f'(x)\,dx = \frac{1}{-a+1}[f(x)]^{-a+1} + c$.

$I = \frac{1}{3}\int \frac{2x}{(x+1)^2}\,dx = \frac{1}{-2+1}(x^2+1)^{-1} + c.$ $\quad f(x) = x^2+1$

$f'(x) = 2x \quad a=-2.$

$\int \frac{2x}{x^2+1}\,dx = \ln|x^2-1| + c \qquad \int \frac{1}{x^2+1}\,dx = \arctan x$