求逆.

$AB = BA = 1 \Rightarrow B = A^{-1}$

2×2

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x & u \\ y & v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A^{-1}

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \qquad \begin{matrix} ax + by = f \\ cx + dy = g \end{matrix} \quad \times\left(-\frac{c}{a}\right)$$

$R_2 - R_2 - \frac{c}{a}R_1$

$$0x + \left(d - \frac{bc}{a}\right)y = g - \frac{c}{a}f$$

$$\begin{bmatrix} a & b \\ 0 & d - \frac{c}{a}b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g - \frac{c}{a}f \end{bmatrix}$$

$R_2 \to R_2 \Big/ \left(d - \frac{c}{a}d\right)$

$$\left[\begin{array}{cc|c} a & b & f \\ 0 & 1 & \frac{ag-f}{ad-bc} \end{array}\right]$$

$R_1 \to R_1 - R_2 \times b$

$$\left[\begin{array}{cc|c} a & 0 & f - b\frac{ag-cf}{ad-bc} \\ 0 & 1 & \frac{ag-cf}{ad-bc} \end{array}\right]$$

$\leftarrow \quad \frac{adf - bag}{ad-bc}$

$\downarrow R_1 \to R_1/a$

$$\left[\begin{array}{cc:cc} 1 & 0 & \frac{df-bg}{ad-bc} \\ & & \\ 0 & 1 & \frac{ay-cf}{ad-bc} \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{\begin{vmatrix} f & b \\ g & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \\ \\ \frac{\begin{vmatrix} a & f \\ c & g \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \end{bmatrix}$$

$$\left[\begin{array}{cc:cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right]$$

$\downarrow R_1 \to R_1/a$

$$\left[\begin{array}{cc:cc} 1 & 1/a & 1/a & 0 \\ c & d & 0 & 1 \end{array}\right]$$

$\downarrow R_2 \to R_2 - R_1 c$

$$\left[\begin{array}{cc:cc} 1 & b/a & 1/a & 0 \\ 0 & d-\frac{bc}{a} & -\frac{c}{a} & 1 \end{array}\right] \xrightarrow{R_2 \to \frac{R_2}{(d-\frac{bc}{a})}} \left[\begin{array}{cc:cc} 1 & \frac{b}{c} & 1/a & 0 \\ 0 & 1 & -\frac{c}{ab-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - R_2 \times \frac{b}{a}} \left[\begin{array}{cc:cc} 1 & 0 & \frac{d}{ab-bc} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-c}{ab-bc} & \frac{a}{ad-bc} \end{array}\right]$$

$I \qquad A^{-1}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ \cdot & \cdot \end{bmatrix}$ but $\det(A) \neq 0$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} x & u & p \\ y & v & q \\ z & w & r \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

↓ $R_1 \to R_1 - R_2 \cdot 3/2$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

↓ $R_3 \to R_1 - R_3$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 3/2 & 1 \end{array}\right)$$

↓ $R_1 \to R_1 + R_3 \times 2$.

$$\left(\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 3/2 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 1 & -3/2 & -1 \end{array}\right)$$ ↷ 互換

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$$\left(\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 3/2 & 0 \\ 0 & 1 & 0 & 1 & -3/2 & -1 \\ 0 & 0 & 1 & 0 & 1/2 & 0 \end{array}\right)$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(-1)^{1+1}\begin{pmatrix} e & f \\ h & i \end{pmatrix} + b(-1)^{1+2}\begin{pmatrix} d & f \\ g & i \end{pmatrix} + c(-1)^{1+3}\begin{pmatrix} d & e \\ g & h \end{pmatrix}$$

$$A^{-1} = \frac{C^t}{\det(A)}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & \cdots & & a_{nn} \end{pmatrix} \qquad C_{ij} = (-1)^{i+j} \begin{vmatrix} a_{21} & a_{23} & \cdots & a_{2n} \\ a_{32} & a_{33} & \cdots & a_{3n} \\ a_{n1} & a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{32} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}.$$

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix} \qquad |A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}.$$

$$AA^{-1} \Rightarrow I$$

$$I_{11} = \frac{1}{|A|}\, a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = 1$$

I_{12} = 算式中含有 2 列相同的列 $\Rightarrow 0$.