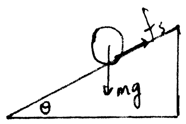
3/13



$$\begin{cases} f_s - mg\sin\theta = ma_{com} & -① \\ f_s \cdot R = -I_{com} \cdot \alpha & -② \\ R\alpha = a_{com} & -③ \end{cases}$$

$< ② : f_s = \frac{I_{com}}{R} \cdot \alpha = -\frac{I_{com}}{R^2} a_{com}$

$\therefore -mg\sin\theta = ma_{com} + \frac{I_{com}}{R^2} a_{com}$

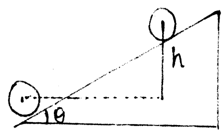
$\therefore a_{com} = \dfrac{-g\sin\theta}{1 + \frac{I_{com}}{mR^2}}$

* Ring : mR^2 Shell : $\frac{2}{3}mR^2$

Disc : $\frac{1}{2}mR^2$ Sphere : $\frac{2}{5}mR^2$

Cylindet : $\frac{1}{2}mR^2$

1.



solid sphere

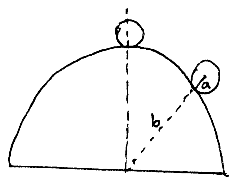
$m = 6\,kg$. $\theta = 30°$. $h = 1.2\,m$.

$V_f = ?$ $f_s = ?$

$mgh = \frac{1}{2}mV^2 + \frac{1}{2}I \cdot (\frac{V}{R})^2$

$= \frac{1}{2}mV^2 + \frac{1}{2} \cdot \frac{2}{5}mV^2$

2.



$mg\cos\theta = \frac{mV^2}{a+b}$

$V = a\omega$

$mg(a+b)(1-\cos\theta) = \frac{1}{2}mV^2 + \frac{1}{2} \cdot \frac{2}{5}ma^2 \cdot \omega^2$

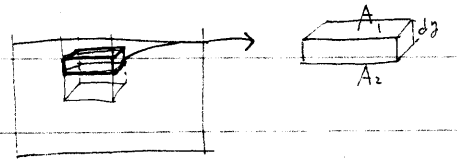
$= \frac{1}{2}mV^2 + \frac{1}{5}mV^2 = \frac{7}{10}mV^2$

$\therefore mg(a+b)(1-\cos\theta) = \cancel{m}V^2 \cdot \frac{1-\cos\theta}{\cos\theta} = \frac{7}{10}\cancel{m}V^2$

$\therefore \cos\theta = \frac{10}{17}$ #

$\boxed{\text{流体}}$

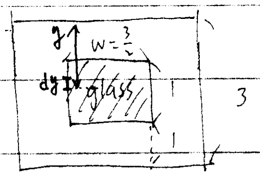
密度不均之压力（均匀：$\rho h g$）



$A_1 = A_2 = A$，且上下面压力相同（∵ 不动）

$\Rightarrow PA = (P + dP)A + \rho g A\,dy \Rightarrow \dfrac{dP}{dy} = -\rho g$

$\Rightarrow \int dP = g \int \rho(y)\,dy$



求 glass 受力 = ?

$$\int_{y=1}^{2} \rho g y w\,dy = \rho g \cdot \frac{3}{2} \int_{y=1}^{2} y\,dy = \frac{9}{4} \rho g$$

$\boxed{\text{gas}}$

利用 $N_2 . O_2$ 28.8 gw/mol.

定 0°C，海平面 $\rho_0 = 1.29\ \mathrm{kgw/m^2}$. $\rho \propto P$

$$\frac{\rho}{\rho_0} = \frac{P}{P_0} \qquad \frac{dP}{dy} = -\rho g = -\frac{P}{P_0}\rho_0 g = -Pg\left(\frac{\rho_0}{P_0}\right)$$

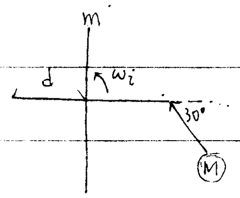
$$\Rightarrow \int_{P_0}^{P(h)} \frac{dP}{P} = -\frac{\rho_0}{P_0} g \int_0^h dy$$

h↑，P(h)↓

$$\Rightarrow \ln\left(\frac{P(h)}{P_0}\right) = -\frac{\rho_0 g}{P_0} h \Rightarrow P(h) = P_0 e^{-\frac{\rho_0 g}{P_0} h}$$

$\frac{\rho_0 g}{P_0} \rightarrow 1.25 \times 10^{-4}\ m^{-1}$

單位 760 mmHg = 760 torr



d=0.5m

$\omega_i = -2$ rad/s ⇒ 求 ω_f = ?

$m = \frac{1}{5}M$

$v_i = 12$ m/s

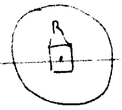
$\boxed{\vec{L}_i = \vec{L}_f}$ M对m 有 $\vec{L} = F \times \vec{p}$

$$I_m = \frac{1}{12}m(2d)^2 \times 2 = \frac{2}{3}md^2$$

$$\left(\frac{2}{3}md^2 + Md^2\right)\omega_f = \frac{2}{3}md^2\omega_i - Mv_i\sin 30°$$

↳ ω_M 和 ω_m 方向相反.

孔方之転，求I.



面密度

$$m = \sigma(\pi R^2 - R^2) \Rightarrow \sigma = \frac{m}{(\pi-1)R^2}$$

$$\begin{cases} m_1 = \text{(disc)} = m\frac{\pi}{\pi-1} \\ m_2 = \text{(square)} = m\frac{1}{\pi-1} \end{cases}$$

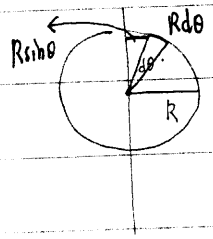
$$I = I_1 - I_2 = \frac{1}{4}\left(\frac{\pi}{\pi-1}m\right)R^2 - \frac{1}{12}\left(\frac{1}{\pi-1}m\right)R^2$$

↳ 和棍同 (▨ = 很多 ☰)

↳ 对圆心転 = $\frac{1}{2}MR^2$

对軸転 = $\frac{1}{4}MR^2$ (一半)

證明 (用 Ring 作 exp).



$$I = 2\int_0^{\pi}(R\sin\theta)^2\lambda R\,d\theta = 2R^3\lambda\int_0^{\pi}\sin^2\theta\,d\theta$$

由 $\int_0^{\pi}\sin^2\theta\,d\theta = \int_0^{\pi}\cos^2\theta\,d\theta$

$$= 2R^3\lambda\int_0^{\pi}\frac{\sin^2\theta + \cos^2\theta}{2}\,d\theta$$

$$= \frac{2\pi R\lambda}{2}R^2 = \boxed{\frac{1}{2}}mR^2$$

↳ 和原 mR^2 比較: 一半.