$A = [a_{ij}]$

$m \times n$

$A + B = [a_{ij} + b_{ij}] = [C_{ij}]$
$\overset{\shortparallel}{C}$

dot product

$$[a_1, a_2 \cdots a_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \cdots a_n b_n$$

$$\underset{m \times n}{A}\ \underset{n \times r}{B} = \underset{m \times r}{C}$$

$$C_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots a_{in} b_{nj}$$

ex

$$A = \begin{bmatrix} 4 & 2 & 9 \\ -3 & -1 & 1 \\ 2 & 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 2 & -4 \\ 0 & 0 & 1 \\ 3 & -4 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4\cdot(-2) + 2\cdot 0 + 9\cdot 3 & & \\ & -3\cdot 2 + (-1)\cdot 0 + (-4)\cdot 1 & \\ & & \end{bmatrix} = \begin{bmatrix} 19 & -28 & -23 \\ 9 & -10 & 10 \\ 2 & -4 & -9 \end{bmatrix}$$

$$BA = \begin{bmatrix} -22 & -10 & -24 \\ 2 & 1 & 2 \\ 22 & 9 & 21 \end{bmatrix}$$

$BA \neq AB$

Identity matrix $\leftarrow n \times n$

$[a_{ij}] = [\delta_{ij}]$

$$\begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ & & & 1 \end{bmatrix} = I \Rightarrow AI = IA$$

Transpose matrix 轉置矩陣

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad A^t = [a^t_{ij}] = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & & & \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

※ $a^t_{ij} = a_{ji}$

1. $(A^t)^t = A$ 2. $(A+B)^t = A^t + B^t$ 3. $(cA)^t = cA^t$

4. $(AB)^t = B^t A^t$

$A^t = A$

$n \times n \to$ 对称 matrix

$\sigma_2^+ = (\sigma^t)^*$ 当 $a_2^+ = a_2$ 為 Hermitian matrix

$$\begin{cases} (cA)^+ = c^* A^+ \\ (A+B)^+ = A^+ + B^+ \\ (cA)^+ = c^* A^+ \\ (AB)^+ = B^+ A^+ \end{cases}$$

$(AB)_{ij} = \sum_k A_{ik} B_{kj}$

$[(AB)^t]_{ij} = \sum_k A_{jk} B_{ki} = \sum_k B_{ki} A_{jk}$

$= \sum_k B^t_{ik} A^t_{kj} = (B^t A^t)_{ij}$

$(AB)^t = B^t A^t$

pauli matrix $\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\sigma_1^2 = \sigma_1\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$. $\sigma_2\sigma_2 = \sigma_3\sigma_3$

$\sigma_1\sigma_2 = i\sigma_3$ $\sigma_2\sigma_3 = i\sigma_1$, $\sigma_3\sigma_1 = i\sigma_2$

$\sigma_2\sigma_1 = -i\sigma_3$ $\Rightarrow$ $\sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij} I$

$A\underset{\sim}{v} = \lambda \underset{\sim}{v}$

$\underset{\sim}{v}$: A's eigenvector

λ = eigenvalue

An $n \times n$ matrix has n^2 elements

$|A| = \det(A)$ 行列式 = determinant

2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

3×3 matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\det(A) = \overset{(-1)^{1+1}}{a_{11}} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \overset{(-1)^{1+2}}{-} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \overset{(-1)^{1+3}}{+} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{33} \end{vmatrix}$$

$$= \overset{(-1)^{1+2}}{-a_{12}} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{2+2} a_{22} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{3+2} a_{32} \begin{vmatrix} a_{11} & a_{13} \\ a_{23} & a_{33} \end{vmatrix}$$

$$= \sum \sigma \begin{pmatrix} 1 & 2 & 3 \\ i & j & k \end{pmatrix} a_{1i} a_{2j} a_{3k}$$

$$= \sum_{ijk} \epsilon_{ijk} a_{1i} a_{2j} a_{3k}.$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} + \lambda a_{11} & a_{22} + \lambda a_{22} & \cdots & a_{2n} + \lambda a_{1n} \\ a_{31} & a_{32} & \cdots & a_{3n} \end{vmatrix}$$

$$= \sum_{k_1 k_2 k_3} \sigma \begin{pmatrix} 1 & 2 & \cdots & n \\ k_1 & k_2 & \cdots & k_n \end{pmatrix} a_{1k_1} (a_{2k_2} + \lambda a_{1k_1}) a_{3k_3} \cdots a_{nk_n}$$

$$= |A|$$